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Dear colleagues,

this year the 26th international Czech-Polish seminar Modern Mathematical Methods in Engineering 2018 took place already in January. The seminar venue has not changed, and we gathered in great numbers in the Horský Hotel Excelsior in Horní Lomná near the village of Jablunkov. The Department of Mathematics and Descriptive Geometry of the VŠB - TU Ostrava and the Ostrava branch of the Union of Czech Mathematicians and Physicists have, as is becoming tradition, organized the seminar. 60 participants of the seminar, amongst them 25 foreign guests from Poland, arrived in the snowy Beskids. The organizers of the seminar were happy to see increased interest shown in the event by young teachers and scientists.

Doc. RNDr. Jindřich Bečvář, CSc. of the Faculty of Mathematics and Physics of the Charles University in Prague dedicated a series of plenary lectures Benefits of mathematics history in Europe to present natural science.

In total 22 talks were given and 21 posters were presented. Research-oriented contributions pertained to mathematical modeling, simulation, coding, statistics, the aplications of mathematics in geology, geodesy and economy. Methodically-oriented contributions introduced to the audience applicational problems in mathematical education, analysed access to academic literature and its correlation with academic results, and discussed the use of Geogebra in mathematical education. An integral part of the seminar were informal discussions in the lobbies, which focused on expert topics or the issues of higher education in both countries.

Conference is supported by a project CZ.11.4.120/0.0/0.0/16_013/0000213 in programme INTERREG V-A Republika Czeska – Polska within the project Cross-border exchange of experience in production engineering using principles of mathematics. This allowed participation of a wide range of those interested, including PhD students, but mainly a much greater involvement of our Polish colleagues from the universities near the border.

The result of the seminar are two volumes of proceeding. Proceedings with all contributions are published on CD, while the contributions in English are published in traditional printed form. The contributions in English that have been provided in camera ready form have gone through standard review procedures. Selected contributions will be published after editing in the journal AEEE (Advances in Electrical and Electronic Engineering). The selection is made by the editorial board of AEEE with regard to their academic focus.

January 2018 On behalf of the organization and program committee Jarmila Doležalová.

CONTENTS

PRODUCT DEFECTIVENESS ANALYSIS USING METHODS AND TOOLS OF QUALITY ENGINEERING

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Abstract: All production companies, regardless of the industry, face the problem of defective products. Removing the causes of the final product's defectiveness is connected with incurring additional costs, which reduces the efficiency of the production processes. That is why companies take measures to identify those causes which bear the most significance to ensuring the required product quality. The present article aims to indicate the tools of quality engineering that can be used to determine product defects and their impact on the quality of the final product. The article is based on a case study analysis of the example rock wool manufacturing company.

Keywords: defect, production, production process, FMEA, Ishikawa diagram, Pareto chart, quality costs

1 Introduction

Every manufacturing company encounters the problem of defective products. Unfortunately, even properly defined requirements, working production system and following procedures does not guarantee manufacturing a product compliant with the previously set requirements [1].

During the production process companies strive to fulfil all the necessary requirements, including quality requirements, while observing restrictions such as production costs. Practice demonstrates that fulfilling all requirements is not possible as no process takes place in ideal conditions. This is connected, among others, with the fact that machines (elements) wear and tear in the production process, unplanned breaks occur due to breakdowns, measurement devices wear out, materials with varying properties are used in the process, etc. All these factors translate into disturbing the work rhythm, unevenness of the production process, with human errors piling on top. That is why the problem of defective products exists in every company, which results in consequences such as increased production costs. In order to avoid this problem and fulfil the clients' expectations to the highest degree possible, measures should be taken that will allow for identifying the places in which defects arise as well as determine their causes.

2 The concept of defect

The concept of defect is defined in various ways. A defect is often identified with improper execution of the production process, which makes the product unusable for the end user. It can take many forms, although most commonly the term refers to the presence of errors or low quality. A defect is a non-compliance with the adopted requirements for the parameters that describe a given product. It is defined as a departure from the adopted requirements.

The concept of defect is connected with the concept of defectiveness, which is related to the technological defects or design flaws of a given product as well as other aspects connected with the necessity of making changes to the product. Defectiveness, therefore, can lead to client dissatisfaction and result in additional costs incurred by the manufacturer. Among the defects we can distinguish:

- important defects, i.e. one that makes it impossible to use the product or hinders its use,
- insignificant defects with relatively low impact on the functioning of the product and do not considerably decrease it usability.
- critical defects, which can lead to the development of conditions in which using the product would be dangerous or decreasing the capacity of certain functions of the product.

The occurrence of the aforementioned defects results in a non-compliant product. Non-compliance is understood as a state of a property that does not meet requirements, while a non-compliant unit is defined as a product with a number of non-compliances exceeding the permissible limit or one which contains a defect that causes its complete unusability [2]. According to the ISO 9000:2005 standard, a non-compliance (3.6.2) – failure to meet a requirement 3.1.2) [3].

Occurrence of a non-compliance does not facilitate quality, can be connected with a failure to meet the requirements related to the standards, quality documentation, legal regulations, requirements of the parties to the contract, client requirements or other parties concerned. Only defects that have been confirmed, that is supported with evidence, can be treated as non-compliances. The following categories of non-compliances are distinguished:

- systemic – defects detected in the quality management system,
- accidental – a requirement is not met, but without any major consequences.

Non-compliances are also divided into:

- small non-compliances – isolated, proven case of a requirement not being met,

- big (critical) non-compliances – defects of the entire system, a systemic non-compliance or alternatively a large number of non-compliances with the quality management system.

Moreover, documenting a non-compliance requires:

- indicating the requirement that has not been met,
- describing the nature of the non-compliance,
- demonstrating proof.

The reference to the requirement should be precise. Formulating the non-compliance should be clear, unambiguous, concise, while the proof needs to be documented and sufficiently detailed. Upon discovering a non-compliance, corrective measures (removing the cause of the non-compliance) are taken.

3 Defect detection

At present, any production defects are detected outside the production line at the last stage of production, which is linked to a great loss of time and additional financial expenses as well as generates undesirable wastes.

Every manufacturer strives to ensure fault-free production and minimise the losses on production lines, which translates into ensuring the economic efficiency of manufacturing activities. Fault-free production at a minimal level of waste generation results in a more balanced and competitive manufacturing industry, which is becoming the standard in modern manufacturing. The primary goal of every company is reliable and environment-friendly production without any defects and wastes. That is why new equipment and program solutions are being developed that bring innovation in terms of technology, modelling and methodology to integrated production quality control systems. At the same time, companies are introducing quality cost controlling for planning and monitoring the cost of measures aimed at ensuring the proper quality of products. These measures aim to reduce production costs, shorten stoppage time and minimise losses while simultaneously making it possible to manufacture safer products of desired quality.

Regardless of the place in which the defect was detected (at the input, in the middle or at the end of the production process) the process needs to be analysed in order to take such measures that will minimise the possibility of the problem occurring in the future. Early detection of a defect leads to streamlining of the production process, higher quality, product durability and has significant impact on production and quality costs.

Detection of defects is possible only through quality checks, which are responsible for assuring the quality of the product in the course of production, after the production process and in the trade [4].

The identified defective (non-compliant) product should be marked in such a way as to prevent accidental use. Dealing with such a product consists in [5]:

- taking measures aimed at removing the non-compliance (correction, corrective measure),
- permitting the use of or repurposing the product for other uses (re-qualification of the product),
- action aimed at preventing the product from being used for its original purpose,
- in the case of correction, the defective product must be verified again.

4 Production process of rock wool

Mineral wool is a commonly used insulating material (Fig. 1). Production of mineral wool is a complex process whose course depends on the type of product produced. The technology of rock wool production uses basalt, gabbro, dolomite, limestone as well as slag and coke (lowers the melting point). The wool production process starts with measuring the correct proportions of raw materials and placing them in a special cast iron oven where the coke (used as fuel) produces a high temperature of about 1400-1500°C during combustion. As a result of the melting, liquid rock mass flows out of the furnace, dropping onto discs that spin at a speed of several thousand revolutions per minute. These discs break up the pig iron, converting it into fibres, which are then cooled with air and collected in a settling chamber in the form of a carpet of wool. During the formation of fibres, binder and hydrophobic agents are added. From the settling chamber, the wool rug is directed onto the technological line, where it is formed by compression and distortion of the fibres in many directions.

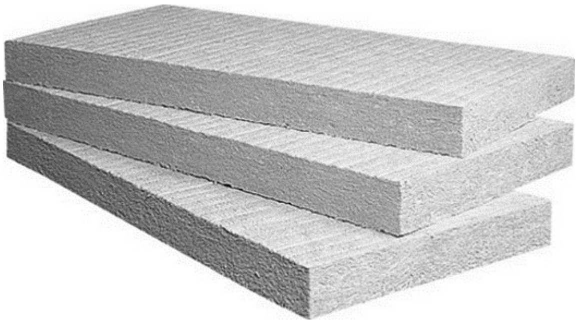

Fig. 1. Rock wool

The next step is a polymerization chamber where the wool is heated to a temperature of about 200°C to fully polymerize the added resins and stabilise the material before its final treatment. After cooling the carpet, at the end of the line the rock wool is cut to the specified dimensions and then packed in foil.
Mineral wool is characterised by the following properties:

- thermal isolation (low heat conductivity coefficient),
- incombustibility and fire resistance,
- sound absorption capacity,
- size and shape stability,
- mechanical strength, elasticity,
- biological and chemical resistance,
- waterproofness and vapour permeability.

5 Analysis of product defect formation [6]

The case presented concerns a company in which studies were conducted to identify defects in the product, determine the causes, and implement preventive measures that reduce the number of defective products. To achieve the goal, we used quality engineering tools in the form of the Pareto chart, the Ishikawa diagram and FMEA.

In the first stage, the most common defects that arise in the production process of rock wool were analysed. Four main groups of defects were identified, outlying eight main causes, namely:

- improper wool colour,
- lack of declared product properties (i.e. the compressive strength and tensile strength),
- slab dimension non-compliance,
- improper softness of the slab (locally).

Table 1 presents the causes, frequency of their occurrence per year as well as their percentage share.

Table 1. List of causes of defects and their number per year

Cause No.	Type of defect	Cause of the defect	No. of defects	Share %
1	Improper wool colour	Too high shaft furnace temperature	30	9.7
2		Clogged defibrator	60	19.4
3	Lack of declared product properties	Too high resin temperature	15	4.8
4		Uneven spraying on the fibre	80	25.9
5		Wrong fibre misaligning machine settings	28	9.1
6	Dimensional non-compliance	Wrong saw blade dimension settings	50	16.2
7		Blunt saw blade	14	4.5
8	Slab locally soft	Improper functioning of the fibre misaligning machine	32	10.4

Next, the causes presented in Table 1 were sorted in decreasing order and the cumulative value was calculated (Table 2).

Table 2. Causes of rock wool defects

Cause No.	Cause of the defect	No. of defects	Share %	Cumulative value %
4	Uneven spraying on the fibre	80	26	26
2	Clogged defibrator	60	19	45
6	Wrong saw blade dimension settings	50	17	62
8	Improper functioning of the fibre misaligning machine	32	10	72
1	Too high shaft furnace temperature	30	10	82
5	Wrong fibre misaligning machine settings	28	9	91
3	Too high resin temperature	15	5	96
7	Blunt saw blade	14	4	100

The above juxtaposition shows that the main causes that generate more than 80% of all defects are:

- uneven spraying on the fibre,
- clogged defibrator,
- wrong saw blade dimension settings,
- improper functioning of the fibre misaligning machine,
- too high shaft furnace temperature.

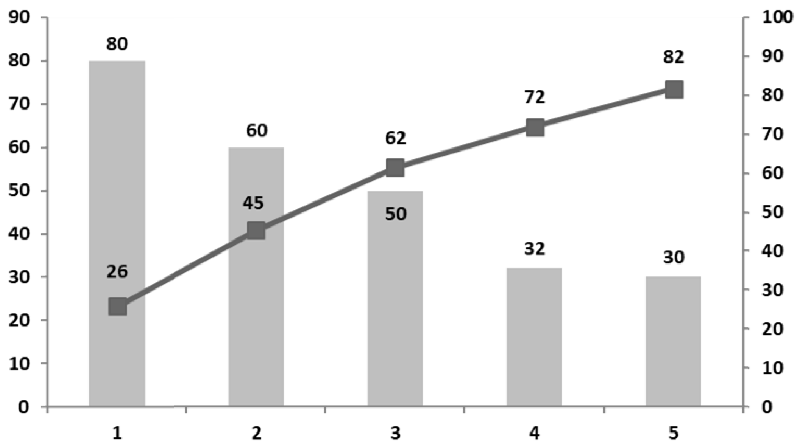

Fig. 2. Pareto-Lorenz diagram for the causes of defects in rock wool

The Pareto-Lorenz diagram (Figure 2) was narrowed for five main causes, which represent more than 80% of all defects (in accordance with the ABC principle).

The use of Pareto analysis allows us to consider the causes of the four major defects in the mineral wool production process. On the other hand, using the Ishikawa Diagram (Figure 3), we can show the reasons for potential defects of the manufactured product in the following areas:

- machine,
- material,
- people,
- method,
- management.

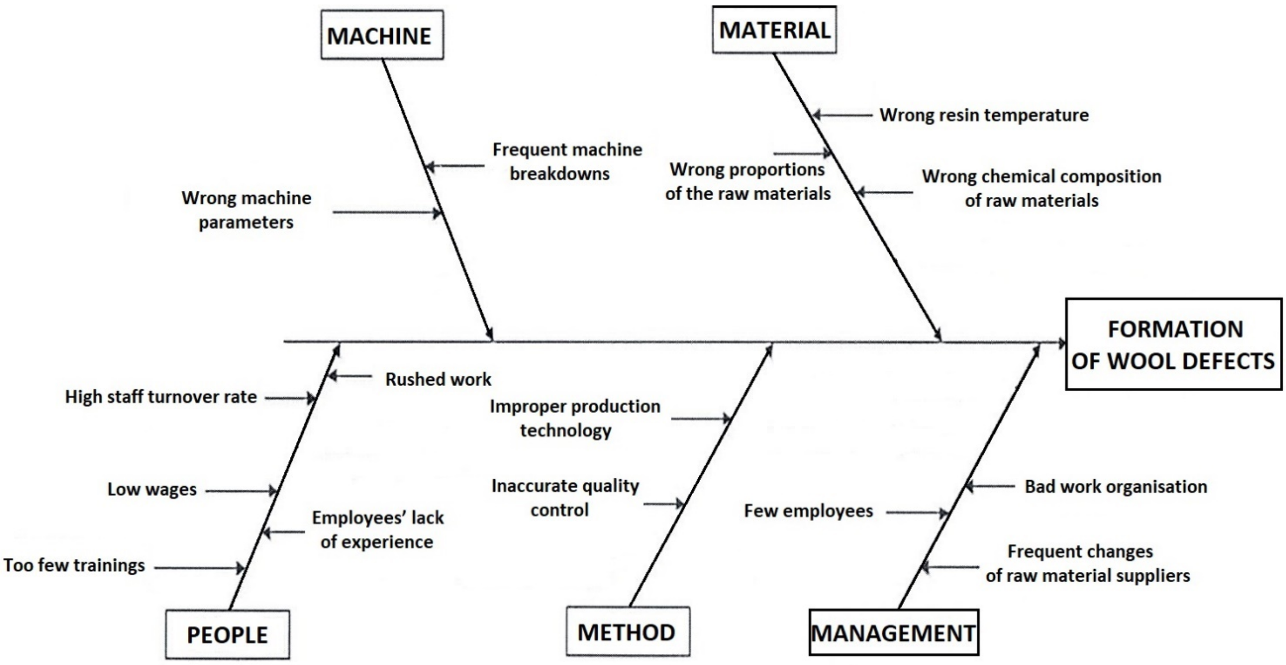

Fig. 3. Ishikawa diagram of the rock wool defect formation causes
Source: graph based on [6]

In this case, the Ishikawa diagram allowed us to clearly classify the causes of mineral wool defects. When analysing the diagram, it can be seen that the formation of defects in the production process of rock wool depends mainly on human factors such as:

- employees' lack of experience,
- small number of trainings,
- rushed work,
- high employee turnover, mainly due to low wages.

Further analysis of the diagram allows us to formulate conclusions about the desired improvement measures. The company should pay more attention to:

- more detailed quality control conducted at the earlier stages,
- motivating employees by directing their actions at ensuring product quality,
- raising the qualifications of production employees,
- increasing the number of production employees along with reducing the turnover rate.

The above list clearly indicates the important role of human resources in the production process. The success of the production process, resulting in fewer defective products, is largely dependent on skilled and experienced workers, which may translate into the financial sphere of production management [7].

For a more in-depth analysis of causes and effects of defects, another quality engineering tool was used: the FMEA (Failure Mode and Effects Analysis), i.e. an analysis of types and effects of possible failures [8, 9]. FMEA for defects in the rock wool mineral production process is presented in Table 3.

Table 3. Analysis of causes and effects of rock wool defects

Defect No.	Potential defect	Potential effects of the defect	Potential causes of the defect	Preventive measures	Zn	Cz	Wy	WPR	Results of action				
									Improvement measures	Zn	Cz	Wy	WPR
1	Improper wool colour	Customer complaints	Too high shaft furnace temperature	Constant monitoring of the lava-smelting furnace	8	3	1	24	Automatic control of the furnace temperature	8	2	1	16
			Clogged defibrator	More frequent cleaning of the machine	8	5	1	40	Checking the condition of the machine before starting a new batch	8	2	1	16
2	Lack of adequate strength properties	- Customer complaints - Product not maintaining stable dimensions and shape during use	Too high resin temperature	Stricter control of the raw material supplier	9	2	1	18	Analysis of the raw material	9	1	1	9
			Uneven spraying on the fibre	Monitoring the technological installation	9	4	2	72	Increasing the number of quality inspections	9	2	1	18
			Wrong fibre misaligning machine settings	Setting the machine correctly	9	5	1	45	Increasing the monitoring of the machine's parameters	9	3	1	27
3	Dimension and shape non-compliance	- Making adjustments - Customer dissatisfaction - Remanufacturing the product - Increase in production costs	Wrong dimension settings on the saw	Correcting the saw settings	5	7	2	70	Purchase of more precise measurement tools	5	3	1	15
			Defects of the cutting tools	Replacement of the cutting knives	5	4	2	40	More frequent maintenance of the cutting tools	5	3	2	30
4	Slab locally soft	- Improper functioning of the fibre misaligning machine - Manufacturing of a new product -Increase in costs - Delays in product shipments	Wrong settings of the fibre misaligning programme	- More precise instructions - Employee training	9	4	6	216	Creating a more intuitive programme, training employees in this regard	9	3	6	162
5	Damaged collective pallet	- Increase in the amount of waste - Increased order completion time - Additional costs	Improper transport, lading and storage of the pallets	Packing the product more thoroughly, proper storage, more careful lading	9	4	2	72	Monitoring the safe packaging of the product and supervision of the warehouse hall	9	2	2	36
6	Lack of moisture resistance	- Increased wetness - Failure to meet requirements - Product cannot be used	Inadequate proportions of added binders and hydrophobic agents	Improving the process of adding binders and hydrophobic agents	10	2	6	120	Increased control of added components and samples for analysis	10	1	6	60

Use of the FMEA can help prevent the effects of defects that may occur during the design and manufacturing stages and indicates potential defects that may occur and result in the production of a defective product. In the case described, the assessment was made on a ten-point scale and included three criteria:

1. defect occurrence risk – Z_n,
2. potential for detecting the cause of the defect – C_z,
3. importance of the defect to the user – W_y.

For these three criteria, the WPR number is calculated, which indicates which defect may occur at the earliest and which should be paid the highest attention.

$$WPR = Z_n \cdot C_z \cdot W_y$$

Table 3 summarizes the WPR values obtained before and after the FMEA. The corrective measures introduced have resulted in a reduction in the WPR values by half or more for each type of defect.

Table 4. WPR values before and after the FMEA

Defect No.	Sum WPR		Valume in %	
	before	after	before	after
1	64	32	9	5
2	135	54	19	8
3	110	45	15	6
4	216	162	30	23
5	72	36	10	5
6	120	60	17	9

Table 4 shows the effects of corrective measures. The data contained in Table 4 are shown graphically in Figure 4.

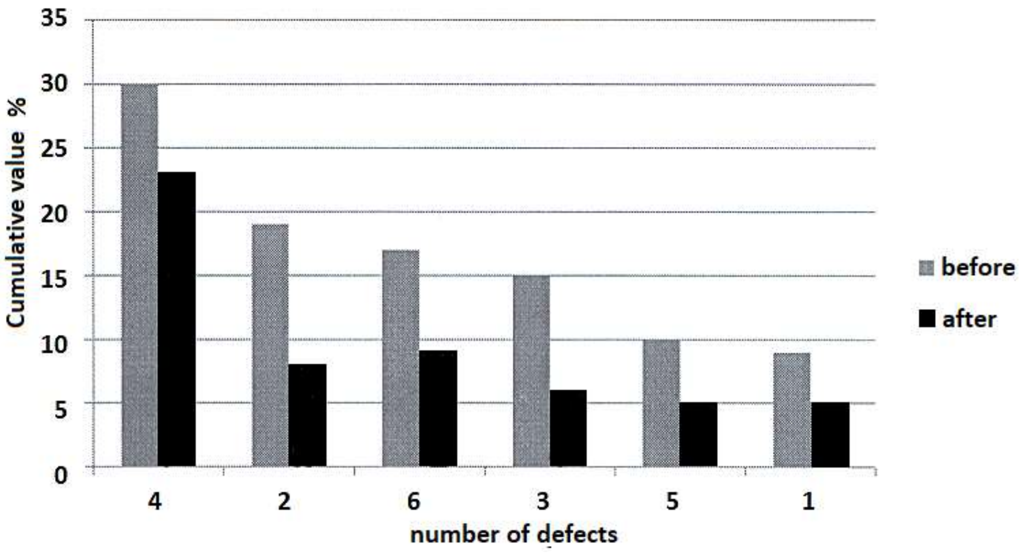

Fig. 4. Ordered diagram of defects “before” and “after” the FMEA
Source: figure based on [6]

The defects have been sorted in descending order, giving a clear picture of their significance. At the same time, it indicates the recommended order in which the defects in the production process should be removed.

Conclusion

The production process of the rock wool is a multistep process and the finished product is affected by many factors from the human factor, the raw materials through the manufacturing process to the storage. This article recommends a method for analysing a manufacturing process aimed at detecting defects and identifying areas for corrective actio. To this end, the following quality engineering tools were proposed: the Pareto-Lorenz diagram, the Ishikawa diagram and the FMEA method. In-depth analysis using these tools identified the most common defects that result in failure to meet specific quality requirements and thus customer expectations. The tools selected also made it possible to identify the causes of defects and their effects on the quality of the final product. Defects discovered after the production process can rarely be corrected, which translates into an increase in the costs of the production process and reduced economic efficiency of the company. For this reason, it is important to make a financial commitment to eliminate the identified non-compliances in order to reduce defect rates and thereby maintain the desired level of production costs. The quality engineering tools proposed can be extended to include quality cost control, which would allow for greater consideration of the issues of product defects and improvement of production processes. In the authors' opinion that the methods indicated can also be used in the area of analyses aimed not only at the technical but also the economic consequences of product defects.

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ANALIZA WADLIWOŚCI PRODUKTU Z WYKORZYSTANIEM METOD I NARZĘDZI INŻYNIERII JAKOŚCI

Streszczenie: We wszystkich przedsiębiorstwa produkcyjnych niezależnie od branży, występuje problemem wyrobów wadliwych. Usunięcie przyczyn powodujących wadliwość produktu końcowego, wiąże się z poniesieniem dodatkowych kosztów, co obniża efektywność procesów produkcyjnych. Dlatego też, przedsiębiorstwa podejmują działania aby rozpoznać przyczyny, które mają najistotniejsze znaczenie dla zapewnienia wymaganej jakości. Celem artykułu jest wskazanie możliwych do wykorzystania narzędzi inżynierii jakości dla określenia wad produktu i ich wpływu na jakość produktu gotowego. Artykuł opiera się na analizie case study w oparciu o przykład producenta wełny mineralnej skalnej.

Słowa kluczowe: wada, produkcja, proces produkcyjny, FMEA, diagram Ishikawy, diagram Pareto, koszty jakości

On solution of contact shape optimization problem by proximal bundle method

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Abstract: From the mathematical point of view, the contact shape optimization is a problem of nonlinear (usually nonsmooth) optimization with a specific structure which can be exploited in its solution. In this paper, we show how to overcome the difficulties related to the nonsmooth cost function by using the proximal bundle method. To illustrate the performance of the presented algorithm, we solve a shape optimization problem associated with the discretized two-dimensional contact problem with Coulomb's friction.

Keywords: nonsmooth optimization, Clarke calculus, proximal bundle method, shape optimization.

1 Introduction

Shape optimization problems arise naturally in mechanical engineering whenever the design requirements include an optimal performance of a machine comprising several bodies in mutual contact. From the mathematical point of view, these problems can be characterized by a locally Lipschitz continuous cost function which is differentiable in most but not all points. Shape optimization problems have the following form:

$$\left.\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \Omega \subset \mathbb{R}^n. \end{array}\right\} \qquad (1)$$

The solution of such problems can be obtained by a suitable iterative algorithm – its typical structure reads as in Tab 1. The hardest difficulty is the direction searching in Step 2 since the cost function f is not differentiable but only locally Lipschitz continuous. This implies that to minimize the function f, we can choose an algorithm from the following two classes: derivative-free methods (like genetic algorithms) and methods that use the subgradient information (like subgradient or bundle methods). Since the subgradient information is available for our problem, we have chosen the latter class of algorithms. In this paper, the proximal bundle method (see [4] or [6]) is presented. This method needs the function value $f(x)$ and one (arbitrary) Clarke subgradient of f at x in every step of the iteration process.

Table 1: Basic iterative algorithm.

Step 0: (Initialization)
Find a feasible starting point $x_1 \in \Omega$ and set $k = 1$.

Step 1: (Stopping criterion)
If x_k is "close enough" to the required solution then STOP.

Step 2: (Direction finding)
Find a feasible descent direction $d_k \in \mathbb{R}^n$:
$f(x_k + td_k) < f(x_k)$ and $x_k + td_k \in \Omega$ for some $t > 0$.

Step 3: (Line search)
Find a step size $t_k > 0$ such that
$t_k \approx \arg\min_{t>0}\{f(x_k + td_k)\}$ and $x_k + td_k \in \Omega$.

Step 4: (Updating)
Set $x_{k+1} = x_k + t_k d_k, k = k + 1$ and go on to Step 1.

Main definitions are introduced in the beginning of this article and then the following section presents description of the proximal bundle method. To show the functionality of the presented algorithm, we solve a shape optimization problem with the discretized two-dimensional contact problem with Coulomb's friction in the last part.

2 Nonsmooth analysis - calculus of Clarke

We start this section with definition of Lipschitz continuity and generalized gradient.

Definition 1 *A function $f\colon \Omega \subset \mathbb{R}^n \to \mathbb{R}$ is said to be Lipschitz continuous on Ω if there exists some constant $L = L(\Omega) > 0$ such that*

$$|f(x) - f(y)| \le L\|x - y\|, \quad \forall x, y \in \Omega. \tag{2}$$

Definition 2 *A function $f\colon \mathbb{R}^n \to \mathbb{R}$ is said to be Lipschitz continuous at $x \in \mathbb{R}^n$ if there exists a neighbourhood U of x and a constant $L = L(U) > 0$ such that*

$$|f(x) - f(y)| \le L\|x - y\|, \quad \forall y \in U. \tag{3}$$

Definition 3 *A function $f\colon \mathbb{R}^n \to \mathbb{R}$ is said to be locally Lipschitz continuous in $\mathbb{R}^n$ if this function f is Lipschitz continuous at $x \in \mathbb{R}^n$ for every $x \in \mathbb{R}^n$.*

Definition 4 *Let $\Omega \subset \mathbb{R}^n$. Then* conv (Ω) *denotes the convex hull of the set Ω, which is defined by*

$$\operatorname{conv}(\Omega) = \left\{ \sum_{i=1}^{n} \lambda_i x_i \;\middle|\; n \in \mathbb{N}, \lambda \in \mathbb{R}^n, x_1, \cdots, x_n \in \Omega, \lambda_i \ge 0, \forall i, \sum_{i=1}^{n} \lambda_i = 1 \right\}. \tag{4}$$

Definition 5 *Let the objective function $f\colon \mathbb{R}^n \to \mathbb{R}$ be locally Lipschitz continuous (in $\mathbb{R}^n$). The generalized gradient of the objective function f at $x \in \mathbb{R}^n$ is the set*

$$\partial f(x) = \operatorname{conv} \left\{ g \in \mathbb{R}^n \;\middle|\; g = \lim_{i\to\infty} \nabla f(x_i), x_i \to x, x_i \notin \Omega_f \right\}, \tag{5}$$

where $\Omega_f = \{x \in \mathbb{R}^n, f \text{ is not differentiable in } x\}$. Each element $g \in \partial f(x)$ is called a subgradient of the objective function f at x.

We now illustrate the previous definition Def. 5. Let us consider the function

$$f(x) = |x-1| \ + \ |x| \ + \ |x+1|.$$

Figure 1 shows the graph of the function f (left) and graph of its general gradient.

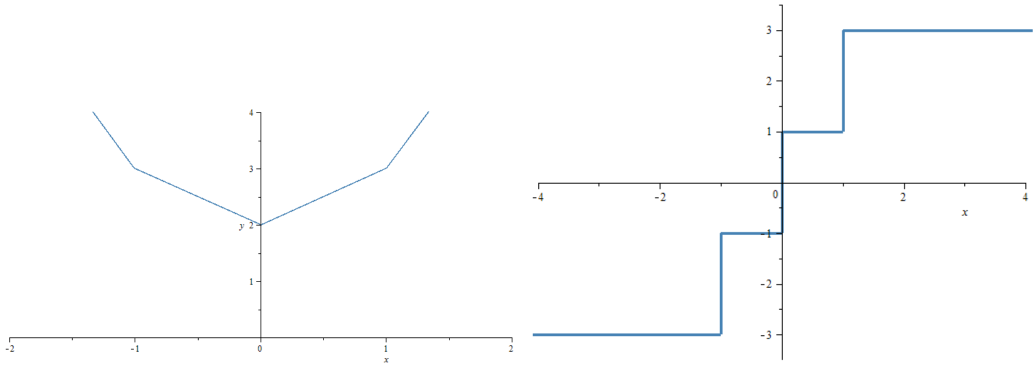

Figure 1: Graph of function f (left) and graph of its general gradient (right).

3 Description of the proximal bundle method

Consider the following nonlinear constrained optimization problem

$$\left.\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & Cx \le b, \\ & x_{\min} \le x \le x_{\max}, \end{array}\right\} \tag{6}$$

where the objective function $f\colon \mathbb{R}^n \to \mathbb{R}$ is locally Lipschitz continuous in $\mathbb{R}^n$, $C \in \mathbb{R}^{m\times n}$ is an constraint matrix, $b \in \mathbb{R}^m$ is a right-hand side vector and $x_{\max} \in \mathbb{R}^n$, $x_{\min} \in \mathbb{R}^n$ are bound vectors. To make these notations simple we suppose that the simple bounds $x_{\min}, x_{\max}$ are included in the linear system $Cx \le b$. For further details on the proximal bundle method the interested reader is reffered to [4].

3.1 Direction finding

Our aim is to solve the problem with respect to $d \in \mathbb{R}^n$

$$\left.\begin{array}{ll} \text{minimize} & f(x_k+d) - f(x_k), \\ \text{subject to} & x_k + d \in \Omega, \end{array}\right\} \tag{7}$$

where $\Omega = \{x \in \mathbb{R}^n \mid Cx \le b\}$ and d is the descent direction .

Suppose that we have some starting point $x_1 \in \Omega$, the current iteration point $x_k \in \Omega$ and that we have subgradients $g_j^f \in \partial f(y_j)$ for all $j \in J_f^k$, where $J_f^k \subset \{1, \ldots, k\}$ is a nonempty index set and where $y_j \in \Omega$ is an auxiliary point. Denoting

$$f_j^k = f\left(y_j\right) + \left(g_j^f\right)^T \left(x_k - y_j\right), \tag{8}$$

the linearization of our cost function is

$$\overline{f}_j(x) = f_j^k + \left(g_j^f\right)^T (x - x_k) \;\text{ for all } j \in J_f^k, \tag{9}$$

but we can rewrite the formulation (8) into its recursive form

$$f_j^{k+1} = f_j^k + \left(g_j^f\right)^T (x_{k+1} - x_k) \;\text{ for all } j \in J_f^k. \tag{10}$$

Moreover, we can employ this linearization for polyhedral approximation of the objective function (e.g. in Fig. 2)

$$\hat{f}^k(x) = \max\{\, \overline{f}_j(x) \mid j \in J_f^k \,\} \tag{11}$$

and then we can define the improved polyhedral function $\hat{H}^k$

$$\hat{H}^k(x) = \hat{f}^k(x) - f(x_k) \;\text{ for all } x \in \mathbb{R}^n. \tag{12}$$

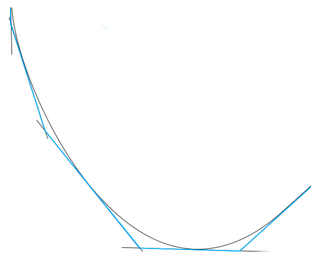

Figure 2: Illustration of the linearization.

By employing the proximal bundle idea [1] and after a series of adjustments, we can rewrite the whole problem (7) into its dual form

$$\left.\begin{array}{ll} \min\limits_{\lambda,\nu} & \frac{1}{2u_k}\|\sum\limits_{j\in J_f^k}\lambda_j g_j^k + \sum\limits_{i\in I}\nu_i C_i\|^2 + \sum\limits_{j\in J_f^k}\lambda_j \alpha_{f,j}^k + \sum\limits_{i\in I}\nu_i\alpha_{C,i}^k \\ \text{subject to} & \sum\limits_{j\in J_f^k}\lambda_j = 1 \text{ and } \lambda,\nu \geq 0, \end{array}\right\} \tag{13}$$

where u_k is the weight, $\alpha_{f,j}^k$ are subgradient errors $\left(\alpha_{f,j}^k = f(x_k) - f_j^k,\ \text{for } j \in J_f^k\right)$ and $\alpha_{C,i}^k$ are errors of the constraints subgradients $\left(\alpha_{C,i}^k = -C_i x_k + b_i,\ \text{for } i \in I = \{1,\dots,m\}\right)$. We denote the solution of the problem (13) as vector (λ^k, ν^k). The descent direction d_k is given as

$$d_k = -\frac{1}{u_k}\left(\sum_{j\in J_f^k}\lambda_j^k g_j^k + \sum_{i\in I}\nu_i^k C_i\right) \tag{14}$$

and the awaited decrease v_k can be computed as

$$v_k = -\frac{1}{u_k}\left\|\sum_{j\in J_f^k}\lambda_j^k g_j^k\right\|^2 - \sum_{j\in J_f^k}\lambda_j^k\alpha_{f,j}^k - \sum_{i\in I}\nu_i^k C_i < 0. \tag{15}$$

[1]The idea of adding a penalty to be able to limit the step length.

3.2 Subgradient aggregation

There is still one hidden but equally important difficulty in the problem (13). Let us consider the index set J_f^k. The simplest way to choose this set seems to let

$$J_f^k = \{1, \ldots, k\}\,. \tag{16}$$

However, this is not the right idea. Because, in every iteration step, the index set will enlarge which causes larger and larger memory requirements. In 1985, Kiwiel presented the subgradient aggregation strategy. The idea is to aggregate the constraints made up by the previous subgradients. This strategy allows us to keep the quantity of constraints bounded. We denote the aggregate subgradient by p_f^k. For more details see [4].

3.3 Nonconvexity

Let us recall that $\alpha_{f,j}^k = f(x_k) - f_j^k$ is the linearization error. If f is convex, then $\alpha_{f,j}^k \geq 0$ for all $j \in J_f^k$ and $\overline{f}_j(x) \leq f(x)$ for all $x \in \Omega$. It means that our linearization approximates the cost function f from bellow and $\alpha_{f,j}^k$ indicates how good our linearization is. But this is true only if the cost function f is convex. Unfortunately, in the nonconvex case, the inequality $\overline{f}_j(x) \leq f(x)$ is not valid at every $x \in \Omega$. The linear approximation can be above the cost function f and the linearization error may takes values less then zero (see Fig. 3).

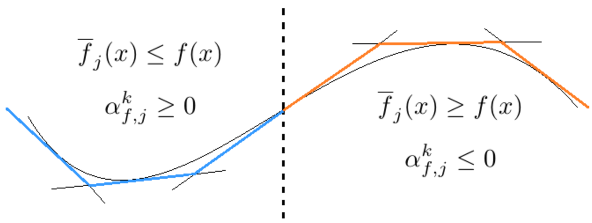

Figure 3: Linear approximation of a nonconvex function.

We have to generalize the subgradient error $\alpha_{f,j}^k$. To achieve this, we will need some information about the distance between the trial point y_j and the actual iteration point x_k.

Definition 6 *Let us define the distance measure at every iteration k by*

$$s_j^k = \begin{cases} \|x_j - y_j\| + \sum_{i=j}^{k-1} \|x_{i+1} - x_i\| & \text{for } j = 1, \ldots, k-1 \\ \|x_k - y_k\| & \text{for } j = k \end{cases} \tag{17}$$

And now we are able to define the subgradient locality measure.

Definition 7 *At every iteration step k, the subgradient locality measure is defined by*

$$\beta_j^k = \max\left\{|\alpha_{f,j}^k|, \gamma\left(s_j^k\right)^2\right\} \text{ for all } j \in J_f^k, \tag{18}$$

where $\gamma \geq 0$ is the distance measure parameter which is equal to zero, when the cost function is convex.

We choose the parameter γ heuristically. We denote the aggregate subgradient locality measure by $\tilde{\beta}_{f,p}^k$.

3.4 Line search

The descent direction d_k is known. But we do not know yet how far we can go in the direction d_k to evaluate the next value x_{k+1}. A solution to this problem was presented by Kiwiel in 1990 in his contribution [3].

3.5 Weight update

One of the last but still very important question is the choice of weight update u_k. We cannot keep u_k constant, because it could make some difficulties (e.g. if the parameter u_k is large, values $|v_k|$ and $\|d_k\|$ will be very small and therefore the decrease will be small). This difficulty was also solved by Kiwiel in 1990. The whole weight update strategy can be found in the book [4] and in the article [3].

3.6 Several conclusion notes about proximal bundle method algorithm

At the beginning of our algorithm, we need to set several parameters such as stopping tolerance $\varepsilon_S > 0$, which is used in the stopping criterion, the maximum number of stored subgradients $M_g \geq 2$ and distance parameter $\gamma > 0$.

In the next step of the algorithm, we should find multipliers λ_j^k by solving the dual problem (13). In the algorithm, there is also implemented the stopping criterion. We need to evaluate whether $w_k \leq \varepsilon_S$, where $w_k = \frac{1}{2}\|p_f^k\|^2 + \tilde{\beta}_{f,p}^k$, holds or not. If so, the algorithm stops and we obtain the desired result. Otherwise the algorithm continues by line search and after finding the step size, we make the linearization update.

The final part of the algorithm consists of the weight update and the index set updating $J_f^{k+1} = J_f^k \cup \{k+1\}$, but if the size of $J_f^{k+1} > M_g$, we set $J_f^{k+1} = J_f^{k+1} \setminus \{\min\ j \ \mid\ j \in J_f^{k+1}\}$. Now it remains to increase the iteration counter k by 1 and to repeat the whole algorithm from the part with the dual problem.

4 Numerical experiment

The proximal bundle method described in the previous section will now be used to solve a model example. We chose the shape optimization of a discretized two-dimensional contact problem with Coulomb friction as the model example. Shape optimization is a part of the optimal control in which the control variables are linked to the geometry of elastic bodies that are in contact. The aim of the problem on the lower level which is contact problem with friction is to find the set of the state variables for the fixed vector of control variables. The state vector contains variables which describe the displacements and the normal stress on the contact boundary. Hereafter the contact problem with Coulomb friction will be considered as the state problem. The mapping describing the solution of the state problem for the prescribed control variable is named as the control–state mapping. A typical feature of the contact shape optimization with Coulomb friction is its nonsmooth character due to the fact that the respective control–state mapping is typically nondifferentiable. Shape optimization of a discretized 2D contact problem with Coulomb friction was considered in [1]. Shape optimization of a discretized 3D contact problem with Coulomb friction was considered in [2]. Sensitivity analysis (computation of the subgradients of the minimized function) with help of calculus of Clarke (for 2D case) and calculus of Mordukhovich (for 3D case) was proposed in [1], [2]. In this contribution, we approximate subgradients only numerically by the forward finite difference approximation.

Example 1 *Now let us deal with the shape optimization of a discretized two-dimensional contact problem with Coulomb friction only briefly. Let $\mathcal{J}$ be a cost function. The shape optimization problem is defined generally as follows*

$$\left.\begin{array}{l} \textit{minimize } \mathcal{J}(\alpha, \mathcal{S}(\alpha)) \\ \textit{subject to } \alpha \in U_{ad}, \end{array}\right\} \tag{19}$$

where the admissible set U_{ad} is given by

$$\begin{aligned} U_{ad} := \; & \{\alpha \in \mathbb{R}^d : \; 0 \le \alpha^i \le C_0, \; i = 0, 1, \ldots, d-1; \; |\alpha^{i+1} - \alpha^i| \le C_1 h, \; i = 0, 1, \ldots, d-2; \\ & C_{21} \le \text{meas}\, \Omega(\alpha) \le C_{22}\} . \end{aligned}$$

We will try to smooth down the peaks of the normal contact stress distribution. To this aim, we should minimize the max-norm of the discrete normal contact stress λ. The objective function $\mathcal{J}$, however, must be continuously differentiable to ensure that the composite function $\mathcal{J}(\alpha, \mathcal{S}(\alpha))$ is locally Lipschitz, so we will use the p power of the p norm of the vector λ with $p = 4$ as the objective function $\mathcal{J}$. The shape optimization problem then reads as follows:

$$\left.\begin{array}{l} \textit{minimize } \|\lambda\|_4^4 \\ \textit{subject to } \alpha \in U_{ad}. \end{array}\right\} \tag{20}$$

The vector α denotes the control vector, u denotes the displacement and λ denotes the normal stress and mapping $\mathcal{S}\colon \alpha \in U_{ad} \subset \mathbb{R}^d \to (u, \lambda) \in \mathbb{R}^{3p}$ denotes the control–state mapping. Number d is the dimension of the control vector α, p is the number of the nodes of the discretized elastic body $\Omega(\alpha)$ and U_{ad} is the set of the admissible control variables. For more detailed description, see [1].

The shape of the elastic body $\Omega(\alpha)$, $\alpha \in U_{ad}$, is defined through a Bézier function F_α as follows (cf. Fig. 4):

$$\Omega(\alpha) = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \in (0, a), \; F_\alpha(x_1) < x_2 < b\},$$

where the vector α contains the control points of the Bézier function F_α.

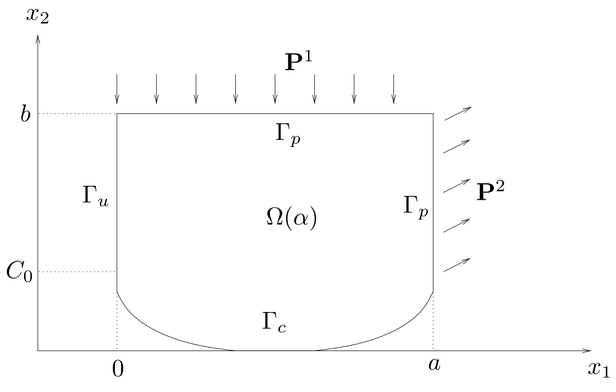

Figure 4: The elastic body and applied loads.

From Fig. 4 we can also see the distribution of external pressures on the boundary Γ_P, given as $P^1 = (0; -200\ MPa)$ on $(0, a) \times \{b\}$, while $P^2 = (100\ MPa; 40\ MPa)$ on $\{a\} \times (0, b)$ Further, Γ_u is the part of the boundary where the zero displacements are prescribed.

The set of the admissible designs U_{ad} and the elastic body $\Omega(\alpha)$ is specified as follows: $a = 2$, $b = 1$ and $C_0 = 0.75$, $C_1 = 1$, $C_{21} = 1.8$, $C_{22} = 2$. The Young modulus $E = 1$ GPa and the Poisson constant $\sigma = 0.3$ are used for the definition of the mapping $\mathcal{S}$. The value of the coefficient of the Coulomb friction is 0.25. The state problem on $\Omega(\alpha)$ is discretized by isoparametric quadrilateral elements of Lagrange type. The total number of nodes (vertices of quadrilaterals) is 3976 for any $\alpha \in U_{ad}$. The dimension of the control vector α, generating the Bézier function and defining $\Omega(\alpha)$, is $d = 8$.

The stopping tolerance was set to $\varepsilon_S = 1 \cdot 10^{-6}$. This required precision was reached after 11 iterations. We depict the initial shape and the distribution of the von Mises stress in the loaded initial body in Fig. 5 . Figure 6 shows the optimal shape and the von Mises stress in the deformed optimal body. Finally, figure Fig. 7 compare the contact normal stresses for the initial and optimal shape, respectively. Note that during the optimization process the initial value $\mathcal{J}(\alpha_0) = 2.8612 \cdot 10^{11}$ of the cost functional dropped to $\mathcal{J}(\alpha_{opt}) = 1.0695 \cdot 10^{11}$. The decrease of the peak stress is also quite significant. The experiment was carried out in Mathworks Matlab.

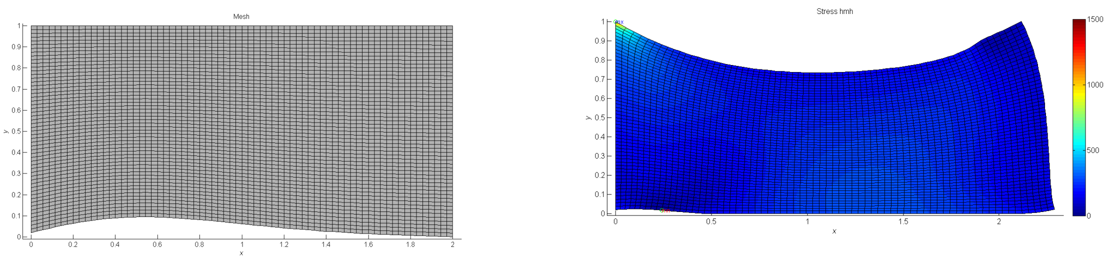

Figure 5: Example, initial design – the initial shape of the body (left) and the distribution of the von Mises stress in the deformed initial body (right).

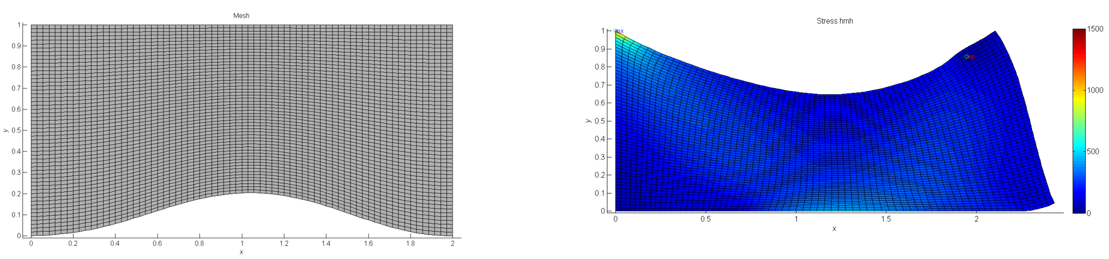

Figure 6: Example, optimal design – the optimal shape of the body (left) and the distribution of the von Mises stress in the deformed optimal body (right).

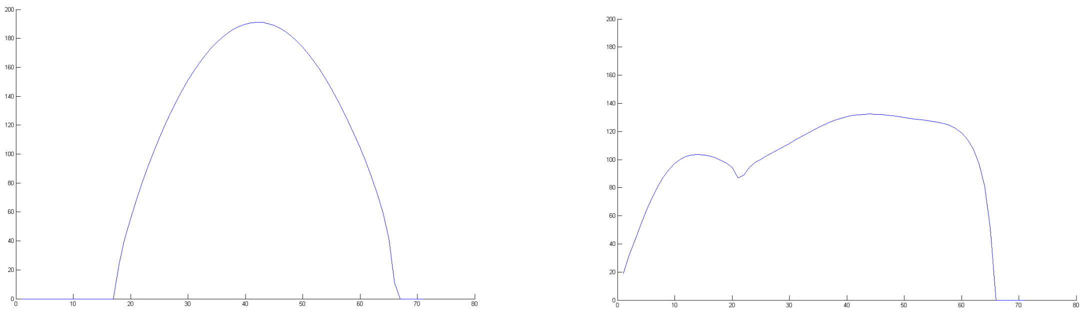

Figure 7: Example, normal stress for initial (left) and optimal (right) design.

Conclusion

In this contribution we have briefly introduced the proximal bundle method for nonsmooth optimization problems with linear constraints and with simple bounds. We outlined the implemented algorithm, which was employed to solve our model example. Then we tried to deal with the shape optimization of a discretized 2D contact problem with Coulomb friction.

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Řešení úlohy tvarové optimalizace pro kontaktní úlohu pomocí proximal bundle metody

Abstrakt: Úlohu tvarové optimalizace pro kontaktní úlohu můžeme popsat jako úlohu nelineární optimalizace. Velmi často jde o úlohu nehladké optimalizace. V tomto příspěvku si ukážeme, jak minimalizovat cenovou funkci, která je nediferencovatelná. K tomu použijeme proximal bundle metodu. V příspěvku popíšeme postup minimalizace nediferencovatelné funkce, včetně linearizace, hledání směru poklesu, výpočtu délky kroku a návrhu ukončující podmínky. Abychom ukázali efektivitu této metody, použijeme ji pro řešení úlohy tvarové optimalizace pro 2D kontaktní úlohu s Coulombovým třením.

Klíčová slova: nehladká optimalizace, Clarkeův kalkul, proximal bundle metoda, tvarová optimalizace.

Student videos for students

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Abstract: Ten years ago we've been making video tutorials at the Department of Mathematics and Descriptive Geometry. These videos are very popular. Based on the surveys we conducted, students would welcome even more examples of practice. Because the students better understand the expressions of other students than the professional interpretation of the teachers, we have addressed our students and offered them the opportunity to shoot more videos of the exercises to practice.

Keywords: Video, students work, tutorial, youtube.

1 Introduction

Students of master's programs come to the VSB - Technical University from various specializations and the level of their knowledge of mathematics is therefore diametrically different. In this article, we will focus on a variety of students from present and combined following bachelors studies of Faculty of Mining and Geology. We run the Engineering Mathematics and Selected chapters in mathematics for this faculty. The lessons take place in Ostrava and on the detached workplace in Most. Both courses require a good knowledge of a Bachelor's Mathematics I. and II. Not every student has the required knowledge to follow these courses.

The combined form of study is divided into 18 hours of classes, which are running in two or three blocks. We have students with different levels of mathematic attending these classes, as they absolved the Bachelor's maths in different schools, where the level was lower than on VSB-TU Ostrava. Students can use the scripts or the web address called "studijni opory" – learning support (www.studopory.vsb.cz). These pages are not only for teaching maths. They were created in 2006 – 2008 in the Learning Support project with preponderant distant elements for subjects of the theoretical basis of the study, also video materials on the web of the department and the collection of unsolved exercises for training. (Picture 1) If the student approaches a difficulty, he has the possibility of personal consultation with his tutor. If the personal meeting cannot be arranged, we can consult the student over the phone or Skype. The students can also visit MSC. The main characteristic of MSC is that we lead consultations in an informal study atmosphere, out of the class. Students come to support centre with problems from the lectures, regular lessons or their individual projects. Our tutors don't solve the problem instead of the student. On the contrary, they guide him or her and help with an advice. If needed, they lead students by questions, in such a way that the student gets to the root of his/her problem and finds a solution. Our goal is that students solve their problems by themselves. We believe that with active independent approach, students will learn much more, than if we just say them how their problem should be solved. We advice them how to study and learn effectively and how to independently approach new topics. Without such approach the overcoming of gaps in knowledge from four years of study at secondary school while also studying the first year at technical university is almost impossible. Our approach can be easily described by famous quotation of Benjamin Franklin: "Tell me and I forget. Teach me and I may remember. Involve me and I learn." (Hamříková, R., Kotůlek, J., Žídek, A., 2017)

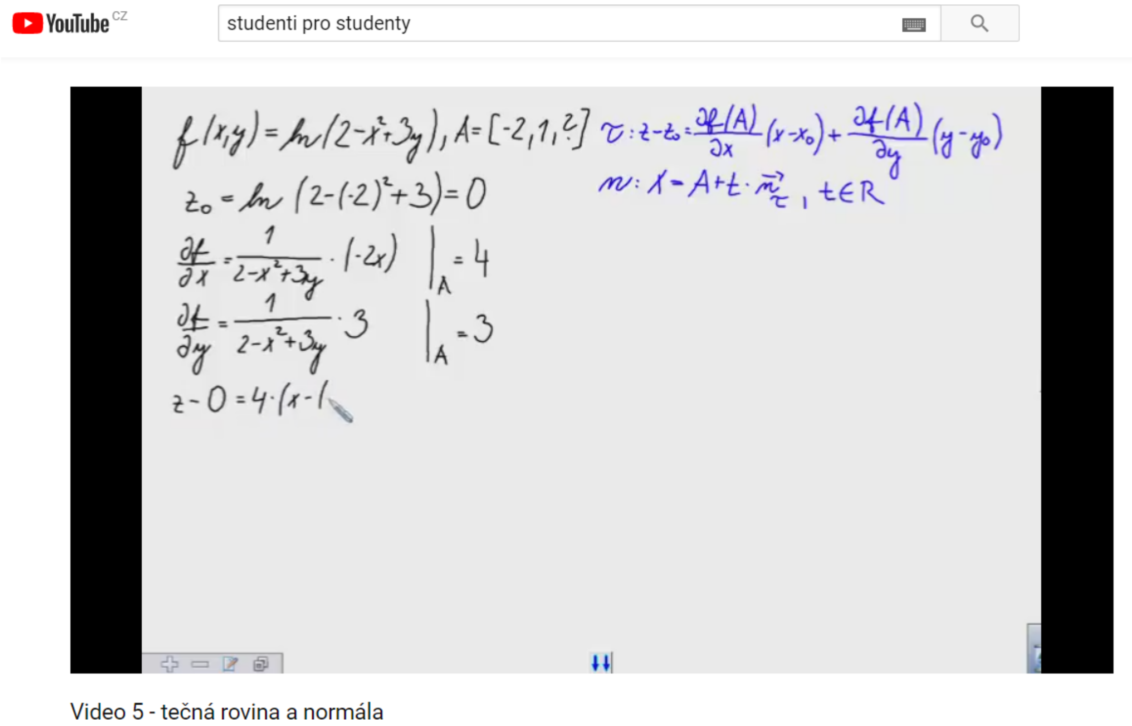
But because most of our students come from distant places and they have to commute to school or study on our detached workplace in Most, personal consultations are almost impossible for them. In these cases, we can help by leading the consultation via Skype or Phone. And because students understand the explanation from another student better than professional definition from teachers, we approached our excellent students and offered them the possibility to take a part in this activity.

2 Commented videos

As a base for video-consultations we used the collection of unsolved exercises for training. From every thematic field, we've chosen one exercise and recorded a video tutorial of solving the problem with the spoken comment. Our students are already used to this form of self-study support, as there is already a bank of exercises from the Bachelors Mathematic I, II and descriptive geometry.

For video recording we use interactive board, program for operating the board, which is called the Device manager, then the Camasia studio for cut and sounding and a graphic calculator GeoGebra for functions and 3D mathematic, which is free to download. This form of study is very popular among our students. We've created a questionary and it was answered by half of our students approx. From their answers, it was clear, that our videos are very important part of their preparation for an exam. They appreciate their accessibility on the internet and our willingness to record another videos.

From our experiences, we already know, that when solving a certain exercise, students play the commented video of a similar problem in the first place and they try to understand each step of the solution. We record our videos such as the exercise is not only solved correctly but also economically. Too long explaining of each step could discourage the more talented students and would cut down the independent activity of another ones. If any student shall need further and closer explaining of any steps, he can send us a question. For communication with students, we use the school e-mail or comment underneath the videos on YouTube.



Picture 1 - commented video

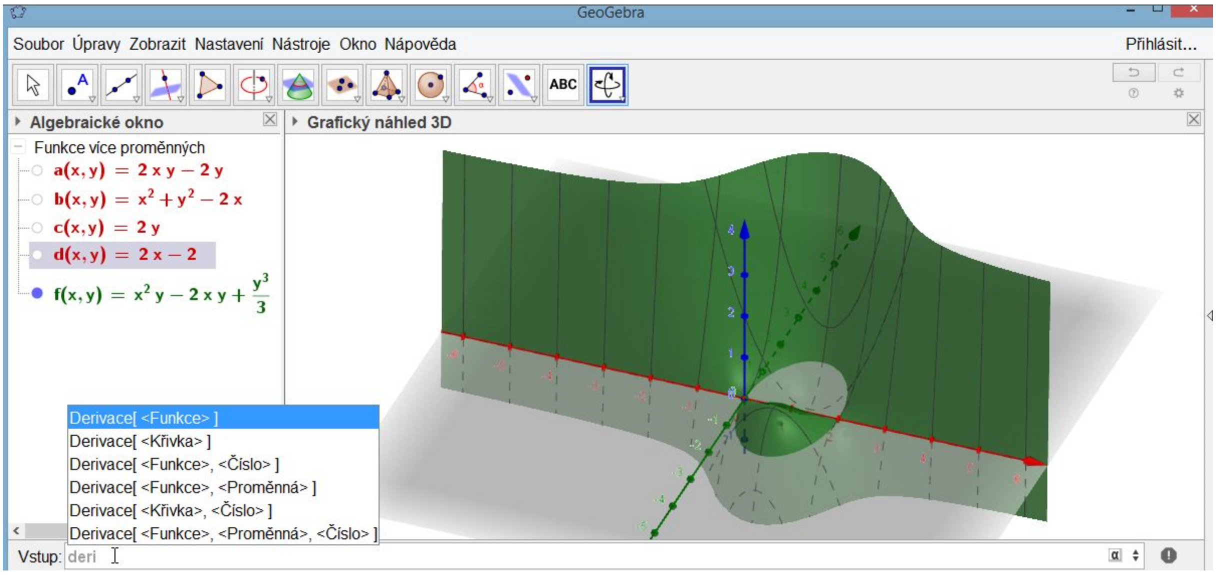
3 How does it work?

Now we will be showing how the usual communication between students and tutors looks like. One of the situation is extremes of a function of two variables (local, dependent and global). In the time allocation of 1 hour are students introduced to the theory, they will have the example of exercise calculated and will receive the link to the web address with materials to self-study. Unfortunately, the majority of students is only re-writing the information from the board and are thinking about the meaning back at home, where there is no one to explain what they didn't understand. Many students also can't attend all classes due to high workloads.

Therefore they often find difficulties when solving the exercises from the collection of unsolved exercises as a preparation for the exam, and they are unable to solve them on their own. The collection has 17 themes. We recorded a video of one example of the theme and the students are to solve another exercises by themselves.

3.1 Student questions

- Q: Can you explain me, how does the function look like?
  A: We will record instructory video of how to work with GeoGebra and make a model of the function.



Picture 2 - Graph in GeoGebra

- Q: How did you find four stationary points?
  A: The video will show you how to solve a combination of two functions of two variables.

$$f(x,y) = x^2\,y - 2\,x\,y + \frac{y^3}{3}$$

$$f'_x = 2\,x\,y - 2\,y \qquad 2\,x\,y - 2\,y = 0$$

$$f'_y = x^2 + y^2 - 2\,x \qquad x^2 + y^2 - 2\,x = 0$$

$$2y(x-1) = 0$$

$$y = 0 \quad x = 1$$

$y = 0$ *dosadíme do druhé rovnice*

$$x^2 - 2x = 0$$
$$x(x-2) = 0$$
$$x = 0 \qquad x = 2$$

$x = 1$ *dosadíme do druhé rovnice*

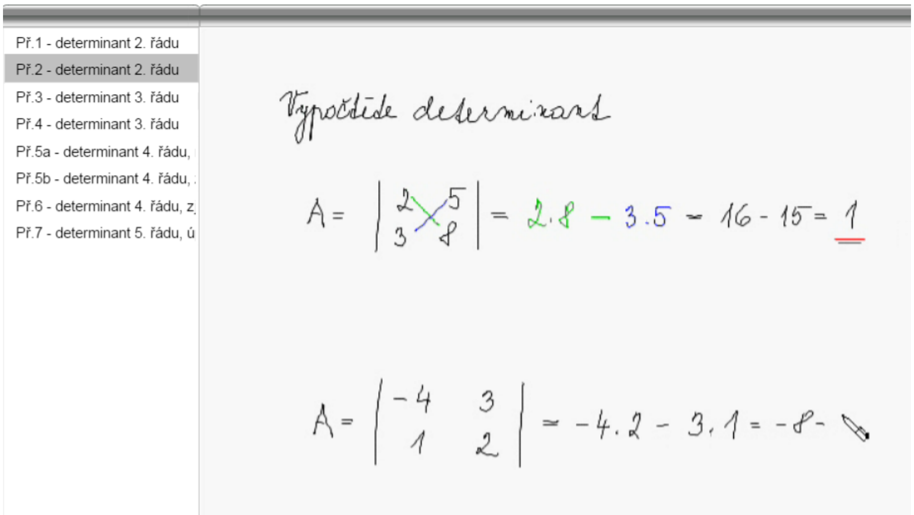
$$1 + y^2 - 2 = 0$$
$$y^2 - 1 = 0$$
$$(y-1)(y+1) = 0$$
$$y = \pm 1$$

$$A_1 = [0,0]$$
$$A_2 = [2,0]$$

$$\boxed{\begin{aligned} A_3 &= [1,1] \\ A_4 &= [1,-1] \end{aligned}}$$

Picture 3 - Video with equation system

- Q: How is determinant being calculated?
  A: Supplementary video with picture of the video.
  http://www.studopory.vsb.cz/studijnimaterialy/Sbirka_uloh/video/deter1/index.html



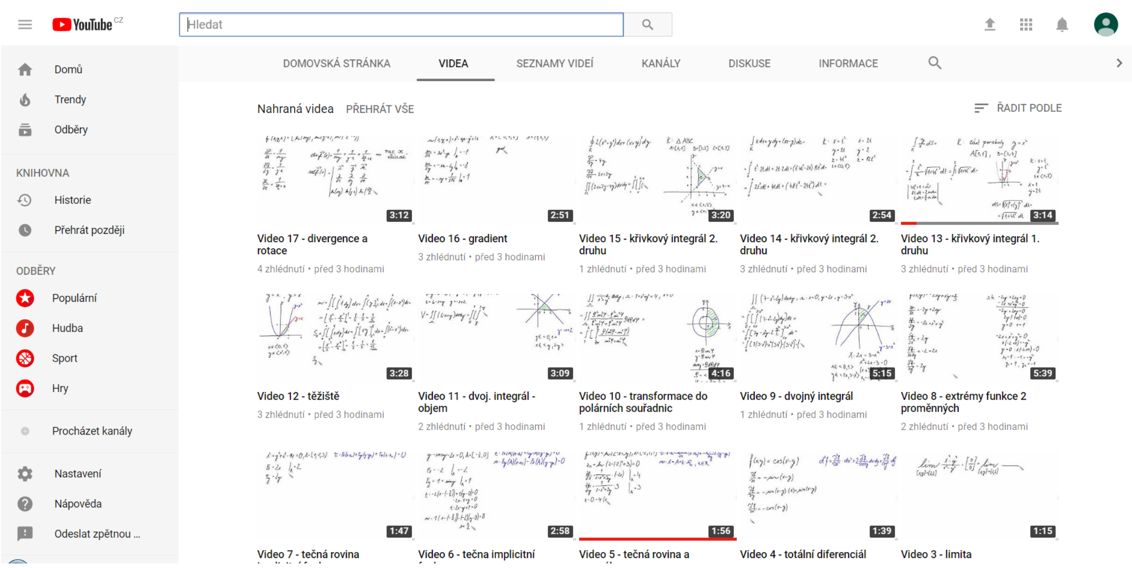
Picture 4 - Studopory

4 Youtube channel

At first, we planned to put those materials on school web, where they also were for two semesters and students were using them actively. After we started to do video-consultations we realized that the space on our web in highly insufficient, so after considering another options we decided to move our materials on YouTube. We don't have many views so far, but we believe that students will watch our materials here too, because the rules of video-consultations are explained in the presentation. All students, which have the certain subject written in their study plan have received an email with a new link to all materials.

Also visit our educational channel on youtube.com.

https://www.youtube.com/channel/UCK_YTyx_ZpwaJXYJ0qniEVg/videos



Picture 5 - Youtube channel

5 Students for students

The fact, that one group of students help another is praiseful. We are glad, that we can at least partially reward them from resources, that we have got from the Program. This is mainly for learning support and talent-management in the field of technology and science in the statutory city of Ostrava for the year 2017 and 1st.trimester of 2018. It's a big benefit for the school that excellent students, who are already co-operating with us, want to stay at our school as post-graduate students. We can also expect, that they will stay as scientists, even though they are from various regions all over the Czech and Slovak Republic. We want to be helpful and prepare them for their pedagogical or scientific path by preparing them for publishing activities and for their participation in professional conferences. Although video tutorials can't replace personal consultation, they show as beneficial for both sides. It is also a way, how to keep intensive contact between the lessons and exams. Yet we see the biggest benefit of working with the students, which are co-operating with us.

Conclusion

Videotutorials can't obviously replace personal consultation, but regarding the limited time schedule of our students, it seems to be beneficial for both sides. It is also a way of how to keeps students in intensive contact between learning blocks and exam. This can also help to eliminate the deficiencies of a large number of students at once. Those students which are using video-consultations are usually well prepared for the exam and they do complete it without problems.

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Studentská videa pro studenty

Abstrakt (Streszczenie): Během letošního roku jsme přizvali ke spolupráci studenty 2. ročníku inženýrského studia. Studenti měli za úkol vypracovat motivační úlohy k tématům inženýrské matematiky, pak jsme společně natočili výuková videa, studenti je doplnili slovním komentářem a zveřejnili jsme je na youtube.com. Naše spolupráce bude pokračovat i nadále.

Klíčová slova (Słowa kluczowe): video, studentská práce, konzultace, tutoriál, youtube

A NOTE ON CIRCULANT MATRICES OF DEGREE l^2

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Abstract: Circulant matrices have quite wide range of application in many different branches of mathematics, such as data and time-series analysis, signal procesing or Fourier transformation. Huge number of results concerning circulant matrices could be found in algebraic number theory, this is because from the ring of circulant matrices of prime degree p one could construct factor ring isomorphic to the p-th cyclotomic field $\mathbb{Q}(\zeta_p)$. In this paper connection between ring of circulant matrices of degree l^2, ie. C_{l^2}, with l equal to odd prime, and the l^2-th cyclotomic field is discused/shown.

Keywords: circulant matrix, cyclotomic field

1 Introduction

A square matrix of degree n is called *circulant matrix* if its each row is cyclic shift of the row above, i.e. it is the matrix of the form

$$\mathbf{A} = \begin{pmatrix} a_0 & a_1 & a_2 & & & \cdots & a_{n-1} \\ a_{n-1} & a_0 & a_1 & a_2 & & \cdots & a_{n-2} \\ & a_{n-1} & a_0 & a_1 & \ddots & & \vdots \\ \vdots & & \ddots & a_0 & \ddots & & \\ & & & \ddots & \ddots & & a_2 \\ & & & & & & a_1 \\ a_1 & \cdots & & & & a_{n-1} & a_0 \end{pmatrix}.$$

The entry a_{jk} on j-th row and k-th column, can be computed from the first row element a_i with $a_i \equiv k - j \pmod{n}$. This means that the circulant matrix is fully determined by its first row and so is often denoted by $\mathbf{A} = circ_n\,(a_0, a_1, \ldots, a_{n-1})$, and this notation will be used in this paper.

The sum and product of two circulant matrices is also circulant and hence the set of all circulant matrices of degree n forms a ring, denoted by $\mathcal{C}_n$.

Another useful and well known fact is that circulant matrices are diagonalizable using the *Fourier matrix* $\mathbf{F}_n$, i.e. the matrix with

$$\mathbf{F}_n = \left(\frac{\zeta_n^{ij}}{\sqrt{n}}\right)_{i,j=0,1,\ldots,n-1,} = \frac{1}{\sqrt{n}}\begin{pmatrix} 1 & 1 & 1 & & & \cdots & 1 \\ 1 & \zeta_n & \zeta_n^2 & & & \cdots & \zeta_n^{(n-1)} \\ & \zeta_n^2 & \zeta_n^4 & & \ddots & & \zeta_n^{2(n-1)} \\ \vdots & & \ddots & & \ddots & & \vdots \\ & & & \ddots & \ddots & & \\ & & & & & & a_1 \\ 1 & \cdots & & & & \zeta_n^{(n-2)(n-1)} & \zeta_n^{(n-1)(n-1)} \end{pmatrix},$$

where ζ_n is a primitive root of unity, i.e. $\zeta_n = \mathrm{e}^{\frac{2\pi\mathrm{i}}{n}} = \cos\frac{2\pi\mathrm{i}}{n} + \mathrm{i}\sin\frac{2\pi\mathrm{i}}{n}$.

The diagonal matrix $\mathbf{D} = \mathbf{F}_n\mathbf{A}\mathbf{F}_n^{-1}$ has entries λ_i equal to

$$\lambda_i = a_0 + a_1\zeta_n^i + a_2\zeta_n^{2i} + \cdots + a_{n-1}\zeta_n^{(n-1)i}. \tag{1}$$

From this we see that λ_i can be viewed as element of the n-th cyclotomic field $\mathbb{Q}(\zeta_n)$. Since the Fourier matrix $\mathbf{F}_n$ is unitary, the determinants $|\mathbf{A}|$ and $|\mathbf{D}|$ are equal and λ_i are eigenvalues of circulant matrix $\mathbf{A}$. Thus we can write $|\mathbf{A}| = \prod_{i=0}^{n-1}\lambda_i$, with $\lambda_0 = a_0 + a_1 + \cdots + a_{n-1}$ and λ_i defined by (1) for $i = 1, 2, \ldots, n-1$.

This observations leads to connection between the ring $\mathcal{C}_n$ of all circulant matrices and the field $\mathbb{Q}(\zeta_n)$. With this connection one can try to use circulant matrices to represent the elements and the arithmetics of the field $\mathbb{Q}(\zeta_n)$.

This is straightforward in the case of $n = l$, where l is odd prime, since in this case all eigenvalues λ_i for $i = 1, 2, \ldots, l-1$ are conjugates and

$$\begin{aligned} |\mathbf{A}| = \prod_{i=0}^{l-1}\lambda_i &= (a_0 + a_1 + \cdots + a_{n-1})\prod_{i=1}^{l-1}\lambda_i = (a_0 + a_1 + \cdots + a_{n-1})\,\mathrm{N}_{\mathbb{Q}(\zeta_l)/\mathbb{Q}}(\lambda_1) = \\ &= (a_0 + a_1 + \cdots + a_{n-1})\,\mathrm{N}_{\mathbb{Q}(\zeta_l)/\mathbb{Q}}\left(a_0 + a_1\zeta_n + a_2\zeta_n^2 + \cdots + a_{n-1}\zeta_n^{(n-1)}\right), \end{aligned}$$

where $\mathrm{N}_{\mathbb{Q}(\zeta_l)/\mathbb{Q}}(\alpha)$ denotes the norm of the element $\alpha \in \mathbb{Q}(\zeta_l)$. For further details see paper [2].

The relation between $\mathcal{C}_n$ and $\mathbb{Q}(\zeta_n)$ where $n = pq$, the product of two different odd primes p, q is described in [3]. The case of $\mathcal{C}_9$ and $\mathbb{Q}(\zeta_9)$ is discussed in [1].

The purpose of this paper is to find a way to represent the field $\mathbb{Q}(\zeta_{l^2})$. Unfortunately for $n = l^2$, the cyclotomic field is not tamely ramified and because of this it does not pose normal integral basis.

2 Basic observations

The degree of the field $\mathbb{Q}(\zeta_{l^2})$ is $[\mathbb{Q}(\zeta_{l^2}) : \mathbb{Q}] = \varphi(l^2) = l(l-1)$, where φ is Euler's totient function[1].

Unfortunatelly, the m-th cyclotomic field $\mathbb{Q}(\zeta_m)$ has a normal basis, i.e. the basis consisting of elements conjugated by the Galois group, if and only if m is squarefree. Surely this is not our case and thus we are bound to work with a power basis.

[1] *Euler's totient function* or *Euler's phi function* $\varphi(n)$ is defined as number of positive integers $\leq n$ that are relatively prime to n, i.e. integers k with $\gcd(k, n) = 1$. Such integers are called the *totatives* of n.

The power basis of $\mathbb{Q}(\zeta_{l^2})$ consists of elements $1, \zeta_{l^2}, \zeta_{l^2}^2, \ldots, \zeta_{l^2}^{l(l-1)-1}$ and hence every element $\gamma \in \mathbb{Q}(\zeta_{l^2})$ can be written in the form $\gamma = c_0 + c_1\zeta_{l^2} + c_2\zeta_{l^2}^2 + \cdots + c_{l(l-1)-1}\zeta_{l^2}^{l(l-1)-1}$.

Let $\mathbf{A}$ be circulant matrix of degree l^2, $\mathbf{A} = circ_{l^2}(a_0, a_1, \ldots, a_{l^2-1})$. Since the eigenvalues of the matrix $\mathbf{A}$ could be viewed as elements of the field $\mathbb{Q}(\zeta_{l^2})$, this relationship leads to the homomorphism between the ring $\mathcal{C}_{l^2}$ and the field $\mathbb{Q}(\zeta_{l^2})$. So define mapping ψ as follows

$$\psi : \mathcal{C}_{l^2} \longrightarrow \mathbb{Q}(\zeta_{l^2}), \quad \psi : \mathbf{A} \longmapsto \sum_{i=0}^{l^2-1} a_i\zeta_{l^2}^i.$$

In order to check that ψ is a homomorphism, we have to prove following equalities

$$\psi(\mathbf{A} + \mathbf{B}) = \psi(\mathbf{A}) + \psi(\mathbf{B}), \text{ and } \psi(\mathbf{A} \cdot \mathbf{B}) = \psi(\mathbf{A}) \cdot \psi(\mathbf{B}),$$

for circulant matrices $\mathbf{A}, \mathbf{B} \in \mathcal{C}_{l^2}$.

The first part is obvious. In second observe that for matrices $\mathbf{A} = circ_{l^2}(a_0, a_1, \ldots, a_{l^2-1})$, $\mathbf{B} = circ_{l^2}(b_0, b_1, \ldots, b_{l^2-1})$ the entries c_k of the product $\mathbf{C} = \mathbf{A} \cdot \mathbf{B} = circ_{l^2}(c_0, c_1, \ldots, c_{l^2-1})$ have form

$$c_k = \sum_{\substack{i+j \equiv k \\ (\mathrm{mod}\ l^2)}} a_i b_j. \tag{2}$$

The product $\gamma = \alpha\beta \in \mathbb{Q}(\zeta_{l^2})$ with elements $\psi(\mathbf{A}) = \alpha = a_0 + a_1\zeta_{l^2} + \cdots + a_{l^2-1}\zeta_{l^2}^{l^2-1}$ and $\psi(\mathbf{B})\beta = b_0 + b_1\zeta_{l^2} + \cdots + b_{l^2-1}\zeta_9^{l^2-1}$ is of the form $\gamma = c_0 + c_1\zeta_{l^2} + \cdots + c_{l^2-1}\zeta_9^{l^2-1}$, with coefficients c_k satisfying the equation (2) again, because here we do not express the elements α, β and γ in power basis and the exponents of ζ_{l^2} are reduced only by relation $\zeta_{l^2}^{l^2} = 1$, i.e. modulo l^2.

The above observations show that ψ is a homomorphism. Surely the image of ψ is entire field $\mathbb{Q}(\zeta_{l^2})$, so ψ is surjective. But the example of the matrices $circ_{l^2}(0, 0, \ldots, 0) \neq circ_{l^2}(1, 1, \ldots, 1)$ with $\psi(circ_{l^2}(0, 0, \ldots, 0)) = 0$ and $\psi(circ_{l^2}(1, 1, \ldots, 1)) = 0$ shows, that ψ is not injective.

In order to express $\psi(\mathbf{A}) \in \mathbb{Q}(\zeta_{l^2})$ with respect to the power basis we have to replace the terms ζ_{l^2} with exponents greater than $l(l-1)$. This is could be done using the cyclotomic polynomial $\Phi_{l^2}(x) = 1 + x^l + x^{(2l)} + \cdots + x^{(l(l-1))}$, the equality $\Phi_{l^2}(\zeta_{l^2}) = 0$, and henceforth $\zeta_{l^2}^{l(l-1)} = \sum_{i=0}^{l-2} \zeta_{l^2}^{il}$. Formally this means use of the isomorphism $\mathbb{Q}(\zeta_{l^2}) \cong \mathbb{Q}[x] / \Phi_{l^2}(x)$.

This means that the image $\psi(\mathbf{A})$ could be written as

$$\psi(circ_{l^2}(a_0, a_1, \ldots, a_{l^2-1})) = a_0 + a_1\zeta_{l^2} + \cdots + a_{l^2-1}\zeta_{l^2-1}^{l^2-1} = \sum_{j=0}^{l-1}\sum_{i=0}^{l-2}(a_{li+j} - a_{l(l-1)+j})\zeta_{l^2}^{li+j}. \tag{3}$$

From equation (3) we can conclude how the kernel of homomorphism ψ looks like. Since for $\mathbf{A} \in \ker(\psi)$ we have $\psi(\mathbf{A}) = 0 \in \mathbb{Q}(\zeta_{l^2})$, the following equalities have to be satisfied

$$a_{li+j} = a_{l(l-1)+j}, \text{ for all } i = 0, 1, \ldots l-2, j = 0, 1, \ldots l-1.$$

This shows that the kernel $\ker(\psi)$ is the set

$$\{circ_{l^2}(x_0, x_1, x_2, \ldots, x_{l-1}, x_0, x_1, x_2, \ldots, x_{l-1}, \ldots, \ldots, x_0, x_1, x_2, \ldots, x_{l-1}); x_i \in \mathbb{Q}\}. \tag{4}$$

The kernel $\ker(\psi)$ is an ideal in $\mathcal{C}_{l^2}$, denote it as $\mathcal{I}_{l^2}$, construct a factor ring $\mathcal{C}_{l^2} / \mathcal{I}_{l^2}$. To prove that this factor ring is a field we have to show that the ideal $\mathcal{I}_{l^2}$ is maximal, i.e. if there is an ideal $\mathcal{J}$ such that $\mathcal{I}_{l^2} \subset \mathcal{J} \subset \mathcal{C}_{l^2}$ then $\mathcal{I}_{l^2} = \mathcal{J}$ or $\mathcal{J} = \mathcal{C}_{l^2}$.

So let $\mathbf{A}$ be matrix such that $\mathbf{A} \notin \mathcal{I}_{l^2}$, $\mathcal{J}$ be an ideal generated by the matrix $\mathbf{A}$ and the ideal $\mathcal{I}_{l^2}$. Finally by $\mathbf{A}^{\times}$ denote circulant matrix from ideal $\mathcal{I}_{l^2}$ with entries x_i of the following form

$$x_0 = \frac{1 - \sum_{j=0}^{l-1} a_{lj}}{l}, \text{ for } i = 0, \text{ and } x_i = -\frac{\sum_{j=0}^{l-1} a_{lj+i}}{l}, \text{ for } i = 1, 2, \ldots, l-1.$$

The matrix $\mathbf{J} = \mathbf{A} + \mathbf{A}^{\times}$ is an element of ideal $\mathcal{J}$, but as the investigation of its eigenvalues shows, $\mathbf{J}$ is invertible and thus $\mathcal{J} = \mathcal{C}_{l^2}$. Eigenvalues of circulant matrices, λ_i, are described in (1). Especially λ_0 is the sum of row entries, i.e. for $\mathbf{J}$ we have

$$\lambda_0 = a_0 + a_1 + \cdots + a_{l^2-1} + lx_0 + lx_1 + \cdots + lx_{l-1} = 1.$$

For λ_1 we have

$$\lambda_1 = \sum_{i=0}^{l^2-1} \left(a_i + x_{i \bmod l}\right) \zeta_{l^2}^{i} = \sum_{j=0}^{l-1} \sum_{k=0}^{l-2} (a_{lk+j} - a_{l(l-1)+j}) \zeta_{l^2}^{lk+j}.$$

At least one coefficient of λ_1 is nonzero since $\mathbf{A} \notin \mathcal{J}$ and so we have $\lambda_1 \neq 0$. The same is true for all λ_i with $\gcd(i, l^2) = 1$.

Finally λ_l can be expressed as

$$\lambda_l = \sum_{i=0}^{l^2-1} \left(a_i + x_{i \bmod l}\right) \zeta_{l^2}^{il} = \sum_{j=0}^{l-1} \sum_{k=0}^{l-1} (a_{li+j} + x_j) \zeta_{l^2}^{lj} = \sum_{j=0}^{l-1} \Big(\big(\sum_{k=0}^{l-1} a_{kl+j}\big) - lx_j \Big) \zeta_{l^2}^{lj} = 1.$$

Similarly we can show that $\lambda_i = 1$ for all i with $\gcd(i, l^2) = l$.[2]

From the above we conclude that the determinant $|\mathbf{J}|$ is nonzero, and hence $\mathbf{J}^{-1} \in \mathcal{C}_{l^2}$ exists. The matrix $\mathbf{J}$ is element of the ideal $\mathcal{J}$ and so for every $\mathbf{C} \in \mathcal{C}_{l^2}$ the product $\mathbf{C} \cdot \mathbf{J}$ is element of $\mathcal{J}$. Especially $\mathbf{J}^{-1} \cdot \mathbf{J} = \mathbf{I}$, i.e. the identity matrix, is in $\mathcal{J}$. This shows that $\mathcal{J} = \mathcal{C}_{l^2}$ and that $\mathcal{I}_{l^2}$ is maximal ideal and ${}^{\mathcal{C}_{l^2}}/_{\mathcal{I}_{l^2}}$ is a field.

There is exactly one matrix of the form $circ_{l^2}\left(c_0, c_1, \ldots, c_{l(l-1)-1}, 0, 0, \ldots, 0\right)$ in every class of the field $\mathcal{C}_{l^2}/\mathcal{I}_{l^2}$. Denote the set of all circulant matrices of such form by $\mathcal{C}_{l^2}^{*}$. Clearly $\psi\left(\mathcal{C}_{l^2}^{*}\right) = \mathbb{Q}\left(\zeta_{l^2}\right)$ and for $\mathbf{A}, \mathbf{B} \in \mathcal{C}_{l^2}^{*}$ also $\psi\left(\mathbf{A} + \mathbf{B}\right) = \psi\left(\mathbf{A}\right) + \psi\left(\mathbf{B}\right)$ holds true. But the product $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ may not belong to $\mathcal{C}_{l^2}^{*}$. Thus to get the ring structure on the set $\mathcal{C}_{l^2}^{*}$ we have to define multiplication in another way. For this purpose, take $\mathbf{A}, \mathbf{B} \in \mathcal{C}_{l^2}^{*}$ and corresponding elements $\alpha, \beta \in \mathbb{Q}\left(\zeta_{l^2}\right)$ and let the product $\mathbf{A} * \mathbf{B}$ be

$$\begin{aligned}
\mathbf{A} * \mathbf{B} &= circ_{l^2}\left(a_0, a_1, \ldots, a_{l(l-1)-1}, 0, \ldots, 0\right) * circ_{l^2}\left(b_0, b_1, \ldots, b_{l(l-1)-1}, 0, \ldots, 0\right) = \\
&= circ_{l^2}\left(a_0, a_1, \ldots, a_{l(l-1)-1}, 0, \ldots, 0\right) \cdot circ_{l^2}\left(b_0, b_1, \ldots, b_{l(l-1)-1}, 0, \ldots, 0\right) \\
&\qquad - circ_{l^2}\left(c_{l(l-1)}, c_{l(l-1)+1}, \ldots, c_{l^2-1}, \ldots, c_{l(l-1)}, c_{l(l-1)+1}, \ldots, c_{l^2-1}\right) = \\
&= circ_{l^2}\left(c_0 - c_{l(l-1)}, c_1 - c_{l(l-1)+1}, c_2 - c_{l(l-1)+2}, \ldots, c_l - c_{l(l-1)}, \ldots, c_{l(l-1)-1} - c_{l^2-1}, 0, \ldots, 0\right) \in \mathcal{C}_{l^2}^{*},
\end{aligned} \tag{5}$$

with $c_k = \sum_{\substack{i+j \equiv k \\ (\mathrm{mod}\ l^2)}} a_i b_j$ as in (2).

[2] The $\gcd(i, j)$ is the *greatest common divisor* of integers i, j, i.e. the largest positive integer that divides both i and j.

With the help of (2) and (4) we can show

$$\begin{aligned}
\psi\left(\mathbf{A} * \mathbf{B}\right) &= \psi\left(\mathbf{A}\cdot\mathbf{B} - circ_{l^2}\left(c_{l(l-1)},\ldots,c_{l^2-1},\ldots,c_{l(l-1)},\ldots,c_{l^2-1}\right)\right) = \\
&= \psi\left(\mathbf{A}\cdot\mathbf{B}\right) - \psi\left(circ_{l^2}\left(c_{l(l-1)},\ldots,c_{l^2-1},\ldots,c_{l(l-1)},\ldots,c_{l^2-1}\right)\right) = \\
&= \psi\left(\mathbf{A}\cdot\mathbf{B}\right) - 0 = \psi\left(\mathbf{A}\right)\cdot\psi\left(\mathbf{B}\right) = \alpha\cdot\beta \in \mathbb{Q}\left(\zeta_{l^2}\right),
\end{aligned}$$

which means that mapping ψ reduced to $\mathcal{C}^*_{l^2}$ as follows

$$\psi : \mathcal{C}^*_{l^2} \longrightarrow \mathbb{Q}\left(\zeta_{l^2}\right), \quad \psi : \mathbf{A} \longmapsto a_0 + a_1\zeta_{l^2} + a_2\zeta^2_{l^2} + \cdots + a_{l(l-1)-1}\zeta_9^{l(l-1)-1}$$

is homomorphism again. Moreover since the kernel is trivial in this case we have also proved that $\left(\mathcal{C}^*_{l^2}, +, *\right) \simeq \mathbb{Q}\left(\zeta_{l^2}\right)$.

3 Representation of the field $\mathbb{Q}\left(\zeta_{l^2}\right)$

In the previous section we have derived straightforward way of representation of $\mathbb{Q}\left(\zeta_{l^2}\right)$ by circulant matrices from $\mathcal{C}^*_{l^2}$. In this section we try to improve this representation, since the determinant of the circulant matrix $\mathbf{C} = circ_{l^2}\left(c_0, c_1, \ldots, c_{l(l-1)}, 0, \ldots, 0\right)$ is not equal to the norm of the corresponding element $\gamma \in \mathbb{Q}\left(\zeta_{l^2}\right)$, $\mathrm{N}_{\mathbb{Q}(\zeta_{l^2})/\mathbb{Q}}(\gamma)$. Also the trace of $\mathbf{C}$ is not equal to the trace of γ, $\mathrm{Tr}_{\mathbb{Q}(\zeta_{l^2})/\mathbb{Q}}(\gamma)$ and the multiplication in $\mathcal{C}^*_{l^2}$ or work with classes in $\mathcal{C}_{l^2}/\mathcal{I}_{l^2}$ could be quite awkward.

Let $\alpha = a_0 + a_1\zeta_{l^2} + \cdots + a_{l(l-1)-1}\zeta_{l^2}^{l(l-1)-1}$ and $\alpha^{-1} = x_0 + x_1\zeta_{l^2} + \cdots + x_{l(l-1)-1}\zeta_{l^2}^{l(l-1)-1}$ is its inverse and corresponding matrices from $\mathcal{C}^*_{l^2}$ as described above, then from the equality

$$circ_{l^2}\left(a_0, a_1, \ldots, a_{l(l-1)-1}, 0, \ldots, 0\right) * circ_{l^2}\left(x_0, x_1, \ldots, x_{l(l-1)-1}, 0, \ldots, 0\right) = circ_{l^2}\left(1, 0, \ldots, 0\right),$$

we obtain equations

$$c_0 - c_{l(l-1)} = 1 \text{ and } c_j - c_{l(l-1)+(j \bmod l)} = 0 \text{ for } j = 1, 2, \ldots, l^2 - 1 \tag{6}$$

once again with $c_k = \sum_{\substack{i+j\equiv k \\ (\mathrm{mod}\ l^2)}} a_i x_j$ as in (2).

Write this system down as $\mathbf{T}_\alpha \cdot \left(x_0, x_1, \ldots, x_{l(l-1)-1}\right)^T = (1, 0, \ldots, 0)^T$, where $\mathbf{T}_\alpha$ is square matrix of degree $l(l-1)$ with entries which are linear combinations of a_i. To get proper form of $\mathbf{T}_\alpha$ think of x_j coefficients in equations (6).

For the first column of $\mathbf{T}_\alpha$ we have to observe x_0. In first equation the term x_0 is multiplied by a_0, in equation with c_1 is x_0 multiplied by a_1 and so on, for equation with c_j the term x_0 is multiplied by a_j, thus the first column of $\mathbf{T}_\alpha$ is $\left(a_0, a_1, \ldots, a_{l(l-1)-1}\right)^T$. Note that first row contains coefficients of α.

Observing x_1 we obtain second column. From (2) we see, that the term x_1 could appear in expression of c_0 only with a_8, which does not occur in α. In the term $c_{l(l-1)}$ we have $a_{l(l-1)-1}x_1$. In equation $c_1 + c_{l(l-1)+1} = 0$ we have c_1 with term a_0x_1 and no possible way to get $c_{l(l-1)+1}$ with x_1. Similarly for all equations with c_j where $\gcd(j, l^2) = 1$. In equations where we have c_j and $\gcd(j, l^2) = l$, there we can find $a_{j-1}x_1$ and also $a_{l(l-1)-1}x_1$ in expression of $c_{l(l-1)}$. So the second column is of the form $\left(-a_{l(l-1)-1}, a_0, a_1, \ldots, a_l - a_{l(l-1)-1}, a_{l+1}, \cdots, a_{l(l-1)-2}\right)^T$, which is $\alpha\zeta_{l^2}$ expressed with respect to the power basis of the cyclotomic field $\mathbb{Q}\left(\zeta_{l^2}\right)$.

In similar way we can show that k-th column of the matrix $\mathbf{T}_\alpha$ consists of coefficients of $\alpha\zeta_{l^2}^{k-1}, k = 1, 2, \ldots, l(l-1)$ expressed with respect to the power basis of the cyclotomic field $\mathbb{Q}(\zeta_{l^2})$.

To every element $\alpha \in \mathbb{Q}(\zeta_{l^2})$ we can assign matrix $\mathbf{T}_\alpha$ or vector $\delta_\alpha = \left(a_0, a_1, \ldots, a_{l(l-1)-1}\right)$. Denote $\mathcal{C}_\mathbf{T}$ set of all such matrices, i.e. $\mathcal{C}_\mathbf{T} = \{\mathbf{T}_\alpha; \alpha \in \mathbb{Q}(\zeta_{l^2})\}$. With this notation we have following theorem

Theorem 1. *For the matrix $\mathbf{T}_\alpha$ it holds*

1. $\mathcal{C}_\mathbf{T} \simeq \mathbb{Q}(\zeta_{l^2})$,
2. $\mathbf{T}_\alpha \cdot \delta_\beta = \delta_{\alpha\cdot\beta}$,
3. $\mathrm{N}_{\mathbb{Q}(\zeta_{l^2})/\mathbb{Q}}(\alpha) = |\mathbf{T}_\alpha|$,
4. $\mathrm{Tr}_{\mathbb{Q}(\zeta_{l^2})/\mathbb{Q}} = \mathrm{Tr}(\mathbf{T}_\alpha)$.

Proof. Multiplication by α defines $\mathbb{Q}$-linear transformation

$$t_\alpha : \mathbb{Q}(\zeta_{l^2}) \longrightarrow \mathbb{Q}(\zeta_{l^2}), \quad x \longmapsto \alpha x.$$

The matrix $\mathbf{T}_\alpha$ is its representation with respect to the power integral basis $1, \zeta_{l^2}, \ldots, \zeta_{l^2}^{l(l-1)-1}$. Hence the items $3, 4$ are just definitions of the norm and the trace in $\mathbb{Q}(\zeta_{l^2})$. The rest follows from the discussion above. $\square$

4 Example

In this section we demonstrate the above results on example. Let now $l = 5$, denote $\zeta = \zeta_{l^2}$ and $\alpha = 1 - \zeta^{11} - \zeta^{17}, \beta = 1 + \zeta^{11} - \zeta^{12} \in \mathbb{Q}(\zeta_{l^2})$. Compute the product $\gamma = \alpha\beta$

$$\begin{aligned}\alpha\beta &= \left(1 - \zeta^{11} - \zeta^{17}\right)\left(1 + \zeta^{11} - \zeta^{12}\right) = 1 + \zeta^{11} - \zeta^{12} - \zeta^{11} - \zeta^{22} + \zeta^{23} - \zeta^{17} - \zeta^{28} + \zeta^{29}\\ &= 1 - \zeta^{12} - \zeta^{22} + \zeta^{23} - \zeta^{17} - \zeta^{3} + \zeta^{4}\\ &= 1 - \zeta^{12} - \left(-\zeta^{2} - \zeta^{7} - \zeta^{12} - \zeta^{17}\right) + \left(-\zeta^{3} - \zeta^{8} - \zeta^{13} - \zeta^{18}\right) - \zeta^{17} - \zeta^{3} + \zeta^{4}\\ &= 1 + \zeta^{2} - 2\zeta^{3} + \zeta^{4} + \zeta^{7} - \zeta^{8} - \zeta^{13} - \zeta^{18}.\end{aligned} \tag{7}$$

So let now $\mathbf{A}, \mathbf{B} \in \mathcal{C}_{l^2}$ be circulant matrices corresponding to the elements α resp. β and compute their product in $\mathcal{C}_{l^2}$, i.e. that means we have

$$\begin{aligned}\mathbf{A} &= circ(1,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,-1,0,0,0,0,0,0,0),\\ \mathbf{B} &= circ(1,0,0,0,0,0,0,0,0,0,0,1,-1,0,0,0,0,0,0,0,0,0,0,0,0),\\ \mathbf{C} &= circ(1,0,0,-1,1,0,0,0,0,0,0,0,-1,0,0,0,0,-1,0,0,0,0,-1,1,0).\end{aligned}$$

Clearly $\mathbf{C}$ is an element of the ring $\mathcal{C}_{25}$ but not an element of the field $\mathcal{C}_{25}^*$. In this form it represents the product $\alpha\beta$ before we reduced the terms $-\zeta^{28}$ and ζ_{29} in equations (7).

Replacing the term ζ^{22} by $(-\zeta^2 - \zeta^7 - \zeta^{12} - \zeta^{17})$ in (7) leads to expressing γ with respect to the power basis, in the matrix case this is done just by substracting proper matrix $\mathbf{C}^{\times}$ from the ideal $\mathcal{I}_{25}$, i.e.

$$
\begin{aligned}
\mathbf{C}^* &= \mathbf{C} - \mathbf{C}^{\times} = \\
&= circ(1,0,0,-1,1,0,0,0,0,0,0,0,-1,0,0,0,0,-1,0,0,0,0,-1,1,0) - \\
&\qquad - circ(0,0,-1,1,0,0,0,-1,1,0,0,0,-1,1,0,0,0,-1,1,0,0,0,-1,1,0) \\
&= circ(1,0,1,-2,1,0,0,1,-1,0,0,0,0,-1,0,0,0,0,-1,0,0,0,0,0,0).
\end{aligned} \tag{8}
$$

Notice here that substracting matrix $\mathbf{C}^{\times}$ from matrix $\mathbf{C}$ in (8) is nothing else as computing the product $\mathbf{A} * \mathbf{B}$ as it was defined in (5).

Now we have $\mathbf{C}^* \in \mathcal{C}^*_{25}$ with the first row entries equal to coefficients of $\gamma = \alpha\beta \in \mathbb{Q}(\zeta_{25})$.

The norms of α, β are $\mathrm{N}_{\mathbb{Q}(\zeta_{l^2})/\mathbb{Q}}(\alpha) = 401$ resp. $\mathrm{N}_{\mathbb{Q}(\zeta_{l^2})/\mathbb{Q}}(\beta) = 101$, but the determinants are $|\mathbf{A}| = -4411$ resp. $|\mathbf{B}| = 1111$. This is because the eigenvalues λ_i with $\gcd(i,l) \neq 1$ belongs to $\mathbb{Q}(\zeta_5)$ and are not conjugates of λ_j with $\gcd(j,l) = 1$ in the field $\mathbb{Q}(\zeta_{25})$.

That is the reason for using matrices $\mathbf{T}_\alpha$ from $\mathcal{C}_{\mathbf{T}}$. Here, in case of $\alpha = 1 - \zeta^{11} - \zeta^{17}$, and $\beta = 1 + \zeta^{11} - \zeta^{12} \in \mathbb{Q}(\zeta_{l^2})$, we have

$$
\mathbf{T}_\alpha \cdot \delta_\beta =
\begin{pmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\cdot
\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
=
\begin{pmatrix} 1 \\ 0 \\ 1 \\ -2 \\ 1 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}
= \delta_\gamma .
$$

This way we obtain $\gamma = \alpha\beta$ only by using basic matrix operations and not the arithmetics of algebraic number fields. Moreover $|\mathbf{T}_\alpha| = 401 = \mathrm{N}_{\mathbb{Q}(\zeta_{25})/\mathbb{Q}}(\alpha) = 401$.

Conclusion

In the presented paper the relation between the ring of circulant matrices, $\mathcal{C}_n$. It is shown that constructing its factor field $\mathcal{C}^*_{l^2} = \mathcal{C}_{l^2} / \mathcal{I}_{l^2}$ yields a field isomorphic to the l^2-th cyclotomic field of degree $\varphi(l^2)$ for odd prime l is shown.

All obtained relations are used to representation of elements of cyclotomic field in order to simplify computations in cyclotomic field.

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O cirkulantních maticích stupně l^2

Abstrakt (Streszczenie): Cirkulantní matice mají širokou škálu aplikací v mnoha různých odvětvích matematiky, jako jsou analýza dat a časový řad, zpracování signálů či Fourierova transformace. Další výsledky využívající vlastností cirkulantních matic můžeme nalézt také v algebraické teorii čísel, což je dáno tím, že z okruhu cirkulantních matic prvočíselného stupně p, lze vytvořit faktorový okruh isomorfní s p-tým cyklotomickým tělesem, $\mathbb{Q}(\zeta_p)$. V článku je ukázán vztah mezi okruhem cirkulantních matic stupně l^2, tj. C_{l^2}, kde l je liché prvočíslo, a cyklotomickým tělesem $\mathbb{Q}(\zeta_{l^2})$.

Klíčová slova (Słowa kluczowe): cirkulantní matice, cyklotomické těleso.

Interactive 3D graphics generated by Asymptote software to support lessons of differential calculus of functions of two variables

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Abstract: The aim of this contribution is to introduce a collection of solved examples which contain interactive 3D graphics to support lessons of differential calculus of functions of two variables. Each example is processed in two versions. The first one consists of examples which are described in details, so they are suitable for self-study. The second includes examples designed for the class lessons, only main steps are shown in this version. Examples were chosen in a way that we were able to show a geometric meaning of all given terms (like partial or directional derivative, local extremum, global extremum, differential of a function, Taylor polynomial, contours of a function) by interactive 3D graphics. Each graphics could be rotated, zoomed, etc. Whole collection is freely available for students and teachers at creator website or at website of the mi21 project of the VŠB – TU Ostrava.

Keywords: interactive 3D graphics, differential calculus, functions of two variables, solved examples

1 Introduction

Imagine situation that we have to explain terms of differential calculus of functions of one variable to students during a lesson. There won't be any difficulties from a point of view of mathematical principles. Moreover, we are able to draw a simple example of geometric meaning of given term, so students could see that situation without any problems. If we move to a world of differential calculus of functions of two variables, situation will be different. Of course, there won't be any difficulties from the point of view of mathematical principles, either, but if we will have to picture given problem, it won't be easy for the most of us. We don't want underestimate anyone of you, someone could be more talented in drawing. Someone could be able to draw an arbitrary geometric meaning of a term of differential calculus of functions of two variables but if someone really could, there won't be available any rotation of a figure or any zoom, etc. That figure has to be redrawn to a new one. Author of this contribution also remember situation in which a teacher tried to illustrate a tangent plane by a paper and notebook. Notebook represent the tangent plane and the paper a function. It was a brave attempt how to picture the geometric meaning of the problem but not quite effective. However, time has changed and it brings to us a more user friendly technical support.

For our purpose we chose a software called Asymptote which is able to generate interactive 3D graphics, for more information see [1], [2] or some examples [3]. Syntax of this software is based on C++, so any of you, who know the basics of C++, could use the Asymptote without any problems. A big, and we have on mind a really big, advantage of the Asymptote is a fact that we could, and still can, include an Asymptote code into a LaTeX document, so we create a PDF document with interactive 3D graphics as a one whole document. PDF document created by this way has to be opened in PDF viewer which is able to active a multimedia content (e.g. Adobe Reader for Windows OS). Otherwise, you won't be able to activate the interactive 3D graphics which is included in the document. To active it a single left click on a mouse will be sufficient.

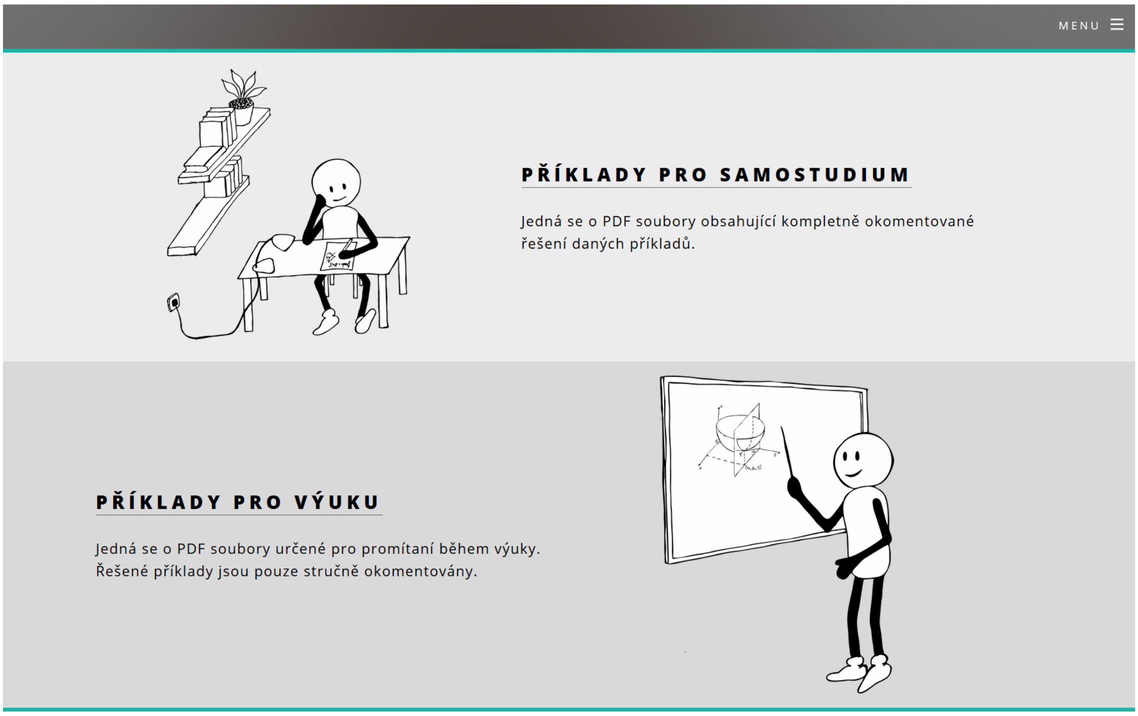

Figure 1: Main menu of the website.

We used the Asymptote to illustrate the geometric meaning of terms of differential calculus of functions consisting of two variables. We created a collection of 24 unique solved examples into which the interactive 3D graphics was added. Examples were chosen in a way that the geometric meaning will be sufficiently illustrative. There are available two versions for each example in the collection. The first one, described in details, is recommended for self-study. The second is suitable for teaching in class lessons, only a necessary steps are shown. Whole collection was uploaded to a creator's website `http://homel.vsb.cz/~fol0037/indexDiff.html/` and to a website of mi21 project `http://mi21.vsb.cz/` of the VŠB – TU Ostrava for better availability from every corner of republic. During creation of the collection we were inspired by [4:76-92], [5] and [6].

A small preview of the collection will be done in the following text.

2 Self-study examples

If you visit a first page of the website `http://homel.vsb.cz/~fol0037/indexDiff.html`, you will find a main menu which is divided into two sections, see Fig. 1. The first one consists of examples designed for self-study ("Příklady pro samostudium" in Czech). The second contains examples suitable for teaching ("Příklady pro výuku" in Czech).

After a click on self-study examples link, the whole collection will appear. It has 8 chapters called: Domain of functions of two variables, contours of functions of two variables, partial derivative, differential of functions of two variables, directional derivative, Taylor polynomial, local extreme and global extreme. In each chapter three examples are included, like you can see in Fig 2.

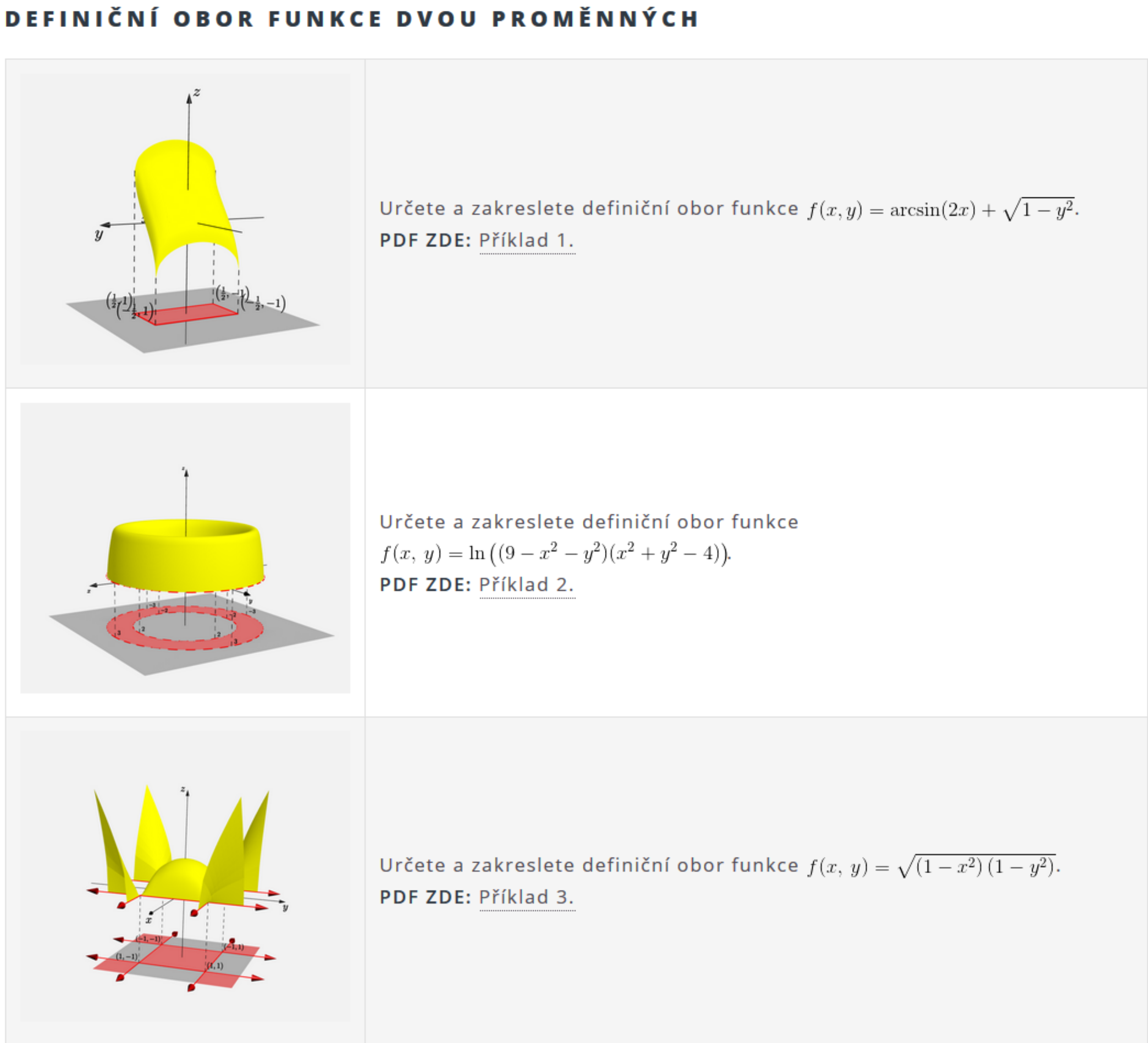

Figure 2: Chapter of domain of function.

There is also available a preview of every example, specifically a task and relevant interactive 3D graphics, within chapters. As it said in a title of this section, these examples were designed for self-study, so they are solved and described in details for easier understanding of given problem, and they were processed in A4 format. We recommend you to visit the website to see individual examples but we note that they are written only in czech language. Due to this fact we don't attached an example of some self-study document page to this contribution. What we can attached is the interactive 3D graphics, so we added here graphics of partial derivative (Fig. 3) and Taylor polynomial (Fig. 4).

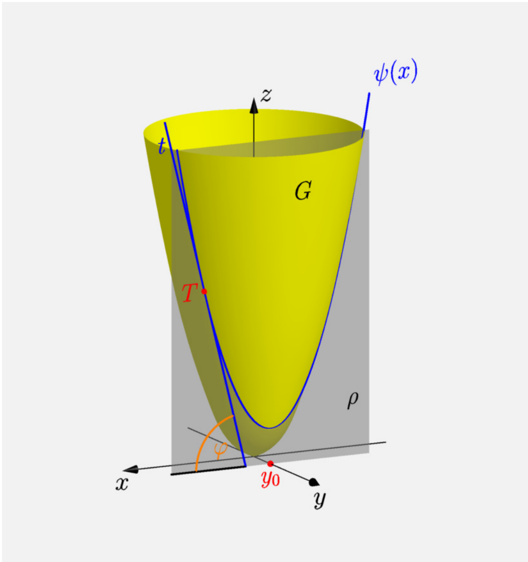

Figure 3: Partial derivative $x^2 + y^2$ by x at $(2; 1)$.

If you have an electronic version of the contribution or you have visited the website, you can activate the graphics by single left click on specific figure but, as we said above, you have to use the PDF viewer which can activate the multimedia content. After the click, the graphics is activated and you can view it in a fullscreen mode by using a menu which will appear after a right click on the graphics. Due to interactivity of the graphics, you are able to rotated it,

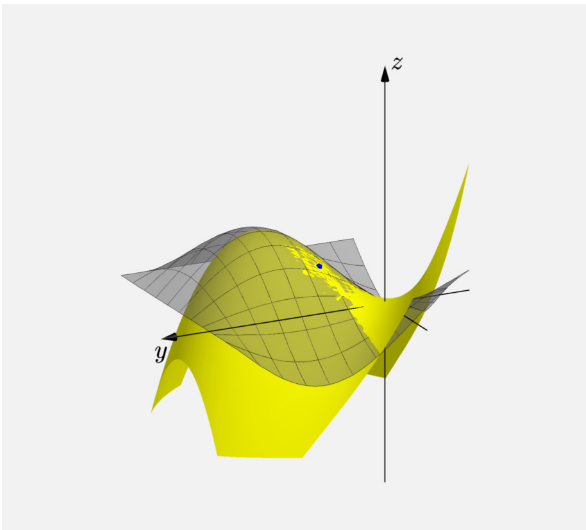

Figure 4: Taylor polynomial of third order of $\sin(x)\sin(y)$ at $(\frac{\pi}{4}; \frac{\pi}{4})$.

zoom it, change a light of scene, etc. Fullscreen preview will be closed by the "escape" key.

3 Examples suitable for teaching

These examples are divided into same chapters as examples for self-study. Also same examples are used so description of them isn't necessary. A difference is in their brevity. In the teaching version only important steps are shown. There isn't any detailed text in this version. We have to emphasize that examples were processed in a format 4:3 which is suitable to be screened during a class lesson. For better idea we added individual slides of a first example of local extremum below. We note that the interactive 3D graphics is also added to this version of examples but we didn't attached it to this example because it is not illustrative here in the contribution as it is in electronic version. The example is written in Czech language but we translated it for purpose of the contribution.

. . . .

Local extremum

Example no. 1

Task:

Find local extremum of given function

$$f(x,y) = \frac{27}{10}x^2y + \frac{14}{10}y^3 - \frac{69}{10}y - \frac{54}{10}x \; .$$

. . . .

Solution:

Partial derivatives of the first order are equal to

$$\begin{aligned} \frac{\partial f}{\partial x}(x,y) &= \frac{54}{10}xy - \frac{54}{10} \; , \\ \frac{\partial f}{\partial y}(x,y) &= \frac{27}{10}x^2 + \frac{42}{10}y^2 - \frac{69}{10} \; . \end{aligned}$$

We express variables x and y from a system of two variables (stationary points).

. . . .

$$\begin{aligned} \frac{54}{10}xy - \frac{54}{10} &= 0 \\ \frac{27}{10}x^2 + \frac{42}{10}y^2 - \frac{69}{10} &= 0 \; , \end{aligned}$$

$$\begin{aligned} 54xy - 54 &= 0 \\ xy - 1 &= 0 \\ x &= \frac{1}{y} \ , \end{aligned} \qquad \begin{aligned} 27x^2 + 42y^2 - 69 &= 0 \\ 27\frac{1}{y^2} + 42y^2 - 69 &= 0 \\ 42y^4 - 69y^2 + 27 &= 0 \end{aligned}$$

$$\text{Substitution: } a = y^2$$

$$42a^2 - 69a + 27 = 0$$

$$a_1 = 1 \quad \Rightarrow \quad y_1 = 1,\ y_2 = -1$$

$$a_2 = \frac{9}{14} \quad \Rightarrow \quad y_3 = \frac{3}{\sqrt{14}},\ y_4 = -\frac{3}{\sqrt{14}} \ .$$

. . . .

We found stationary points $(1,1)$, $(-1,-1)$, $\left(\frac{\sqrt{14}}{3}, \frac{3}{\sqrt{14}}\right)$ and $\left(-\frac{\sqrt{14}}{3}, -\frac{3}{\sqrt{14}}\right)$.

Partial derivatives of the second order are equal to

$$\frac{\partial^2 f}{\partial x^2}(x,y) = \frac{54}{10}y \ ,$$
$$\frac{\partial^2 f}{\partial y^2}(x,y) = \frac{84}{10}y \ ,$$
$$\frac{\partial^2 f}{\partial xy}(x,y) = \frac{54}{10}x \ .$$

. . . .

We assembly a determinant $J(x,y)$

$$J(x,y) = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2}(x,y) & \frac{\partial^2 f}{\partial xy}(x,y) \\ \frac{\partial^2 f}{\partial xy}(x,y) & \frac{\partial^2 f}{\partial y^2}(x,y) \end{vmatrix} \ .$$

In our case $J(x,\ y)$ is in a form

$$J(x,y) = \begin{vmatrix} \frac{54}{10}y & \frac{54}{10}x \\ \frac{54}{10}x & \frac{84}{10}y \end{vmatrix} = \frac{4536y^2 - 2916x^2}{100} \ .$$

. . . .

Points $(1,1)$ and $(-1,-1)$ satisfies

$$J(x,y) = \frac{1620}{100} > 0 \ ,$$

i.e. there is extremum.

Points $\left(\frac{\sqrt{14}}{3}, \frac{3}{\sqrt{14}}\right)$ and $\left(-\frac{\sqrt{14}}{3}, -\frac{3}{\sqrt{14}}\right)$ satisfies

$$J(x,y) = -\frac{1620}{100} < 0 \ ,$$

i.e. there is no extremum.

. . . .

Point $(1,1)$ satisfy

$$\frac{\partial^2 f}{\partial x^2}(1,1) = \frac{54}{10} > 0 \ ,$$

so local minimum is in this point.

Point $(-1,-1)$ satisfy

$$\frac{\partial^2 f}{\partial x^2}(-1,-1) = -\frac{54}{10} < 0 \ ,$$

so there is local maximum in this point.

. . . .

As you can see, there are shown only important steps in this example. We presume that a teacher will be able to say some details by yourself.

Conclusion

We introduced you the collection of 24 solved examples which are exceptional because interactive 3D graphics is added to them. Examples are divided into 8 chapters corresponding to terms of differential calculus of functions consisting of two and more variables. We also noticed that the collection was uploaded to the creator's website and website of the mi21 project.

The collection is already used to support lessons of differential calculus at the VŠB – TU Ostrava university for few years. It was created as author's bachelor thesis under leadership of RNDr. P. Vondráková, Ph.D. The collection was lately extended in a frame of internal SGS project in cooperation with Ing. N. Plívová.

In a future work, we think of translation this collection into English and its usage for lessons of foreign students.

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Interaktivní 3D grafika generovaná programem Asymptote pro podporu výuky diferenciálního počtu funkcí dvou proměnných

Abstrakt: Cílem příspěvku je představení sady řešených příkladů s interaktivní 3D grafikou pro podporu výuky diferenciálního počtu funkcí dvou proměnných. Každý z příkladů je k dispozici ve dvou verzích. Podrobnější verze je určená pro samostudium, stručnější pro promítání během výuky. Příklady jsou zvoleny tak, aby bylo možno geometrický význam všech pojmů demonstrovat pomocí interaktivní 3D grafiky. S grafy, které ilustrují pojmy jako parciální a směrové derivace, diferenciál, Taylorův polynom, lokální a globální extrémy a mnoho dalších, je možno interaktivně pracovat (otáčet, přibližovat,...). Sbírka je volně k dispozici studentům i pedagogům pro využití při výuce a to na webových stránkách autora a webových stránkách projektu „Matematiky pro inženýry 21. století“ (`mi21.vsb.cz`) VŠB – TU Ostrava.

Klíčová slova: iteraktivní 3D grafika, diferenciální počet, funkce dvou proměnných, řešené příklady

CUSTOMER SATISFACTION SURVEY AND A POSSIBILITY OF USING QFD METHOD FOR SATISFACTION IMPROVEMENT - THE ANALYSIS OF THE CASE

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Abstract: In the thesis the information is presented regarding methods of measuring the customer satisfaction. The customer satisfaction questionnaire has been presented for the survey, as well as the results of a questionnaire survey regarding level of satisfaction of recipients of hydraulic components. The possibility of using QFD method in order to improve customer's satisfaction has been analyzed. The analysis has been carried out for the hydraulic coupler to identify key aspects of the product for the customer. There has been made an attempt to formulate examples of solutions that influence customer's satisfaction in a given organization.

Keywords: quality management, customer satisfaction survey, questionnaire survey, QFD method**.**

1 Introduction

Contemporary market is characterized by sudden changes in customers' behavior and demands, which makes business entities increasingly seeking to acquire new customers and keep current ones. Economic, social and technological factors have a great impact on this fact. The source of those factors are: competition, continuous improvement and development, product and service satisfaction surveys and gaining customers' loyalty. On the other hand, from the customers' view there is a bigger awareness and knowledge about possibility of choosing from a number of offers available on the market [2].

One of the most important factors influencing the long term success of an organization is the satisfaction of its customers. These customers are the source of a positive feedback about organization and they make bigger and more frequent purchases. There is a close dependence between customers' loyalty and their level of satisfaction. Keeping a regular customer is cheaper than acquiring a new one, and the most effective way, especially in the long run, is to make sure that the customers are satisfied. Customer satisfaction survey is a way to measure customer satisfaction and changes in their level of satisfaction. But it is not its only function. The research provides information about customers' expectations and the level of fulfillment of those expectations. The results allow to identify the weaknesses and to implement proper corrective solutions. Customer satisfaction survey has a significant impact on the outcome and success of the company. The continuous monitoring of customer satisfaction level helps to prevent situations in which previously loyal customers leave the company, and a new group of customers needs to be acquired [6].

The aim to this paper is to present the results of customer satisfaction survey and the possibility of using QFD method to improve their satisfaction on the example of a manufacturer of hydraulic components.

2 Methods of customers satisfaction measurement

Satisfaction surveys are individual and adapted to specific research problem, industry, product type and organization. The choice of method that needs to be applied depends on many factors, such as [8]:

- Specific research problem (what the organization really wants to learn)
- Type of product offered (whether it is a product or service)
- Type of customer (individual, institutional)

It is necessary to analyze advantages and disadvantages of different methods of customer opinion survey from the point of view of the needs of organization. On this basis, there should be chosen a method that suits best organization's capabilities and priorities. Direct methods can be used to evaluate customer satisfaction. Direct methods allow to measure customer's perception in to what extend the company delivers the desired value with the offered product. They allow to acquire information about direct customer response [1]. How the customers perceive the product will affect their further behavior, thus their level of satisfaction.

By applying direct methods, a company has to be ready to respond to customers' suggestions and comments, because as the customers spend their time defining their level of satisfaction in a survey, they also expect company's response. Direct methods include[8]:

- written and oral complaints and customers' suggestions
- critical accidents
- service quality method (servqual)
- surveys
- focused discussion groups and user groups
- direct and phone interviews
- losing of customer analysis

The use of direct methods requires greater involvement of a company than with indirect methods, but it enables better understanding of customers' perception and its market consequences [3].

However, in indirect methods customers' feedback is skipped, by measuring different satisfaction indicators that typically reflect their behavior on the market. By using indirect methods, it is assumed that clients behave in a manner consistent with their level of satisfaction, and the adapted indicator is closely linked to it.

Direct methods include [9]:

- mysterious customer
- customer retention rate
- benchmarking
- analysis of sales trends, market share trends and investments return trends
- first line support reports

To sum up, it must be remembered that the data obtained from direct methods is a better indicator of a market state than indirect tools such as sales trends or market shares. Indirect methods are less effective because they do not reflect the type and intensity of emotions experienced by the customer [7].

3 Application of QFD method

One of the methods used to translate customers' expectation and preferences into technical product characteristics is QFD method - Quality Function Deployment. Currently, this method is considered highly helpful when translating customer requirements into technical functions of the product at the stage of product quality improvement as well as its design [4].

The main goal of QFD method is to shift customers' expectations and needs to the technical specifications of the product. QFD gives the ability to translate market information provided by users and clients of the product or service, into the technical language used by professionals and designers creating the product or service [10]. A very characteristic feature of this method is a matrix, which is also called an analytical-graphic tool „quality home". Quality home is formed in several stages [11]:

1. Customer requirements identification
2. Identifying the importance of individual requirements of customer
3. Product comparison with the competition
4. Determination of technical parameters of the product
5. Identification of dependency between technical parameters of the product
6. Determination of the relationship between customer's requirements and technical factors of the product
7. Determining the importance of technical parameters
8. Identification of target values of technical parameters
9. Determining the difficulty index

Very characteristic for the quality house is presenting the relationship and connection between technical parameters of the product presented in the columns of the matrix and specific customer requirements shown in subsequent rows of the matrix. It should be noted that the QFD method is used to transfer all market requirements of a particular product to the conditions that must be fulfilled by the company at all stages of a product development. While taking those steps, there should be taken into account as many factors and indicators as possible, which may influence the quality of the production stages of the product, the quality of the product itself or the processes that are part of the product's manufacturing process [4].

4 Description of the studied object

The research was carried out in a large manufacturing company. The company is a manufacturer of hydraulic components (metal fasteners used in hydraulic and refrigeration systems) such as:

- Hoses for hydraulic systems;
- Hoses for refrigeration systems;
- Plastic hoses;
- Hydraulic couplings;
- Quick connectors;
- Flexible hoses;
- Other hydraulic components.

Company is also manufacturing tools such as machines for self-connecting hydraulic hoses with connectors using appropriate pressure, which are sold to related companies (mostly retailers) and unrelated ones. The company is thus a manufacturer and a distributor for following geographical areas: Europe, Middle East, Africa, South and North America.

5 Survey results and use of QFD method for improving customer satisfaction

Based on data acquired from marketing department, it was found out that the company has gained 83 new customers in year 2015, and only 49 in 2016. Based on this data authors want to determine what might have affected such a large reduction of new customers.

In order to determine the level of customer satisfaction there has been developed a survey questionnaire containing questions regarding level of satisfaction from cooperation with the company during implementation of a placed order (Table 1). The Likert scale has been used in a questionnaire, where the customer was able to specify the level of satisfaction of the ordering process. The questionnaire was delivered by e-mail directly to the customer with a request for participation in a survey and providing a feedback. The survey was not anonymous due to the possibility of a faster response to detected irregularities reported by the customer. The responses were averaged.

Table 1. Questionnaire form

Lp.	Question	Grading scale					Averange	Averange %
		5	4	3	2	1		
1.	Did order was compatible with your requirements ?							
2.	Has the order been delivered in time?							
3.	Did the sales department run smoothly?							
4.	Are you satisfied of the received price offer?							
5.	Will the next order be established Do you cooperate with the company?							

Source: Own study based on data obtained from the company

KEY:
5 - I totally agree
4 - I rather agree
3 - Neutral
2 - I rather disagree
1 - I totally disagree

The results were collected from randomly selected 100 customers of the company regarding orders from years 2015 and 2016. The summary results of the questionnaire survey are presented in table 2.

Table 2. The summary results of the questionnaire survey regarding years 2015 and 2016

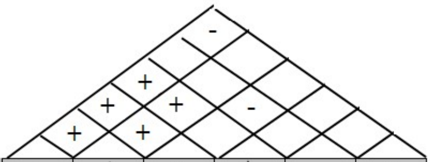
No.	Question	Averange		Averange %	
		2015	2016	2015	2016
1.	Did order was compatible with your requirements ?	**4,7**	**4,5**	**94%**	**90%**
2.	Has the order been delivered in time?	**4,8**	**4,7**	**96%**	**94%**
3.	Did the sales department run smoothly?	**4,9**	**4,6**	**98%**	**92%**
4.	Are you satisfied of the received price offer?	**3,9**	**3,8**	**78%**	**76%**
5.	Will the next order be established Do you cooperate with the company?	**4,0**	**3,9**	**80%**	**78%**

Source: Own study based on data obtained from the company

From the data for year 2015 shown in Table 2 it can be concluded that the smallest amount of customers is satisfied with the price offer obtained during process of order implementation, as this question (number 4) has received average rating 3,9. The next in order was question 5 confirming cooperation with company during the next order which was rated 4,0. The highest rating was obtained by sales consultant cooperation and timely delivery. From table 2, presenting data regarding orders from 2016 it is clear that the question 4, which is satisfactory sale offer obtained at the finalization stage of the order, has received the lowest rating of average 3,8, as well as question 5 which was regarding future cooperation - average rating of 3,9. It can be seen that in comparison with the previous year the lowest evaluated areas of order process in the company are repeated; moreover, both values dropped by 0,1 points. The highest rating has been obtained in question 2 regarding timely delivery, and question 3 regarding cooperation with consultants and sales department, which is the same as in year 2015. However, the decreasing amount of positive evaluations is noticeable.

Due to those ratings, the company decided to introduce changes in stages of order process, starting from the order acceptance to the delivery of product to the customer. For this purpose, it was decided to use QFD method to identify the most important features of one of the product offered by the company. For the analysis, a hydraulic coupler has been chosen based on the largest decrease in the number of orders in 2016, compared to 2015.

The survey that has been carried out using QFD method allowed to identify the most important aspects of the product from the point of view of the customer. They also made it possible to identify the weakest parts from the point of view of contractors of the company, so that the company would be able to adjust and modify the directions of its product development, here: hydraulic couplings.



Direction of optimization ↑ • ↓		↑	↑	↑	↑	•	↑			
Technical parameters / **Customer requirements**	**Evaluation of validity**	Dimensions of the product [mm]	Type of substitutes	Maximum pressure	Temperature range	Thread type	Manufacturer's warranty	**Profile image** 1	2	3
Weight of the product	2	●	●					2	2	2
Resistance to corrosion	7		■				■	3	2	3
Price competitiveness	8	●	■	■	■	▲	■	2	3	5
Resistance to high pressures	9	●	■	■		●	■	4	4	3
Availability	8	●	●	●	●	▲		3	3	5
Organoleptic appearance	6	▲	▲			▲		4	3	3
Easy to assemble and disassemble	8					●		5	5	5
Versatile use	6	■	●	■	■	■		3	3	3
Mechanical wear time	8	▲	▲	▲	▲		■	3	2	2
Technical significance		149	278	239	158	127	288			
Relative technical significance [%]		12%	22%	19%	13%	10%	23%			
Critical features			II	III			I			
Indicators of difficulty execution		2	5	4	3	1	5			
Target values of technical parameters		62 x 30 x 5	Plastic	0 — 40 bars	-30°C — +130°C	No change	36 months			
Comparison with competitors in terms of technical parameters	1	2	1	2	2	3	2			
	2	2	1	2	1	3	1			
	3	2	3	1	1	3	1			

■ - 9

● - 3

▲ - 1

Fig. 1. Analysis of the hydraulic coupling
Source: Own study based on data obtained from the company

The conducted survey enabled identification of key aspects of the product from the point of view of the customer, as well as the distinction of those qualities which are satisfactory for the customers and fulfill their requirements. The level of customer satisfaction is a very important factor for every business, therefore any signals of irregularities reported by regular or potential customers concerning products or services should not be ignored. Having this in mind, the survey that has been carried out with the use of a questionnaire, and later, of the QFD method, allowed for the development of sample solutions that would significantly affect the level of customer satisfaction. These include:

- introduction of individual approach to customer orders,
- use of substitute materials for the special orders of the recipient,
- possibility to extend the warranty period for steel components.

The presented solutions are exemplary suggestions that result from „quality home" and they consequently determine the possibility of noticeable increase in a satisfaction from the products of the analyzed company, from both regular and potential customers.

Conclusion

Customer satisfaction should be at the center of an interest of a company. Customer plays an important role in the process of continuous improvement of individual components as well as the whole system. Companies should therefore consider the level of customer satisfaction as a priority.

In the analyzed company it was crucial to identify the characteristics of the product that affect the significant increase or, as in the studied case, decrease in customer satisfaction in two periods in 2015 and 2016. As it was exposed, regular customers had doubts when it came to choosing the studied manufacturer of the hydraulic components from all competitors on the market, which resulted in decreasing number of new business partners.

Among the most important features affecting the level of customer satisfaction in the studied company, the recipients indicated the individual approach to the needs and requirements of the customer, which very often involves the production of products with non-standard shapes, or dimensions, or using special material substitutes. Very crucial is also the warranty period for the products, which in the opinion of contractors should be significantly extended.

By introducing proposed solutions, the company should gain more flexibility in discussions regarding the selection of hydraulic components from the sales offer with potential partners, as well as with the regular customers. This will also allow to adapt to the requirements of the customers in relation to the finished product.

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BADANIE SATYSFAKCJI KLIENTA I MOŻLIWOŚĆ ZASTOSOWANIA METODY QFD DO POPRAWY TEGO ZADOWOLENIA - ANALIZA PRZYPADKU

Streszczenie: W pracy przedstawiono informacje dotyczące metod pomiaru zadowolenia klienta. Zaprezentowano formularz do badania ankietowego satysfakcji klientów. Przedstawiono wyniki badania ankietowego dotyczącego poziomu satysfakcji odbiorców elementów hydraulicznych. Poddano analizie możliwość zastosowania metody QFD w celu poprawy zadowolenia klienta. Przeprowadzono analizę dla złączki hydraulicznej w celu identyfikacji kluczowych aspektów wyrobu dla klienta. Podjęto próbę sformułowania przykładowych rozwiązań wpływających na poziom zadowolenia klienta w badanej organizacji.

Słowa kluczowe: zarządzanie jakością, badanie satysfakcji klienta, badanie ankietowe, kwestionariusz ankiety, metoda QFD.

QUALITY ENGINEERING TOOLS IN PRODUCTION PROCESS IMPROVEMENT

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Abstract: This article highlights the important role of quality engineering tools in process improvement. As an example, paper production has been selected for analysis. In order to improve the situation, two tools were proposed. In the first place, the ABC method was used to identify the defects that generate the highest costs in the manufacturing process. The Ishikawa Diagram was used for these defects, which allowed to determine the causes of critical defects occurring in the analyzed manufacturing process.

Keywords: quality, tools, engineering, Pareto analysis, Ishikawa diagram, continuous improvement.

1 Introduction

Quality engineering allows to analyze the manufacturing processes to maximize the quality of these processes and the products that result from them. In order to gain competitive advantage, companies use different approaches and management concepts. Some of them have focused their attention on continuous improvement as the basis of many concepts, including quality management or total quality management. Quality management in the enterprise will not bring the expected results without the practical application of methods and tools to improve company's processes and products [4]. The choice of these instruments should not be reflexive, but rather depend on the situation with which we are dealing. The aim of this paper is to present the possibility of improving the production process in company X using selected quality engineering tools. In order to improve current situation in the production process of the selected company the ABC method and the cause-effect diagram were proposed.

2 Using quality tools in paper production process

Analyzed company X produces paper directly from wood which may make the production system complex and time consuming. It consists of seven processes. It starts with the proper processing of wood, its grinding and transformation into pulp. There are then processes to convert the pre-paper pulp into ready-made office paper bundles. Production ends with receiving a paper office bale from the paper machine, which is then transferred to further processing (cutting) or to the warehouse. In the discussed process, defects were identified at subsequent stages of the manufacturing process. The defects at the level of individual operations are shown in Table 1.

Table 1. Defects occurring at the level of particular operations of the wood production process

PROCESS		OPERATION		DEFECT	
A	Sorting of wood	A1	Weighing of the wood	A1	Maximum permissible wood weight exceeded
		A2	Measuring the diameter of a tree	A2	Too little wood diameter
B	Removal of bark from wood rollers	B1	Mechanical removal of bark from wood rollers	B1	Not properly cleaned wood rollers
C	Cutting rollers on chips	C1	Mechanical cutting of wood trunks on wood chips	C1	Too big chips
		C2	Sorting of wood chips in sorting sieves	C2	Not properly sorter chips
D	Spinning on paper pulp	D1	Mechanical milling of wood chips using water	D1	Too thick paper pulp
		D2	Mass heating and dissolving	D2	Incorrect mass consistency
E	Bleaching process	E1	Bleaching with bleach	E1	Inadequately bleached mass
F	Machining process in paper machine	F1	Formation	F1	Insufficiently filled form
		F2	Ironing the ribbon	F2	Too thick ribbon
		F3	Drying	F3	Wet paper ribbon
G	Bale forming process	G1	Cut and wind the paper ribbon on the roll	G1	Inappropriate cutting of paper

Source: [2]

A Pareto analysis was used as a tool to prioritize defects in paper production, the use of which is described in the next chapter.

2.1 Application of Pareto analysis

Pareto analysis uses the empirically established regularity that approximately 70% -80% of the effects are caused by about 20% -30% of the causes. This rule, known as 20-80 rule or Pareto rule, was discovered by the Italian economist and sociologist Vilfredo Pareto (1848-1923). As part of his study, he analyzed the distribution of income in Italian society [6]. The Pareto Diagram represents in decreasing order the relative contribution of each factor (cause) to the total effect (on the occurrence of the problem). It enable to focus on corrective or improvement actions for the most important reasons. The Pareto diagram is often supplemented by the Lorenz graph, which presents the dependencies analyzed in the cumulative diagram [1]. According to the Pareto concept all elements of the studied area are divided into three groups A, B, C [6]. Accordingly, Pareto analysis is also called ABC analysis. The characteristics of each group are as follows [5]:

1. Group A - the most important elements belong to the group. By taking about 5% -20% of the total number of elements, they contribute to about 75% -80% of the value of the analyzed phenomenon. The actions taken should focus mainly on factors in this group. Such activities bring the greatest benefits.
2. Group B – elements of medium significance belong to the group. By taking about 20% -30% of the total number of elements, they contribute to about 10% -20% of the value of the analyzed phenomenon. The actions taken on the elements of this group will result in much less effect than the actions taken on the elements from group A.

3. Group C - the elements of the least importance. By taking about 50% -75% of the total number of elements, they contribute only 5% -10% of the value of the analyzed phenomenon. Actions taken in relation to elements belonging to this group may often not be economically justified.

In the analyzed example, it was decided to get to know the defects generating the highest costs in the paper production process. For this purpose, the ABC analysis was used. Firstly, data were collected on the number of defects occurring in particular operations of the production process and generated costs related to their correction. Based on the collected data, the total and cumulative cost of the correction was calculated for each defect. This allowed us to classify the individual defects into a particular group to determine the defects that are the most costly for the company X. To attribute defects to individual groups, it was assumed that:

- Group A generates 80% of the total cost of the correction, i.e. 1 248 470 PLN x 80% = 998 776 PLN,
- Group B generates together with group A 95% of the total cost of the correction, i.e. 1 248 470 PLN x 95% = 1 186 046. 5, PLN
- Group C is the remaining defects.

The results of the ABC analysis for the identified defects in the paper production process at Company X are shown in Table 2.

Table 2. Classification of defects using ABC analysis

	Defect	Number of occurrences	Unit cost of correction [PLN]	Total cost of correction [PLN]	Cumulative cost of correction [PLN]	Group
F2	Too thick ribbon	68	3100	210800	210800	A
C2	Not properly sorter chips	56	3000	168000	378800	A
D2	Incorrect mass consistency	27	4850	130950	509750	A
G1	Inappropriate cutting of paper	39	3000	117000	626750	A
F1	Insufficiently filled form	40	2900	116000	742750	A
B1	Not properly cleaned wood rollers	50	2200	110000	852750	A
E1	Inadequately bleached mass	29	3780	109620	962370	A
D1	Too thick paper pulp	20	4600	92000	1054370	B
C1	Too big chips	25	3600	90000	1144370	B
F3	Wet paper ribbon	12	5000	60000	1204370	C
A1	Maximum permissible wood weight exceeded	25	1500	37500	1241870	C
A2	Too little wood diameter	33	200	6600	1248470	C

Source: [2]

Graphical presentation of Pareto analysis along with Lorenz graph for the cumulative cost of defect correction is presented in Figure 1.

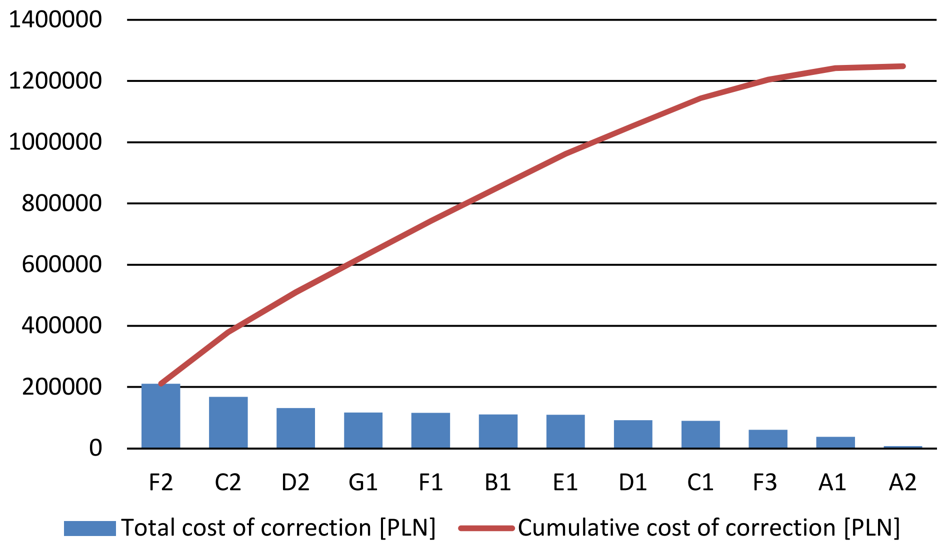

Fig. 1. Pareto-Lorenz chart for identified defects in the paper production process
Source: [2]

The analysis shows that the defects that generate the highest costs (80% of total costs), i.e. the defects included in Group A in Company X, are:

- F2 - too thick ribbon,
- C2 - not properly sorted chips,
- D2 - bad consistency of mass,
- G1 - inadequate paper cutting,
- F1 - insufficiently filled form,
- B1 - inaccurately cleaned wood rollers,
- E1 - insufficiently whitened mass.

The results also show that two of the identified defects of A Group are related to the same stage of the production process - paper machining. These are defects F1 (insufficiently filled form) and F2 (too thick ribbon).

After identifying the defects generating the highest costs in the production process, it was decided to identify the main causes of their occurrence. For this purpose, a cause-effect diagram, also known as the Ishikawa Diagram, was used.

2.2 Application of Ishikawa Diagram

The Ishikawa Diagram allows to present in a structured way, in graphical form, a set of factors affecting the outcome of the process, a set of causes generating a problem [1]. Using the cause-effect diagram, the knowledge of experts, operators, employees is used to produce a diagram, which organizes the knowledge of a specific, strictly defined problem and gives it a clear structure. The cause-effect diagram is known as the Ishikawa diagram or the fishbone diagram. The fish head is the goal (effect) we have achieved, and the reasons that interfere or help it are grouped into groups of interrelated issues, presented on the main axes [3]. Usually, problems are sorted according to the "5M" concept (Method, Material, Man, Machine, Management). So, the Ishikawa diagram allows to identify the causes of the problem, allows to prioritize them (assess their impact on the appearance of nonconformities), and facilitate the determination of appropriate corrective measures for the problem being analyzed.

In the example shown, the analysis using the Ishikawa Diagram was subject to all the defects from A Group. For the purposes of this article, diagrams for F2 and C2 defects are presented. Enterprise X was involved in a brainstorming session in which employees directly involved in the production process were involved to identify possible causes of defects in the paper production process. The identified causes were grouped into categories according to the 5M concept. Complete cause-effect diagrams for the two selected defects from A Group are shown in Figures 2 and 3.

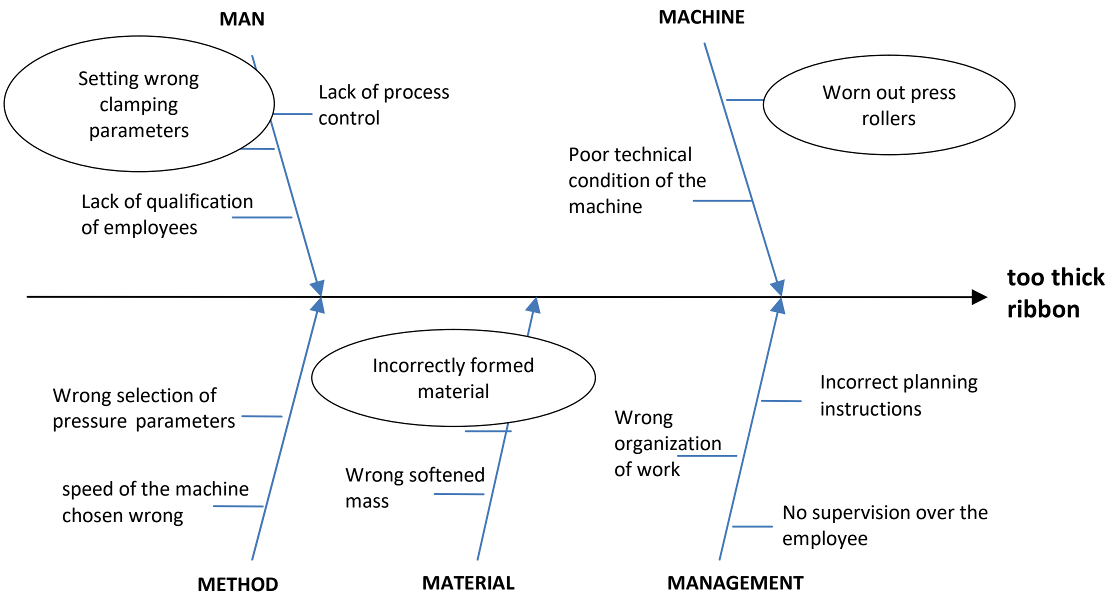

Fig. 2. Ishikawa Diagram for F2 defect - too thick ribbon
Source: [2]

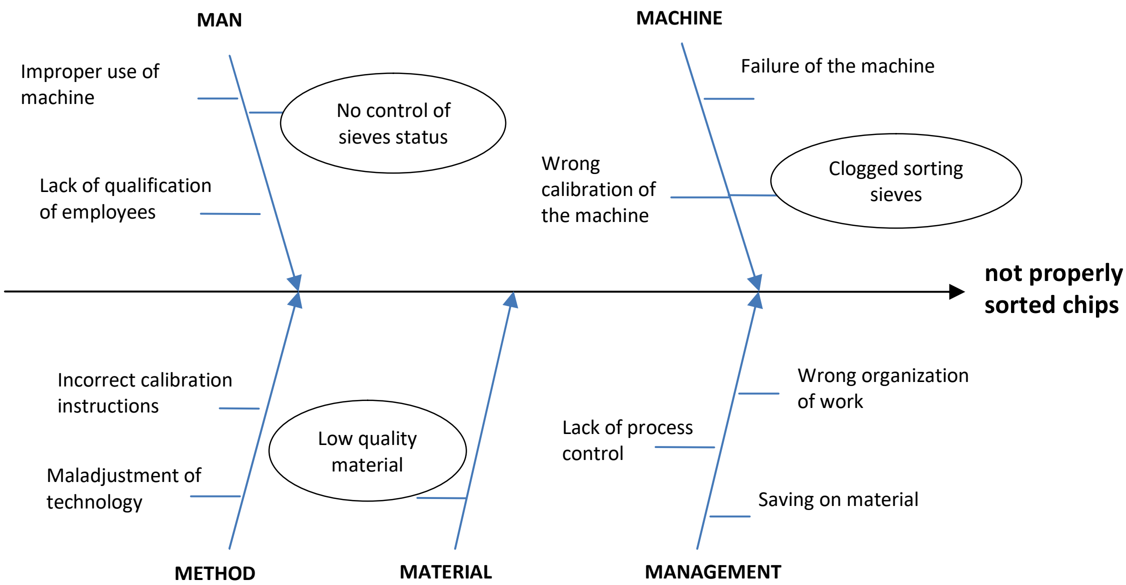

Fig. 3. Ishikawa Diagram for C2 defect - not properly sorted chips
Source: [2]

The analysis of the diagram allowed for the identification of critical causes, i.e. those having the greatest impact on the development of defects. For critical causes, corrective measures have been proposed to prevent re-occurrence of defects in the paper production process which generate the highest costs. The use of the Ishikawa diagram for the F2 defect (too thick ribbon) has allowed to determine the following critical causes: category people - setting wrong clamping parameters, machine category – worn out press rollers and material category - incorrectly formed material. Critical causes for C2 defect (not properly sorted chips) have been identified: in the category of people - no control of sieves, in the machine category - clogged sorting sieves, and in the material category - low quality material. After some time the corrective measures have been applied for the identified critical causes for A Group defects the Pareto analysis and the Ishikawa diagram should be repeated to identify new problems that have not been previously identified.

Conclusion

For companies that aim to produce high quality products, an indispensable part of running a business should be continuous improvement of the processes and products that result from them. In the process of improvement, a variety of methods and tools are used to address the different stages of the product life cycle. Quality engineering comes with a set of tools that help to improve the technical aspects of the quality of processes and products. This article presents an example of using two selected tools to improve the paper production process. In the first place, using the Pareto analysis, the defects were classified in terms of their relevance to the company considering the costs they generated. In the second step, the cause of these defects was identified using the Ishikawa diagram and the critical causes of the defects in the paper production process were identified. Correctly applied corrective actions will improve the situation and reuse of the proposed tools will allow continuous improvement of the production process.

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NARZĘDZIA INŻYNIERII JAKOŚCI W DOSKONALENIU PROCESU PRODUKCYJNEGO

Streszczenie: W niniejszym artykule wskazano na istotną rolę narzędzi inżynierii jakości w doskonaleniu procesów. Jako przykład do analizy wybrano proces produkcji papieru. W celu usprawnienia zaistniałej sytuacji zaproponowano zastosować dwa narzędzia. W pierwszej kolejności wykorzystano metodę ABC aby zidentyfikować te wady, które generują najwyższe koszty w procesie wytwórczym. Następnie dla tych wad zastosowano diagram Ishikawy, który pozwolił na określenie przyczyn krytycznych wad występujących w analizowanym procesie wytwórczym.

Słowa kluczowe: jakość, narzędzia, inżynieria, analiza Pareto, diagram Ishikawy, ciągłe doskonalenie.

IDENTIFICATION OF GAS HAZARDS IN A INDUSTRIAL COMPANY WITH USING CHAIN OF EVENTS ANALYZES

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Abstract: The purpose of this article is to present the process of hazard identification as a sequence of events that leads to an accident and / or material loss at the workplace. An event chain can be described as an ordered set of circumstances conducive to the emergence of a threat. The article presents an analysis of the use of elements of the event theory to identify hazards in a chemical plant on the example of a gas explosion event. An analysis of the circumstances surrounding the emergence of a gas explosion hazard by identifying the causes of direct and indirect events and the main conditions leading to the event is also presented. Also identified are events that are responsible for initiating a gas explosion hazard.

Keywords: chain of events, sequences of events, gas hazards, security deficits.

1 Introduction

For risk assessment with work in an industrial plant there is a need to identify all potential hazards. In the analysis of such risks, the most difficult to detect are certain sequences of events or conditions that collectively lead to an increased probability of undesirable results. The cause of this effect is a chain of events leading to loss (loss). In the chain of events, all the parameters of the work environment and the human factor, which are an important factor of potential hazards in the workplace, are taken into account.

The use of events theory elements to identify hazards depends on the analysis of a chain of events culminating in the accident or material damage or occupational disease. Analysis of such a chain of events makes it possible to identify favorable conditions for the creation of hazardous situations. That is why a deep analysis of the circumstances preceding the hazardous event must be made.

2 Environment parameters and gas security

The work environment is defined as a set of objects associated crew organized to produce specific values in the work process. Parameters of the work environment that relates to its objects have the biggest influence work safety. The direct or indirect effects of work environmental parameters on crew and the operations of an industrial plant can be expressed as follows:

- physical parameters associated with a mining environment such as the magnitudes of critical temperatures, pressures, concentration of toxic or explosive gases, velocities of ventilation flows, etc.;
- geometrical parameters including the area of the fire zone, the size of the gas installation, the volume used in the production of gases, the location of the gas tanks,
- pollution of the ventilation air stream by gases and/or dusts [5].

In industrial areas, in particular in the chemical industry where gas hazards are prevalent, the crew may encounter direct or indirect contact with such events as: sudden release of toxic or explosive gas into the environment, explosion or fire [7].

When the parameters of the work environment, where crew is located, are approximately constant or slightly changing, then it may be called normal conditions. Normal working conditions generally entail a relatively constant relationship between the conditions of the work environment and the location. Emergency conditions, on the other hand, usually entail sudden and significant changes in the conditions of the work environment, including such events as a sudden increase in temperature, air pressure, increase of increase in concentration toxic and explosive gases. The intrinsic nature of the chemical plant is such that providing completely comfortable conditions is impossible [7]. Therefore, existing safety standards in the chemical plant represent a compromise between working comfort and production requirements. It is, however, expected that full safety measures be provided for all of the hazards known to be associated with an ongoing chemical plant operation. Security standards for the conditions of the work environment are determined by mandatory safety regulations [12, 13].

3 Elements of technical prevention used in gas safety

In enterprises where gas appliances are used, a gas hazard signaling system is required. The principle of operation and elements that create the gas signaling system at the workplace are consistent with the fire alarm system. The purpose of such a system is primarily to detect and signal dangerous concentrations of monitored gas. The remaining tasks of the system are to alert employees of potentially explosive and fire hazards (fire protection devices) and to initiate countermeasures to reduce the risk [8].

The gas signaling system consists of the following elements: signaling panel, gas sensors (in the form of electrochemical sensors), alarms, manual fire alarm and guard lines. All elements forming the signaling system are subject to mandatory certification.

The gas fire alarm control panel is a decision-making device that coordinates the operation of the entire signaling system. The main tasks of modern signaling panels are:

- receiving signals from attached detectors and manual fire alarm detectors,
- determining which of the received signals meet the criteria of the alarm and informing people in an optical and acoustic manner of danger,
- transmission by the transmission equipment of the alarm signal to the monitoring station or to the fire brigade,
- indication of the location of the hazard,
- depending on the functionality, the commissioning of neutralizing devices,
- supervision of the functioning of the whole plant, including control of cooperating fire protection devices and signaling of damage,
- logging events occurring in the system [2, 8].

Activation of the alarm signaling should be initiated within a maximum of 10 seconds after starting the manual fire alarm or after the detector has started. This time is necessary for the exchange of information between the control panel and the fire detectors on the surveillance line. The function of the control panel is also the activation of external alarms. Other common functions of the control panel are: detection and indication of the danger area and control of the system reliability (detection and reporting of defects). Alarm control panel may have a two-stage alarms - alarm cycle and secondary alarm, if the installation relates to a highly toxic gas or explosive [8].

Gas sensors according to International Union of Pure and Applied Chemistry are devices that process chemical information (concentration of a particular component of a sample) into an analytically useful signal. The chemical sensor contains two basic elements: a chemically selective detector layer and a transducer element. The transmitter's main task is to convert the measured parameter into an electrical, optical or acoustic signal. The increasing popularity of chemical sensors is mainly due to the choice of the optimum method of measurement and the optimum solution for the technological process [7, 8].

All components of the gas signaling system in an industrial plant are operated as intrinsically safe. For the safe operation of the gas system and monitoring system, there is a requirement for periodic calibration of sensors. In order to ensure gas safety and avoid major industrial accidents or reduce the impact of an accident, the following technical security systems are most commonly used:

- sprinkler system in production halls,
- water supply network with water hydrants - terrestrial,
- fire alarm system, lightning protection and static electricity,
- safety valves on technical gas installations,
- emergency stop buttons for technological processes and fire extinguishers,
- double supply of particularly important components of the production plant,
- installations (nitrogen or other) for safety and firefighting purposes,
- pressure switches for pumps and handheld gas analyzers for staff equipment [7, 8, 9].

In addition to technical security systems, the company also has a number of warning signs and signals, the main task of which is to inform employees of the hazards present in the workplace. All premises where gaseous hazards are present should be marked with special safety markings or colors in accordance with the general health and safety at work regulations. Additional security is the use of sound and light signals located inside and outside the objects [12].

4 Events theory elements with relation to the work environment

Events in the work environment can be attributed to the elements of the sequence of events. Events occurring in the work environment are assigned two logical values, 1 or 0. The logical value 1 is assigned to the occurring events (true events), while the logical value 0 is assigned to events that do not occur. The description of events in the work environment uses basic logical functions, such as conjunction, alternative, negation, implication and equivalence. In addition, logical laws are used to describe events according to mathematical logic [1, 11].

The working environment can be considered as a set of elementary events. All events occurring in the environment can be divided into static events, signifying states, and kinetic events, signifying changes in these states. The kinetic events are the cause and static events are results of a sequence of conditions [3]. Besides the elementary events are macro- and micro-complex events, with varying degrees of complexity consisting of environmental subsets. Complex events consist of certain number of static and kinetic events occurring simultaneously and/or one after the other.

They represent a specific process taking place in the work environment. In certain circumstances, crew activities can directly or indirectly cause of an activation of a specified hazard [6, 10].

The sequence of events determines the principle: every effect is clearly and sufficiently appointed by the general causes and conditions in which it occurs. A sequence of events illustrates causes and effects in the work process. A set of events immediately preceding the change (qualitative and quantitative) presents a sufficient conditional sequence of events. A sufficient condition-specific effect consists of:

- principal cause and conditions (fixed);
- side conditions (random).

Principal conditions occur whenever they are a necessary condition for a result representing a qualitative change [1, 3, 10]. For example, spark or high temperature and explosive gas cloud are the cause and the main condition which are necessary to initiate a gas explosion. Side conditions in a sequence of conditions are random variables that can make the accident more or less likely or affect the size, the course and range of the event. For example, when gas explodes, side conditions determining its strength and range are: the size of the room that determines the growth of the dangerous concentration, the proportion of other volatile components, etc.

Phenomena occurring in the work environment can be described by using the chain events model. A model of such an events chain is well illustrated by dominoes blocks, standing side by side. Knocking over all of the dominoes requires the toppling of the first block, which knocks over the second domino, and so on, until the last. In order for the dominoes to fall, the toppling of the first domino must appear as a factor initiating the entire sequence of events [4, 6].

Relevant combinations of necessary event sequences in the work environment of the chemical plant can be events both in terms of work environment parameters (materials factors) and the human factors (actions and decisions). The scope of activity of the chemical plant is the cause of the hazards of the specified work environment parameters, their change, the processes that affect them and finally, the activities and states on the side of the crew, are the effects of their action. For example, the effect of an action might be: crew members present in a particular place, use ordered technologies under specified conditions. Uncontrolled event sequences occurring in the workplace, on the side of the work environment parameters and the human factors side, can lead to the initiation of the full hazard, that is to say, to undesired processes immediately preceding the harmfulness . The necessary events chain preceding the accident shows the arrangement of subsequent indirect effects and necessary reasons for remaining in the causal relationship [4].

These processes, in which events are considered due to their arrangement, can be assigned to an image geometry, called a graph. A graph is a topological mapping of an events sequence, defining unequivocally the relationships between the individual events. In the graph, nodes represent the necessary conditions of the events sequence, and the branches oriented towards the implication represent indirect results, that can turn into causes in in the nodes and/or principal conditions of the event sequence [1].

5 The results of the analysis of the chain of events for the gas explosion

In order to identify the cause of an explosive gas event with the effect of an accident and material failure SM, an undesired sequel of events prior to a gas explosion should be analyzed. As mentioned above, there is a risk of material damage associated with accidental hazards, which may accompany some accidents. Accident at work WY or / and material injury SM implies trauma UR and a chain of conditions necessary of sequence of events in the full-risk phase. The essential components of the conditions necessary for the initiation of the full-risk phase are:

- uncontrolled processes, uncontrolled parameter changes or uncontrolled crew operations,
- activities currently performed by the crew,
- technical condition of the gas installation,
- the influence of the crew on the course of technological processes,
- the influence of the management on the maintenance of the chemical plant [14, 15].

To determine whether a particular event is an essential component of the chain of necessary conditions, it should be considered whether without this event it would be possible to consider the consequence of events.

For the analyzed hazard: gas explosion EG, essential chain components of the conditions necessary in the full hazard phase are:

- gas leaks on the valve - WGz,
- unsealing of the tank - RZ,
- leakage of other components of the gas installation – NI.

A gas explosion may occur when two conditions are present: the occurrence of a spike ZI or a high temperature ZT with the simultaneous leakage of gas from the gas system. Event string Π (EG) accompanying the hazard of gas explosion EG is the following set of events Zz:

$$EG \Rightarrow WY \wedge SM \Rightarrow UR \Rightarrow \Pi(EG) \equiv Zz \equiv \{[ZI \quad \vee ZT] \wedge [WGz \ \vee RZ \vee NI \quad]\}$$

The expanded chain of events is as follows:

$$\begin{aligned} EG \Rightarrow WY \wedge SM \Rightarrow UR \Rightarrow \Pi(EG) \equiv \{Zz \\ \equiv \{[ZI \quad \vee ZT] \wedge [WGz \quad \equiv \{Ae \vee Uz \vee Bs \equiv \{no\}\}] \vee [RZ \\ \equiv \{Pp \wedge Bsy \equiv \{ac \vee no\} \wedge Wc \equiv \{np \vee ns\}] \vee [NI \equiv \{Og \wedge Bsy\}]\} \Rightarrow \\ no \wedge ac \wedge np \wedge ns, \end{aligned}$$

where
Ae - electrovalve failure
Uz - external damage
Bs - control error
no - operator inattention
Wc - increase in pressure or temperature in the tank
Bsy - no pressure sensor signal
Pp - production process
eg - abnormal chemical process
ns - incorrect control
ac - sensor failure
Og - the presence of gas in the installation.

The analysis of the event chain for the incident and material loss in the form of gas explosion EG has identified three direct causes: WGz, RZ, NI and five indirect causes: Uz, Bs, Wc, Bsy, Ae. In the event chain, three main conditions were also identified for the analyzed event: Pp, Og, WG, and four first cause: ac, np, ns no. The above analysis points to human errors committed in the control and control process of the production process and minor faults, such as the failure of the gas concentration signaling sensor, which are the first cause of the analyzed event and can lead to serious consequences.

Conclusion

Building a security system aimed at eliminating harmfulness and identifying a relative hazard requires that all components of the essential necessary sequences of events preceding the effects (losses) in the chemical plant be identified. For this purpose, an analysis of the chain of preconditions preceding the damage is carried out. The use of elements of event theory to identify hazards very clearly shows the complexity of the causes of the damage (loss). Such analysis gives a broad understanding of the factors (indirect and direct) influencing such events as: accident, material loss at the workplace. When analyzing the chain of events, the threats that cause the intermediate and final effects and the causal link between the causes of the losses in the chemical plant are identified.

The unwanted sequel of the events preceding the gas explosion at the chemical plant is the essence of the gas hazard present, posing a certain risk to the production process. The risk consists of 8 causes (indirect causes and principal conditions) which may be determined by the 15 possible security deficits (deviations from the prescribed safety levels on the parameters of the work environment) and 5 possible deviations on the human side.

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IDENTYFIKACJA ZAGROŻEŃ GAZOWYCH W PRZEDSIĘBIORSTWIE PRZEMYSŁOWYM Z WYKORZYSTANIEM ANALIZY ŁAŃCUCHA ZDARZEŃ

Abstrakt (Streszczenie): Celem artykułu jest przedstawienie procesu identyfikacji zagrożeń jako łańcucha następstwa zdarzeń, które prowadzi do wypadku i/lub straty materialnej na stanowisku pracy. Łańcuch zdarzeń może być opisany jako uporządkowany zbiór okoliczności sprzyjających pojawieniu się zagrożenia. W artykule przedstawiono analizę zastosowania elementów teorii zdarzeń do identyfikacji zagrożeń w zakładzie chemicznym na przykładzie zdarzenia wybuchu gazu. Zaprezentowano również analizę okoliczności sprzyjających powstaniu zagrożenia wybuchem gazu poprzez wyznaczenia przyczyn bezpośrednich i pośrednich zdarzenia oraz warunków głównych prowadzących do zdarzenia. Wskazano również praprzyczyny zdarzenia, które stanowią czynnik inicjujący zagrożenie wybuchem gazu.

Klíčová slova (Słowa kluczowe): łańcuch zdarzeń, następstwo zdarzeń, zagrożenia gazowe, deficyty bezpieczeństwa.

Systems of independent sets - results and conjectures

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Abstract: The system of independent sets is such a system of sets of natural numbers where the asymptotic density of the intersection of its arbitrary finite subsystem is equal to the product of asymptotic densities of sets which belong to this subsystem. Basic properties of systems of independent sets and some results and conjectures concerning them are described here.

Keywords: asymptotic density, system of independent sets.

1 Introduction

Terms probability, independence of random events and independence of random variables belong among basic terms of the probability theory, which are plentifully used in practice.

Lots of mathematical theorems and ideas are based on the assumption of independence of random events and random variables. For example, such assumptions are important in definitions of chi-squared distribution, binomial distribution etc. The well known central limit theorem, which describes distribution of the sum of independent random variables, is used in practice for approximation of various calculations.

From the other side, one of the most frequently used version of chi-square test verifies the hypothesis about independence of random variables. Concepts asymptotic density and system of independent sets are analogies of terms probability and independent events in number theory (see [8], Chapter 4).

The classical probability that a randomly chosen number from the set $\{n \in \mathbb{N} :\ n \leq n_0\}$ belongs to a set $A \subseteq \mathbb{N}$ can be often approximated for large number

n_0 by the asymptotic density of the set A.

Asymptotic density of the set $A \subseteq \mathbb{N}$ is defined as follows.

Definition 1.1. *Let $A \subseteq \mathbb{N}$. Number of elements of the set $A \subseteq \mathbb{N}$ which are less or equal to n is denoted by $A(n)$. Number $\bar{d}(A)$, where*

$$\bar{d}(A) = \limsup_{n\to\infty} \frac{A(n)}{n},$$

is called upper asymptotic density of set A and number $\underline{d}(A)$ where

$$\underline{d}(A) = \liminf_{n\to\infty} \frac{A(n)}{n}$$

is called lower asymptotic density of set A.

If $\bar{d}(A) = \underline{d}(A)$, then value $d(A) = \bar{d}(A) = \underline{d}(A) = \lim_{n\to\infty} \frac{A(n)}{n}$ is called asymptotic density of set A.

We immediately obtain from the definition the following basic properties of asymptotic density, which are further used:

- The upper and lower asymptotic density of set $A \subseteq \mathbb{N}$ certainly exists, but the asymptotic density of A may not.
- $0 \leq \underline{d}(A) \leq \bar{d}(A) \leq 1$
- $d(\mathbb{N}) = 1$

2 Systems of independent sets – results

The term "system of sets with multiplicative asymptotic density" was introduced in paper [3] in 2008. A more general term "system of independent sets" was later introduced in paper [5]. Its basic properties and particular examples were studied in [4], [6] and [5]. System of independent sets is an analogy of systems of independent events which are known from probability theory – the probability of their intersection is equal to the product of their probabilities. The system of independent sets is such a system of sets of natural numbers where the asymptotic density of the intersection of its arbitrary finite subsystem is equal to the product of asymptotic densities of sets which belong to this subsystem.

Definition 2.1. *Let I be a set and $\{A_i\}_{i\in I}$ be a system of sets where $A_i \subseteq \mathbb{N}$ and $d(A_i)$ exists for every $i \in I$. System of sets $\{A_i\}_{i\in I}$ is a system of independent sets if for every finite set $F \subseteq I$ holds*

$$d\left(\bigcap_{i\in F} A_i\right) = \prod_{i\in F} d(A_i).$$

A system of sets with the multiplicative asymptotic density was in [3] defined for $I = \mathbb{N}$. According to the above mentioned definition, systems of independent sets, contrary to systems of sets with the multiplicative asymptotic density, may also be definite, or uncountable. However, this generalization keeps in effect the Theorem 2.1 from article [3] also for systems of independent sets. We can reformulate it this way:

Theorem 2.1. *System of sets $\left\{A_i\right\}_{i\in I}$ is a system of independent sets if and only if the system $\left\{\mathbb{N} - A_i\right\}_{i\in I}$ is a system of independent sets.*

This criterion was generalized in [5] as follows:

Theorem 2.2. *The system $\{A_i\}_{i\in I}$ is a system of independent sets if and only if the system $\{B_i\}_{i\in I}$ where for every $i \in I$ holds $B_i = A_i$ or $B_i = \mathbb{N} - A_i$ is a system of independent sets.*

In paper [7], we have among others studied properties of a system of certain sets where set of natural numbers which are expressible in the form of sum of two squares of integers is included (you can see [2], [9], [10] for more information about this interesting set).

Let us denote $p_1 < p_2 < \cdots < p_i < \dots$ the sequence of prime numbers in form $p_i = 4k+3$, where $k \in \mathbb{N}\cup\{0\}$ and $A^{2j}_{p_i} = \{m.p_i^{2j} \mid j \in \mathbb{N}\cup\{0\}, m \in \mathbb{N}, \gcd(m, p_i) = 1\}$. It means, that $A^{2j}_{p_i}$ is a set of those natural numbers, which have not the prime number p_i with odd exponent in their canonical decompositions. It is well known (see [1]), that set D cotaining all natural numbers, which are expressible in the form of sum of two squares of integers is equal to the intersection of sets $A^{2j}_{p_i}$, $i = 1, 2, \dots$, ie. $D = \cap_{i=1}^{\infty} A^{2j}_{p_i}$.

We have proved following two statements in [7]:

Lemma 2.1. *Let us denote $p_1 < p_2 < \cdots < p_i < \dots$ the sequence of prime numbers in form $p_i = 4k + 3$ where $k \in \mathbb{N} \cup \{0\}$ and $A^{2j}_{p_i} = \{m.p_i^{2j} \mid j \in \mathbb{N} \cup \{0\}, m \in \mathbb{N}, \gcd(m, p_i) = 1\}$. Then for asymptotic density of the set $A^{2j}_{p_i}$ holds*

$$d(A^{2j}_{p_i}) = \frac{p_i}{p_i + 1}.$$

and

Lemma 2.2. *Let $A^{2j}_{p_i}$ be sets defined in the same way as in previous lemma. Then for every $n \in \mathbb{N}$ holds that*

$$d(\bigcap_{i=1}^{n} A^{2j}_{p_i}) = \prod_{i=1}^{n} d(A^{2j}_{p_i}) = \prod_{i=1}^{n} \frac{p_i}{p_i + 1}.$$

We can see that the asymptotic density of the intersection of sets $A_{p_i}^{2j}, \ldots, A_{p_n}^{2j}$ is equal to the product of their asymptotic densities. If we would make minor alterations in the proof of Lemma 2.2, then we would find that for arbitrary finite subsystem of system $\left\{A_{p_i}^{2j}\right\}_{i=1}^{\infty}$ holds that the asymptotic density of intersection of its sets is equal to the product of their asymptotic densities.

This led us to study sets

$$A_a^{\alpha_n} = \{a^{\alpha_n} m : \ m, n \in \mathbb{N}; a \nmid m\}$$

where $\{\alpha_n\}_{n=1}^{\infty}$ is given increasing sequence of non-negative integers and $a \in \mathbb{N} - \{1\}$.

It has been shown in paper [4] that asymptotic densities of sets $A_a^{\alpha_n}$ and $A_b^{\beta_n}$ exist and following theorem has been proven in [5].

Theorem 2.3. *Let $a, b \in \mathbb{N}$, $a, b \neq 1$, $\gcd(a,b) = d$ and $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ be increasing sequences of natural numbers. If $A_a^{\alpha_n} = \{a^{\alpha_n} m : \ m, n \in \mathbb{N}; a \nmid m\}$ and $A_b^{\beta_n} = \{b^{\beta_n} m : \ m, n \in \mathbb{N}; b \nmid m\}$, then*

$$d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{d^{\min\{\alpha_i, \beta_j\}}}{a^{\alpha_i} b^{\beta_j}} \left(1 - d(C_{ij}) - d(D_{ij}) + d\left(C_{ij} \cap D_{ij}\right)\right),$$

where

$$\begin{aligned} d\left(C_{ij}\right) &= \tfrac{\gcd(a \cdot \gcd(a^{\alpha_i}, b^{\beta_j}), b^{\beta_j})}{a \cdot \gcd(a^{\alpha_i}, b^{\beta_j})} = \tfrac{\gcd(a \cdot d^{\min\{\alpha_i, \beta_j\}}, b^{\beta_j})}{a \cdot d^{\min\{\alpha_i, \beta_j\}}}, \\ d\left(D_{ij}\right) &= \tfrac{\gcd(b \cdot \gcd(a^{\alpha_i}, b^{\beta_j}), a^{\alpha_i})}{b \cdot \gcd(a^{\alpha_i}, b^{\beta_j})} = \tfrac{\gcd(b \cdot d^{\min\{\alpha_i, \beta_j\}}, a^{\alpha_i})}{b \cdot d^{\min\{\alpha_i, \beta_j\}}}, \qquad (1) \\ d\left(C_{ij} \cap D_{ij}\right) &= d\left(C_{ij}\right) d\left(D_{ij}\right) \gcd\left(\tfrac{1}{d(C_{ij})}, \tfrac{1}{d(D_{ij})}\right). \end{aligned}$$

Example 2.1. *Let us consider the sets $A = \{6m \mid m \in \mathbb{N}\}$ and $B = \{15m \mid m \in \mathbb{N}\}$. It is clear, that their intersection is a set $A \cap B = \{30m \mid m \in \mathbb{N}\}$ and $d(A \cap B) = \frac{1}{30}$. We can also determine this value using the Theorem 2.3, but the procedure will be quite fiddly. The price, however, is in its universality. We can use it to determine (and we can be sure of its existence) the asymptotic density of the intersections of the more general sets than sets of multiples of given numbers.*

Let $A = \{6m \mid m \in \mathbb{N}\}$ and $B = \{15m \mid m \in \mathbb{N}\}$. Hence $A = A_a^{\alpha_n}$ a $B = A_b^{\beta_n}$, where $a = 6$, $b = 15$ and for every $n \in \mathbb{N}$ holds $\alpha_n = n$ and $\beta_n = n$.

According to the Theorem 2.3 holds

$$d(A \cap B) = \sum_{i=1}^{\infty} \sum_{i=j}^{\infty} d(X_{ij}), \qquad (2)$$

where

$$d(X_{ij}) = \frac{\gcd(a^{\alpha_i}, b^{\beta_j})}{a^{\alpha_i} b^{\beta_j}} \left(1 - d(C_{ij}) - d(D_{ij}) + d\left(C_{ij} \cap D_{ij}\right)\right), \tag{3}$$

By inserting specific values, we receive the following

$$\begin{aligned}
\frac{\gcd(a^{\alpha_i}, b^{\beta_j})}{a^{\alpha_i} b^{\beta_j}} &= \frac{\gcd(6^i, 15^j)}{6^i 15^j} &&= \frac{3^{\min\{i,j\}}}{6^i 15^j} \\
d\left(C_{ij}\right) &= \frac{\gcd(6 \cdot \gcd(6^i,\ 15^j), 15^j)}{6 \cdot \gcd(6^i, 15^j)} &&= \frac{\gcd(2 \cdot 3^{\min\{i,j\}+1},\ 5^j 3^j)}{2 \cdot 3^{\min\{i,j\}+1}} \\
d\left(D_{ij}\right) &= \frac{\gcd(15 \cdot \gcd(6^i, 15^j),\ 6^i)}{15 \cdot \gcd(6^i, 15^j)} &&= \frac{\gcd(5 \cdot 3^{\min\{i,j\}+1},\ 2^i 3^i)}{5 \cdot 3^{\min\{i,j\}+1}}
\end{aligned} \tag{4}$$

From (4) and (1) then follows

$$\frac{\gcd(a^{\alpha_i}, b^{\beta_j})}{a^{\alpha_i} b^{\beta_j}} = \begin{cases} (\textit{for } i > j) = \frac{3^j}{6^i 15^j} &= \left(\frac{1}{6}\right)^i \left(\frac{1}{5}\right)^j \\ (\textit{for } i = j) = \frac{3^i}{6^i 15^i} &= \left(\frac{1}{30}\right)^i \\ (\textit{for } i < j) = \frac{3^i}{6^i 15^j} &= \left(\frac{1}{2}\right)^i \left(\frac{1}{15}\right)^j \end{cases}$$

$$d\left(C_{ij}\right) = \begin{cases} (\textit{for } i > j) = \frac{\gcd(2 \cdot 3^{j+1}, 5^j 3^j)}{2 \cdot 3^{j+1}} &= \frac{1}{6} \\ (\textit{for } i = j) = \frac{\gcd(2 \cdot 3^{i+1}, 5^i 3^i)}{2 \cdot 3^{i+1}} &= \frac{1}{6} \\ (\textit{for } i < j) = \frac{\gcd(2 \cdot 3^{i+1}, 5^j 3^j)}{2 \cdot 3^{i+1}} &= \frac{1}{2} \end{cases}$$

$$d\left(D_{ij}\right) = \begin{cases} (\textit{for } i > j) = \frac{\gcd(5 \cdot 3^{j+1}, 2^i 3^i)}{5 \cdot 3^{j+1}} &= \frac{1}{5} \\ (\textit{for } i = j) = \frac{\gcd(5 \cdot 3^{i+1}, 2^i 3^i)}{5 \cdot 3^{i+1}} &= \frac{1}{15} \\ (\textit{for } i < j) = \frac{\gcd(5 \cdot 3^{i+1}, 2^i 3^i)}{5 \cdot 3^{i+1}} &= \frac{1}{15} \end{cases}$$

$$d\left(C_{ij} \cap D_{ij}\right) = \begin{cases} (\textit{for }, i > j) = \frac{1}{6}\frac{1}{5}\gcd(6,5) &= \frac{1}{30} \\ (\textit{for } i = j) = \frac{1}{6}\frac{1}{15}\gcd(6,15) &= \frac{1}{30} \\ (\textit{for } i < j) = \frac{1}{2}\frac{1}{15}\gcd(2,15) &= \frac{1}{30} \end{cases}$$

Substituting into the equation (3) we receive

$$d(X_{ij}) = \begin{cases} (\text{for } i > j) & = \left(\frac{1}{6}\right)^i \left(\frac{1}{5}\right)^j \frac{10}{15} \\ (\text{for } i = j) & = \left(\frac{1}{30}\right)^i \frac{12}{15} \\ (\text{for } i < j) & = \left(\frac{1}{2}\right)^i \left(\frac{1}{15}\right)^j \frac{7}{15} \end{cases} \tag{5}$$

For $k \in \mathbb{N}$ let us denote $S(k) = \sum_{j=1}^{\infty} d(X_{kj})$. Hence

$$\begin{aligned} S(1) &= 0 + d(X_{11}) + \sum_{j=2}^{\infty} d(X_{1j}) = \\ &= 0 + \left(\frac{1}{30}\right)^1 \frac{12}{15} + \left(\frac{1}{2}\right)^1 \frac{7}{15} \sum_{j=2}^{\infty} \left(\frac{1}{15}\right)^j = \\ &= 0 + \left(\frac{1}{30}\right)^1 \frac{12}{15} + \left(\frac{1}{2}\right)^1 \frac{7}{15} \left(\frac{1}{15}\right)^2 \frac{15}{14} = \\ &= \frac{1}{36} \end{aligned}$$

$$\begin{aligned} S(2) &= d(X_{21}) + d(X_{22}) + \sum_{j=3}^{\infty} d(X_{2j}) = \\ &= \left(\frac{1}{6}\right)^2 \left(\frac{1}{5}\right)^1 \frac{10}{15} + \left(\frac{1}{30}\right)^2 \frac{12}{15} + \left(\frac{1}{2}\right)^2 \frac{7}{15} \sum_{j=3}^{\infty} \left(\frac{1}{15}\right)^j = \\ &= \left(\frac{1}{6}\right)^2 \left(\frac{1}{5}\right)^1 \frac{10}{15} + \left(\frac{1}{30}\right)^2 \frac{12}{15} + \left(\frac{1}{2}\right)^2 \frac{7}{15} \left(\frac{1}{15}\right)^3 \frac{15}{14} \end{aligned}$$

$$
\begin{aligned}
S(3) &= d(X_{31}) + d(X_{32}) + d(X_{33}) + \sum_{j=4}^{\infty} d(X_{3j}) = \\
&= \left(\frac{1}{6}\right)^3 \frac{10}{15} \left[\left(\frac{1}{5}\right)^1 + \left(\frac{1}{5}\right)^2\right] + \left(\frac{1}{30}\right)^3 \frac{12}{15} + \left(\frac{1}{2}\right)^3 \frac{7}{15} \sum_{j=4}^{\infty} \left(\frac{1}{15}\right)^j = \\
&= \left(\frac{1}{6}\right)^3 \frac{10}{15} \left[\left(\frac{1}{5}\right)^1 + \left(\frac{1}{5}\right)^2\right] + \left(\frac{1}{30}\right)^2 \frac{12}{15} + \left(\frac{1}{2}\right)^2 \frac{7}{15} \left(\frac{1}{15}\right)^4 \frac{15}{14} \\
&\vdots
\end{aligned}
$$

In general, for $k \geq 2$ *we can write*

$$
\begin{aligned}
S(k) &= \sum_{j=1}^{k-1} d(X_{kj}) + d(X_{kk}) + \sum_{j=k+1}^{\infty} d(X_{kj}) = \\
&= \sum_{j=1}^{k-1} \left(\frac{1}{6}\right)^k \left(\frac{1}{5}\right)^j \frac{10}{15} + \left(\frac{1}{30}\right)^k \frac{12}{15} + \sum_{j=k+1}^{\infty} \left(\frac{1}{2}\right)^k \left(\frac{1}{15}\right)^j \frac{7}{15} = \\
&= \frac{10}{15} \left(\frac{1}{6}\right)^k \sum_{j=1}^{k-1} \left(\frac{1}{5}\right)^j + \left(\frac{1}{30}\right)^k \frac{12}{15} + \frac{7}{15} \left(\frac{1}{2}\right)^k \left(\frac{1}{15}\right)^{k+1} \frac{15}{14} = \\
&= \frac{10}{15} \left(\frac{1}{6}\right)^k \left[\frac{1}{5} \frac{1 - \left(\frac{1}{5}\right)^{k-1}}{1 - \frac{1}{5}}\right] + \left(\frac{1}{30}\right)^k \frac{24}{30} + \frac{1}{30} \left(\frac{1}{2}\right)^k \left(\frac{1}{15}\right)^k = \\
&= \frac{1}{6} \left(\frac{1}{6}\right)^k \left[1 - 5\left(\frac{1}{5}\right)^k\right] + \left(\frac{1}{30}\right)^k \frac{25}{30} = \\
&= \frac{1}{6} \left(\frac{1}{6}\right)^k - \frac{5}{6} \left(\frac{1}{30}\right)^k + \left(\frac{1}{30}\right)^k \frac{5}{6} = \\
&= \frac{1}{6} \left(\frac{1}{6}\right)^k
\end{aligned}
$$

Hence

$$
\begin{aligned}
d(A \cap B) &= \sum_{i=1}^{\infty} \sum_{i=j}^{\infty} d(X_{ij}) = \sum_{i=1}^{\infty} S(i) = S(1) + \sum_{i=2}^{\infty} \frac{1}{6} \left(\frac{1}{6}\right)^i = \\
&= \frac{1}{36} + \frac{1}{6} \frac{1}{6^2} \frac{1}{\left(1 - \frac{1}{6}\right)} = \frac{1}{36} \left(1 + \frac{1}{5}\right) = \mathbf{\frac{1}{30}}.
\end{aligned}
$$

3 Systems of independent sets – conjectures

We have defined sets

$$A_a^{\alpha_n} = \{a^{\alpha_n} m : \ m, n \in \mathbb{N}; a \nmid m\}$$

where $\{\alpha_n\}_{n=1}^{\infty}$ is given increasing sequence of non-negative integers and $a \in \mathbb{N} - \{1\}$.

Sequence $\{\alpha_i\}_{i=1}^{\infty}$ is an increasing sequence of non-negative integers, therefore we can choose $\alpha_i = i - 1$. Then $A_a^{\alpha_i} = \mathbb{N}$ and for an arbitrary set $A_b^{\beta_j}$ then holds

$$d\left(A_a^{\alpha_i} \cap A_b^{\beta_j}\right) = d\left(\mathbb{N} \cap A_b^{\beta_j}\right) = d\left(A_b^{\beta_j}\right) = d\left(\mathbb{N}\right) d\left(A_b^{\beta_j}\right) = d\left(A_a^{\alpha_i}\right) d\left(A_b^{\beta_j}\right).$$

We can see that sets $A_a^{\alpha_i}$ and $A_b^{\beta_j}$ form a system of independent sets. But this case is trivial and not very interesting. If sequences $\{\alpha_i\}_{i=1}^{\infty}$ and $\{\beta_j\}_{j=1}^{\infty}$ are increasing sequences of natural numbers, the above described construction of a system of independent sets fails because in this case $A_a^{\alpha_i} \subseteq \{am : m \in \mathbb{N}\} \neq \mathbb{N}$ and $A_b^{\beta_j} \subseteq \{bm : m \in \mathbb{N}\} \neq \mathbb{N}$.

The question is whether there exist numbers $a, b \in \mathbb{N} - \{1\}$ and sequences of natural numbers $\{\alpha_i\}_{i=1}^{\infty}$ and $\{\beta_j\}_{j=1}^{\infty}$ so that sets $A_a^{\alpha_i}$ and $A_b^{\beta_j}$ form a system of independent sets.

The asymptotic density of the intersection of set $A_a^{\alpha_n} = \{a^{\alpha_n} m : \ m, n \in \mathbb{N}; a \nmid m\}$ and of the set $A_b^{\beta_n} = \{b^{\beta_n} m : \ m, n \in \mathbb{N}; b \nmid m\}$, where $\{\alpha_n\}_{n=1}^{\infty}$ and $\{\beta_n\}_{n=1}^{\infty}$ are given increasing sequences of natural numbers and $\gcd(a, b) = d \in \mathbb{N}$ was investigated in paper [5].

It was stated that in case of $\gcd(a, b) = 1$ sets $A_a^{\alpha_n}$ and $A_b^{\beta_n}$ form a system of independent sets, but in general case $\gcd(a, b) = d \in \mathbb{N}$, sets $A_a^{\alpha_n}$ and $A_b^{\beta_n}$ may not form a system of independent sets.

We introduce following conjectures now.

Conjecture 3.1. *Let $a, b \in \mathbb{N}$; $a, b \neq 1$; $1 < \gcd(a, b) = d \neq \min\{a, b\}$ and $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ be increasing sequences of natural numbers. If $A_a^{\alpha_n} = \{a^{\alpha_n} m : \ m, n \in \mathbb{N}; a \nmid m\}$ and $A_b^{\beta_n} = \{b^{\beta_n} m : \ m, n \in \mathbb{N}; b \nmid m\}$, then*

$$d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) > d\left(A_a^{\alpha_n}\right) d\left(A_b^{\beta_n}\right).$$

and

Conjecture 3.2. *Let $a, b \in \mathbb{N} - \{1\}$, $1 < \gcd(a, b) = \min\{a, b\}$.*

1. *There exist sets $A_a^{\alpha_n}$ and $A_b^{\beta_n}$ fulfilling $d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) > d\left(A_a^{\alpha_n}\right) d\left(A_b^{\beta_n}\right)$.*
2. *There exist sets $A_a^{\alpha_n}$ and $A_b^{\beta_n}$ fulfilling $d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) < d\left(A_a^{\alpha_n}\right) d\left(A_b^{\beta_n}\right)$.*
3. *There exist sets $A_a^{\alpha_n}$ and $A_b^{\beta_n}$ fulfilling $d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) = d\left(A_a^{\alpha_n}\right) d\left(A_b^{\beta_n}\right)$.*

Conclusion

In next paper, using above mentioned results, we are ready to prove that sets $A_a^{\alpha_n}$ and $A_b^{\beta_n}$ do not form a system of independent sets in case $1 < \gcd(a,b) \neq \min\{a,b\}$. In particular, we will prove that inequality $d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) > d\left(A_a^{\alpha_n}\right) d\left(A_b^{\beta_n}\right)$ holds in this case.

Finally, we will prove that in case $1 < \gcd(a,b) = \min\{a,b\}$, all three possibilities may occur and these are $d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) > d\left(A_a^{\alpha_n}\right) d\left(A_b^{\beta_n}\right)$, $d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) < d\left(A_a^{\alpha_n}\right) d\left(A_b^{\beta_n}\right)$ and also $d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) = d\left(A_a^{\alpha_n}\right) d\left(A_b^{\beta_n}\right)$.

Hence in case $1 < \gcd(a,b) = \min\{a,b\}$ there exist sets $A_a^{\alpha_n}$ and $A_b^{\beta_n}$, which certainly do not form a system of independent sets, however there also exist sets $A_a^{\alpha_n}$ and $A_b^{\beta_n}$ which form a system of independent sets.

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Systems of independent sets and systems of conditionally independent sets

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Abstract: System of conditionally independent sets is a generalization of the term system of independent sets. Definitions and examples of such systems and the overview of generalizations of results which led to the introduction of the term system of conditionally independent sets is described here. Some known results which could be generalized are mentioned.

Keywords: asymptotic density, system of independent sets, system of conditionally independent sets.

1 Introduction

1.1 Denotation

We use common denotation in this paper. Denotation $a \mid b$ means that number a divides b and $a \nmid b$ means, that number a does not divide b. We denote the greatest common divisor of numbers $a_1, \ldots, a_n$ by $gcd(a_1, \ldots, a_n)$, the least common multiple of numbers $a_1, \ldots, a_n$ by $\mathrm{lcm}(a_1, \ldots, a_n)$, the number of elements of the set $A \subseteq \mathbb{N}$ which are less or equal to n by $A(n)$, the lower asymptotic density of the set A by $\underline{d}(A)$, the upper asymptotic density of the set A by $\overline{d}(A)$, the asymptotic density of the set A by $d(A)$, and the difference of sets A and B by $A - B$. Moreover, for given increasing sequence of non-negative integers $\{\alpha(k)\}_{k=1}^{\infty}$ and $a \in \mathbb{N} - \{1\}$ we denote

$$A_a^{\alpha(k)} = \{a^{\alpha(k)}m :\, m, k \in \mathbb{N}, a \nmid m\}.$$

1.2 Asymptotic density

The upper asymptotic density of the set $A \subseteq \mathbb{N}$ is defined as the number $\bar{d}(A) = \limsup\limits_{n\to\infty} \frac{A(n)}{n}$ and the lower asymptotic density of the set $A \subseteq \mathbb{N}$ is defined as the number $\underline{d}(A) = \liminf\limits_{n\to\infty} \frac{A(n)}{n}$ and the number $d(A) = \lim\limits_{n\to\infty} \frac{A(n)}{n}$ we call asymptotic density of the set A (if it exists).

Some of well known properties of asymptotic density, which we use below, are summed up in Lemma 1.1.

Lemma 1.1 *If $A \subseteq \mathbb{N}$, then holds:*

- *The upper and lower asymptotic density of set $A \subseteq \mathbb{N}$ exist, but the asymptotic density of A needs not.*
- $0 \leq \underline{d}(A) \leq \overline{d}(A) \leq 1$.
- $d(\mathbb{N}) = 1$.
- *If $d(A)$ exists, then $d(\mathbb{N} - A) = 1 - d(A)$,*
- *Let $c \in \mathbb{N}$, $B = \{ca : a \in A\}$. If $d(A)$ exists, then $d(B)$ exists too and*

$$d\left(B\right) = \frac{d(A)}{c}.$$

Proofs of these statements are trivial and follow immediately from the definition of the asymptotic density.

2 Systems of independent sets

The system of independent sets was defined as follows in [4].

Definition 2.1 *Let I be a set and $\{A_i\}_{i \in I}$ be a system of sets where $A_i \subseteq \mathbb{N}$ and $d(A_i)$ exists for every $i \in I$. System of sets $\{A_i\}_{i \in I}$ is a system of independent sets if for every finite set $F \subseteq I$ holds*

$$d\left(\bigcap_{i \in F} A_i\right) = \prod_{i \in F} d(A_i).$$

Asymptotic density of a set A_i can be interpreted as an estimation of the probability that we randomly choose an element from A_i in the set $\{1, 2, \ldots, n\}$ where n is "large" (see [7]). From this point of view, we can say that the events "to choose an element from A_i" are independent (within the meaning of the probability theory) in case of $\{A_i\}_{i \in I}$ is a system of independent sets.

Trivial example of system of independent sets is a system formed by sets $A_i \subseteq \mathbb{N}$, $i \in I$ where $d(A_i) = 0$. In this case, it is obvious that for every finite set $F \subseteq I$ holds

$$d\left(\bigcap_{i \in F} A_i\right) = \prod_{i \in F} d(A_i) = 0$$

We will use following criterion (see [2]) to create more examples of systems of independent sets.

Theorem 2.1 *System of sets $\left\{A_i\right\}_{i \in I}$ is a system of independent sets if and only if the system $\left\{\mathbb{N} - A_i\right\}_{i \in I}$ is a system of independent sets.*

Example 2.1 *System of sets* $\{A_i\}_{i\in I}$ *where sets* A_i *are defined below is a system of independent sets.*

1. $A_i = A_{p_i}^k = \{p_i m : m \in \mathbb{N}\}$

where $\{p_i\}_{i\in\mathbb{N}}$ *is a sequence of pairwise distinct prime numbers,* $I = \mathbb{N}$.

It is obvious (see Lemma1.1) that

$$d(A_i) = \frac{1}{p_i}$$

and $\bigcap\limits_{i\in F} A_i = \left\{ m \prod\limits_{i\in F} p_i : m \in \mathbb{N} \right\}$. *Hence*

$$d\left(\bigcap_{i\in F} A_i\right) = \frac{1}{\prod_{i\in F} p_i} = \prod_{i\in F} d(A_i).$$

2. $A_i = \mathbb{N} - A_{p_i}^k = \{m : m \in \mathbb{N}, \gcd(p_i, m) = 1\}$

where $\{p_i\}_{i\in\mathbb{N}}$ *is a sequence of pairwise distinct prime numbers,* $I = \mathbb{N}$.

This system of independent sets we obtain from previous item using Theorem 2.1.

3. $A_i = A_{p_i}^{2k} = \{p_i^{2k} m \mid k, m \in \mathbb{N}, \gcd(p_i, m) = 1\}$

where $\{p_i\}_{i\in\mathbb{N}}$ *is a sequence of pairwise distinct prime numbers,* $I = \mathbb{N}$. *It was proved in [5] that*

$$d(A_{p_i}^{2k}) = \frac{p_i}{p_i + 1}$$

and

$$d(\bigcap_{i\in F} A_{p_i}^{2k}) = \prod_{i\in F} \frac{p_i}{p_i + 1} = \prod_{i\in F} d(A_{p_i}^{2k}).$$

(This result can be used to determine the asymptotic density of the set B_2 *of natural numbers which are sum of two squares of integers (see [1]). But there are more general results describing* $B_2(n)$ *(see [8]).)*

4. $A_i = \mathbb{N} - A_{p_i}^{2(k-1)} = \{p_i^{2k-1} m \mid k, m \in \mathbb{N}, \gcd(p_i, m) = 1\}$

where $\{p_i\}_{i\in\mathbb{N}}$ *is a sequence of pairwise distinct prime numbers,* $I = \mathbb{N}$.

This system of independent sets we obtain from previous item using Theorem 2.1.

Asymptotic densities of sets $A_i^{\alpha(k)}$ and of their intersections were studied in [3]. It was found :

Theorem 2.2 *Let* $p \in \mathbb{N}$, $p > 1$, *and let* $\{\alpha(k)\}_{n=1}^{\infty}$ *be an increasing sequence of non-negative integers. If we denote* $A_p^{\alpha(k)} = \{p^{\alpha(k)} m \mid m, k \in \mathbb{N}, p \nmid m\}$, *then*

$$d(A_p^{\alpha(k)}) = \left(1 - \frac{1}{p}\right) \sum_{j=1}^{\infty} \frac{1}{p^{\alpha(j)}}.$$

and

Theorem 2.3 *Let $P = \{p_1, p_2, \ldots, p_r\}$ be a set of pairwise co-prime natural numbers where $1 \notin P$ and $\{\alpha_1(k)\}_{k=1}^{\infty}, \{\alpha_2(k)\}_{k=1}^{\infty}, \ldots, \{\alpha_r(k)\}_{k=1}^{\infty}$ are increasing sequences of non-negative integers. Then*

$$d\left(\bigcap_{i=1}^{r} A_{p_i}^{\alpha_i(k)}\right) = \prod_{i=1}^{r} d(A_{p_i}^{\alpha_i(k)}).$$

Theorem 2.2 and Theorem 2.3 imply:

Theorem 2.4 *Let $\{p_i : i \in I\}$ be a nonempty set of pairwise co-prime natural numbers where for every $i \in I$ holds $p_i > 1$ and $\{\alpha_i(k)\}_{k=1}^{\infty}$ is an increasing sequence of non-negative integers. Then $\left\{A_{p_i}^{\alpha_i(k)}\right\}_{i\in I}$ is a system of independent sets.*

Asymptotic density of the intersection of sets $A_a^{\alpha(n)} = \{a^{\alpha(n)}m : m, n \in \mathbb{N}, a \nmid m\}$ and $A_b^{\beta(n)} = \{b^{\beta(n)}m : m, n \in \mathbb{N}, b \nmid m\}$ where $\{\alpha(n)\}_{n=1}^{\infty}$ and $\{\beta(n)\}_{n=1}^{\infty}$ are given increasing sequences of natural numbers, $a, b \in \mathbb{N} - \{1\}$ and $\gcd(a, b) = d \in \mathbb{N}$ was studied in [4]. It was found that

$$d\left(A_a^{\alpha_n} \cap A_b^{\beta_n}\right) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \frac{d^{\min\{\alpha_i,\beta_j\}}}{a^{\alpha_i} b^{\beta_j}} \left(1 - d(C_{ij}) - d(D_{ij}) + d\left(C_{ij} \cap D_{ij}\right)\right) \tag{1}$$

where

$$d\left(C_{ij}\right) = \frac{\gcd(a\cdot\gcd(a^{\alpha_i}, b^{\beta_j}), b^{\beta_j})}{a\cdot\gcd(a^{\alpha_i}, b^{\beta_j})} = \frac{\gcd(a\cdot d^{\min\{\alpha_i,\beta_j\}}, b^{\beta_j})}{a\cdot d^{\min\{\alpha_i,\beta_j\}}},$$

$$d\left(D_{ij}\right) = \frac{\gcd(b\cdot\gcd(a^{\alpha_i}, b^{\beta_j}), a^{\alpha_i})}{b\cdot\gcd(a^{\alpha_i}, b^{\beta_j})} = \frac{\gcd(b\cdot d^{\min\{\alpha_i,\beta_j\}}, a^{\alpha_i})}{b\cdot d^{\min\{\alpha_i,\beta_j\}}},$$

$$d\left(C_{ij} \cap D_{ij}\right) = d\left(C_{ij}\right) d\left(D_{ij}\right) \gcd\left(\frac{1}{d(C_{ij})}, \frac{1}{d(D_{ij})}\right).$$

This result implies that in case of a and b are not co-primes sets $A_a^{\alpha(n)}$ and $A_b^{\beta(n)}$ need not create system of independent sets.

The aim is to find result analogical to (1) for finite intersection of sets $A_{p_i}^{\alpha_i(k)}$, $i \in I$. It means, we are looking for the generalization of Theorem2.3 where numbers p_i need not be co-primes. The problem is that it is not possible to obtain this result analogically as (1) was obtained in [4].

Nevertheless we claim that the asymptotic density of the set $X = \bigcap_{k=1}^{p} A_{a_k}^{\alpha_k(n)}$ exists for arbitrary $p \in \mathbb{N}$ and it holds:

Conjecture 2.1 *Let $a_1, a_2, \ldots, a_p \in \mathbb{N} - \{1\}$. And let us define for every $k \in \{1, 2, \ldots, p\}$ sets $A_k^{\alpha_k} = \left\{a_k^{\alpha_k(i_k)} m : i_k \in \mathbb{N}, m \in \mathbb{N} - \mathbb{N}_{a_k}\right\}$, where $\mathbb{N}_{a_k} = \{a_k m : m \in \mathbb{N}\}$ and $\{\alpha_1(i_1)\}_{i_1=1}^{\infty}$, $\{\alpha_2(i_2)\}_{i_2=1}^{\infty}$, $\ldots$, $\{\alpha_p(i_p)\}_{i_p=1}^{\infty}$ be increasing sequences of natural numbers.*

If $X = \bigcap_{k=1}^{p} A_k^{\alpha_k}$, then $d(X)$ exists and

$$d(X) = \sum_{i_1=1}^{\infty} \cdots \sum_{i_p=1}^{\infty} d\left(X_{i_1,\ldots,i_p}\right),$$

where

$$d\left(X_{i_1,\dots,i_p}\right)=\frac{1}{\operatorname{lcm}\left(a_1^{\alpha_1(i_1)},\dots,a_p^{\alpha_p(i_p)}\right)}\left(1-\sum_{s=1}^{p}\left((-1)^{(s-1)}\sum_{\{j_1,\dots,j_s\}\subseteq\{1,\dots,p\}}d_{i_1,\dots,i_p}^{j_1,\dots,j_s}\right)\right),$$

$$d_{i_1,\dots,i_p}^{j_1,\dots,j_s}=\frac{1}{\operatorname{lcm}(\delta_{j_1,i_1,\dots,i_p},\delta_{j_2,i_1,\dots,i_p},\dots,\delta_{j_s,i_1,\dots,i_p})}$$

and

$$\delta_{j_m,i_1,\dots,i_p}=\frac{\operatorname{lcm}\left(a_{j_m}^{\alpha_{j_m}(i_{j_m})+1},\operatorname{lcm}\left(a_1^{\alpha_1(i_1)},\dots,a_p^{\alpha_p(i_p)}\right)\right)}{\operatorname{lcm}\left(a_1^{\alpha_1(i_1)},\dots,a_p^{\alpha_p(i_p)}\right)}.$$

3 Systems of conditionally independent sets

The system of independent sets was defined as follows in [6].

Definition 3.1 *Let I be a set, $B\subseteq\mathbb{N}$, $d(B)\neq 0$ exists and $\{A_i\}_{i\in I}$ be a system of sets where $A_i\subseteq\mathbb{N}$ and $d(A_i\cap B)$ exists for every $i\in I$. The system of sets $\{A_i\}_{i\in I}$ is a system of conditionally independent sets under condition B if and only if for every finite nonempty set $F\subseteq I$ holds*

$$\frac{d\left(\bigcap_{i\in F}(A_i\cap B)\right)}{d(B)}=\prod_{i\in F}\frac{d(A_i\cap B)}{d(B)}.$$

We also say, that sets A_i, where $i\in I$ are conditionally independent.

We can see that systems of independent sets are a special case of systems of conditionally independent systems. This is the trivial case where the condition is the set $B=\mathbb{N}$.

Several examples and ways of construction of systems of conditionally independent sets were found in [6]. Following theorems were proved:

Theorem 3.1 *Let $a,b_1,b_2\in\mathbb{N}$, $\gcd(a,b_1)=\gcd(a,b_2)=1$ and $p_1<p_2<\dots<p_i<\dots$ be sequence of prime numbers fulfilling $p_i\equiv b_1(\mod a)$ for every $i\in\mathbb{N}$. Let us denote*

$$A_i=\{p_im_2\mid m_2\in\mathbb{N},m_2\equiv b_2(\mod a)\}$$

for every $i\in\mathbb{N}$. Then $\{A_i\}_{i\in\mathbb{N}}$ is a system of conditionally independent sets under condition $B=\{m_1m_2\mid m_1,m_2\in\mathbb{N};m_1\equiv b_1(\mod a);m_2\equiv b_2(\mod a)\}$.

This theorem shows particular system of conditionally independent sets.

Theorem 3.2 *Let I contain at least two elements, $\{A_i\}_{i\in I}$ is system of independent sets, $K\subset I$ is finite nonempty set and $B=\cap_{i\in K}A_i$ where $d(B)\neq 0$. Then $\{A_i\}_{i\in I-K}$ is a system of independent sets and also it is a system of conditionally independent sets under condition B.*

If we have given system of independent sets, then this theorem allows us to create system of conditionally independent sets.

Another way to create a system of conditionally independent sets from the system of independent sets $\{A_i\}_{i\in I}$ is that we put all the elements $a\in\bigcup_{i\in I}A_i$ to the transformation $a^*=ca+d$, where c and d are given natural numbers.

Theorem 3.3 *Let $\{A_i\}_{i\in I}$ is a system of independent sets, $c \in \mathbb{N}$, $d \in \mathbb{N} \cup \{0\}$, $B = \{ca + d \mid a \in \mathbb{N}\}$ and for every $i \in I$ holds $B_i = \{ca + d \mid a \in A_i\}$. Then $\{B_i\}_{i\in I}$ is a system of conditionally independent sets under condition B.*

But there was not introduced any criterion for systems of conditionally independent sets in [6]. Nevertheless, we are convinced now that Theorem2.1 can be generalized following way:

Conjecture 3.1 *System of sets $\left\{A_i\right\}_{i\in I}$ is a system of conditionally independent sets under condition B if and only if the system $\left\{\mathbb{N} - A_i\right\}_{i\in I}$ is a system of conditionally independent sets under condition B.*

Conclusion

We have described basic known properties of systems of independent sets and systems of conditionally independent sets. We have also introduced two conjectures. The first one generalizes result describing the asymptotic density of intersection of two sets of type $A_a^{\alpha(n)}$ and describes asymptotic density of intersection of arbitrary finite number of sets of type $A_a^{\alpha(n)}$. The second one generalizes a criterion of systems of independent sets and it is a criterion of systems of conditionally independent sets.

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PROFITABILITY OF GAS MICROCOGENERATION USE IN BUILDINGS WITH CALCULATED ENERGY PERFORMANCE

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Abstract: Various aspects of high-efficiency gas microcogeneration application in buildings with varying demand for heat and power were presented. Recent changes the European and national legislation were described concerning the methodology of defining energy characteristics. For chosen referential objects a profitability of applying the MCHP XRGI gas microcogeneration by determining the time of the return on investment was defined.

Keywords: gas microcogeneration, energy performance of buildings, time of return on investment.

1 Introduction

When taking into consideration the requirements of energy performance of buildings resulting from the EU Directive 2010/31/ [3] of 19 May 2010 on energy performance of buildings and the subsequent amendment to the Construction Law [6], including the so-called second methodology of determining energy performance of buildings [4] they can be described as exhausting, but at the same time allow drafting acorrect energy characterization of a building, resulting in obtaining a proper energy performance certificate. For efficient use of primary energy for newly-designed as well as modernized energy supply systems in buildings, it is necessary to rely not only on the energy performance of the building in question. It is essential also to show that the technology applied is economically sound.

2 Methodology for calculation of energy performance of buildings

Directive of the European Parliament and of the Council on the energy performance of buildings states, that methodology for calculating energy performance should not only be based on the season in which the heating is required, but also cover the annual energy performance of the building. This methodology should take into account existing European standards [3]. Member States should set minimum requirements for the energy performance of buildings and building elements.

For the purpose of optimizing energy use of technical systems of the building, system requirements in respect of the overall energy performance, the proper installation, and the appropriate dimensioning, adjustment and control of the technical systems installed in already-existing buildings shall be set.

System requirements were set for installation, replacement and upgrading of technical building systems and shall be applied insofar as they are technically, economically and functionally feasible.

The system requirements shall cover at least the following:

(a) heating systems;
(b) hot water supply systems;
(c) air-conditioning systems;
(d) large ventilation systems;
or a combination of the above.

2.1 Energy performance certificates

The energy performance of a building shall be determined on the basis of the calculated or actual annual energy that is consumed in order to meet the different needs associated with its typical use and shall reflect the heating energy needs and cooling energy needs (energy needed to avoid overheating) to maintain the envisaged temperature conditions of the building, and domestic hot water needs.

The energy performance of a building shall be expressed in a transparent manner and shall include an energy performance indicator and a numeric indicator of primary energy use, based on primary energy factors per energy carrier.

The methodology shall be laid down taking into consideration at least the following aspects:

(a) the following actual thermal characteristics of the building including its internal partitions:
(i) thermal capacity;
(ii) insulation;
(iii) passive heating;
(iv) cooling elements; and
(v) thermal bridges;
(b) heating installation and hot water supply, including their insulation characteristics;
(c) air-conditioning installations;
(d) natural and mechanical ventilation which may include air-tightness;
(e) built-in lighting installation (mainly in the non-residential sector);
(f) the design, positioning and orientation of the building, including outdoor climate;
(g) passive solar systems and solar protection;
(h) indoor climatic conditions, including the designed indoor climate;
(i) internal loads.

For the purpose of the calculation, buildings should be adequately classified into the following categories:

(a) single-family houses of different types;
(b) apartment blocks;
(c) offices;
(d) educational buildings;
(e) hospitals;
(f) hotels and restaurants;
(g) sports facilities;
(h) wholesale and retail trade service buildings;
(i) other types of energy-consuming buildings.

In Poland energy performance is provided based on the regulation concerning the methodology of calculating energy performance of both the housing unit of the building and the part of the building constituting the self-contained technical-functional whole, and the method of issuing and template of energy performance certificates [4].

The regulation in question is defining: the methodology of calculating energy performance, the method of issuing energy performance certificates, and template of energy performance certificates. This regulation is also providing definitions of: heating system, built–in illumination installation system, simple technical system, complex system, non-renewable primary energy, renewable primary energy, final energy, additional final energy.

This regulation also gives definitions of functional energy for heating of the building, cooling of the building, for warming the water, and emission of pollutant into the atmosphere mentioned in act on the management system of emissions of greenhouse gases and other substances [2].

National regulation shows two methods of evaluation of energy performances:

- calculation method (newly-designed and existing buildings) - method based on the standard use and on climatic data from the database of the nearest meteorological station,
- consumption method (existing buildings, in use for at least 3 years) - the method based on the actual consumed quantity of energy or energy carrier [4, 6].

The regulation has already been amended in order to eliminate the simplified method of determining energy performance of residential buildings.

Types of buildings, for which energy characteristics are provied:

- residential buildings,
- apartment blocks,
- public buildings,
- individual recreation buildings,
- farm buildings,
- production buildings,
- storage buildings.

3 MCHP XRGI technology of gas microcogeneration

Microcogeneration system MCHP XRGI offers an economically viable solution to lower energy costs in an environmentally-friendly way. Using the principle of combined heat and power generation, the XRGI system achieves an extremely high level of efficiency (up to 96%) with the primary energy input, and thus helps protect the environment and lower energy costs. The special feature of this tried-and-tested method is that it allows the heat produced during electricity generation to be used rather than releasing it to the atmosphere with harmful consequences for the climate. This is why combined heat and power (CHP) generation is seen as the sustainable energy production of the future: it actively contributes to protecting the environment. That is also reason why this technology has been welcomed by environmental associations and supported by many of European governments. Cogeneration occupies a special place among the environmentally-friendly methods of energy production. Unlike solar and wind power, cogeneration does not depend on the weather. Combined heat and power units save resources in all weather conditions and provide reliable supply of electricity and heat. The XRGI system consist of main components shown in Figure 1 [1]:

Fig. 1. MCHP XRGI unit– (left to right: power unit, Q-heat distributor, iQ-control panel, storage tank)

Source: [1]

Power unit has the following functions: heat generation, electricity generation, safety and output regulation. The Q-Heat Distributor allows for regulation of engine water temperature, connection of XRGI system to storage tank and central heating system, the use of highly efficient pumps with controlled output based on current needs, creation of storage strategies based on current needs, service notification and error messages.

The iQ-Control Panel has the following functions: integration into electrical network, electrical safety features, control of the XRGI system, status and output display, remote data transmission.

The storage tank with external Storage Control ensures that the XRGI system saves engine heat until it is required. The storage tank retains surplus heat for times when heat consumption is high. The XRGI system thus operates for longer periods of time, making it more efficient. Installation of a storage tank is essential for the XRGI system to operate properly.

Figure. 2 shows a basic chart of heat and electricity production with the use of MCHP XRGI and an additional (for example already-existing) boiler.

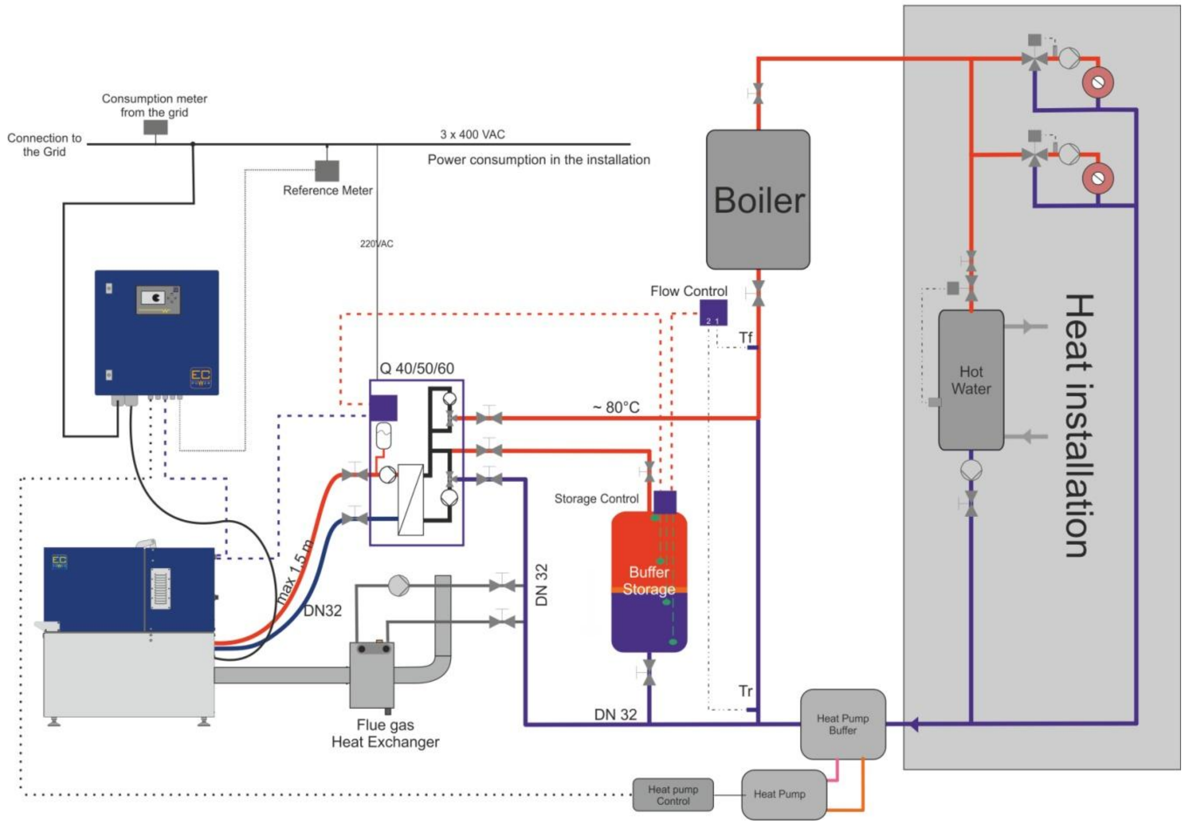

Fig. 2. Chart of heat and electricity production with the use of MCHP XRGI unit
Source: [1]

Technical parameters of the MCHP series of microcogenerators are shown in Table 1. Some of these units' parameters are very important. For example: low sound pressure level (only 49 dB), service intervals (10000 operating hours for units 6 and 9, and 6000 to 8000 operating hours for units 15 and 20), very high total efficiency (102-106% with condensation), small size allowing installation in the existing boiler rooms, high durability and reliability.

Table 1. Technical parameters of the MCHP XRGI series of gas microcogenerators

Cogeneration units XRGI 6 - 9 - 15 - 20				
	XRGI 6	XRGI 9	XRGI 15	XRGI 20
Electrical output (modulated)	2,5 - 6,0 kW	4,0 - 9,0 kW	6,0 - 15,2 kW	10,0 - 20,0 kW
Thermal output	8,5 - 13,5 kW	14,0 - 20,0 kW	17,0 - 30,0 kW	25,0 - 40,0 kW
Total efficiency (including optional condenser)	102%	104%	102%	106%
Gas engine	Layout-series			
Number of cylinders	3	3	4	4
Displacement [ccm]	952	952	2237	2237
Fuel	Natural gas, LPG			
Cooling	Water			
CO emission	<150 mg/m^3	<50 mg/m^3	46/89$^{*)}$ mg/m^3	25/49$^{*)}$ mg/m^3

NO_X emission	<350 mg/m^3	<100 mg/m^3	49/314$^{*)}$ mg/m^3	26/84$^{*)}$ mg/m^3
Generator	Asynchronous, three-phase, cos φ- 0,8			
Voltage	400 V	400 V	400 V	400 V
Nominal / maximum current	12/12 A	20/20 A	27/27 A	40/40 A
Service interwal	10 000 h	10 000 h	8 500 h	6 000 h
Flow temperature	80 - 85 ºC			
Return temperature	5 - 75 ºC			
Sound presure	< 49 dB(A), from 1 m distance			

$^{*)}$ At partial/full load
Source: [1]

3.1 Principles of adjusting the power of the cogeneration system to the needs of the facility.

The microcogeneration system should be properly adjusted to the needs of the facility for electricity and heat and their changes during the year. Correct choice of cogeneration unit or units ensures their continuous operation up to 24 hours a day throughout the year. Owing to such use, the highest operating savings and the shortest investment return periods are achieved. In order for the cogeneration system to work for the maximum number of hours per year, it is necessary to select cogeneration based on the electrical and thermal power values that are constant throughout the year. Basing on the smallest basic power consumption means that these values will be constant throughout the year, and that at this level there will be a constant delivery of both streams of energy produced by the co-generator.

4 Reference facilities

By using the EU directive and the Polish regulation guidelines, energy demands for selected groups of buildings can be determined in accordance with the applicable methodology. In order to analyze the utility of gas microcogeneration in buildings with various levels of demand for heat and electricity, the following reference objects were defined:
a) 180 m^2 residential building,
b) 330 m^2 residential building,
c) 330 m^2 residential building with a seasonal swimming pool of 30 m^2,
d) 330 m^2 residential building with a 30 m^2 of year-round swimming pool,
e) guest house of 380 m^2,
f) terraced houses: 20 x 180 m^2,
g) terraced houses: 40 x 180 m^2,
h) hotel with an area of 1000 m^2,
i) hotel with an area of 1000 m^2, including 80 m^2 of a year-round swimming pool,
j) hotel with an area of 2000 m^2,
k)hotel with an area of2000 m^2, including 80 m^2 of a year-round swimming pool,
l) hotel with an area of 4000 m^2,
m) hotel with an area of 4000 m^2, including 120 m^2 of a year-round swimming pool,
n) 3000 m^2 public swimming pool complex with 312 m^2 of a year-round swimming pool,
o) 2000 m^2 production plant with a continuous demand for 70 kW of thermal energy of,
p) 2000 m^2 production plant with a continuous demand for 140 kW of thermal energy,

Table 2 shows the comparison of operating savings and investment return periods (without subsidization, with funding for investment and with subsidy for generated energy) for all of the analyzed facilities [5].

Table 2. Comparison of operational costs, investment return periods and reduction of CO_2 for referential models considering use of the MCHP XRGI. Red is for return of investment in time longer than 10 years, yellow for 5-10, and green for less than 5 years.

Referential model	Code in prosument classification	Implemented microcogeneration unit	Operaational savings PLN/year	Simple investment return time, [years]			
				Without subsidy	With 30% subsidy	With surcharge of 0,10 zł/kWh	With 30% subsidy and surcharge of 0,10 zł/kWh
180 m^2 residential building,	PME 1	1 × MCHP XRGI 6	700,-	168,5	118,0	119,2	83,5
330 m^2 residential building,	PME 1	1 × MCHP XRGI 6	1.745,-	67,6	47,3	47,8	33,5
330 m^2 residential building with a seasonal swimming pool of 30 m^2,	PME 1	1 × MCHP XRGI 6	2.689,-	43,9	30,7	31,0	21,7
330 m^2 residential building with 30 m^2 of year-round swimming pool,	PME 1	1 × MCHP XRGI 6	4.324,-	27,3	19,1	19,3	13,5
Guest house of 380 m^2,	AG 1	1 × MCHP XRGI 6	4.967,-	23,8	16,6	16,7	11,6
Terraced houses: 20x180m^2,	PME 2	1 × MCHP XRGI 9	14.155,-	9,2	6,5	6,6	4,6
Terraced houses: 40x180m^2,	PME 2	1 × MCHP XRGI 20	33.214,-	5,7	4,8	4,1	3,4
Hotel with an area of 1000 m^2	AG 1	1 × MCHP XRGI 6	8.420,-	14,0	9,8	9,9	6,9
Hotel with area of 1000 m^2 including 80 m^2 of a year-round swimming pool	AG 1	1 × MCHP XRGI 15	25.590,-	6,3	4,4	4,5	3,1
Hotel with an area of 2000 m^2,	AG 1	1 × MCHP XRGI 9	17.552,-	7,4	5,2	5,4	3,6
Hotel with an area of 2000 $m^{2,}$ including 80 m^2 of a year-round swimming pool	AG 1	1 × MCHP XRGI 20	39.801,-	4,8	3,3	3,4	2,3
Hotel with an area of 4000 m^2	AG 1	1 × MCHP XRGI 20	42.414,-	4,5	3,1	3,2	2,2
Hotel with area of 4000 m^2, including 120 m^2 of a year-round swimming pool	AG 1	2 × MCHP XRGI 20	74.338,-	5,0	3,5	3,5	2,4
3000 m^2 public swimming pool complex, with 312 m^2 of a year-round swimming	PISE 3	3 × MCHP XRGI 20	142.655,-	4,0	2,8	2,9	2,0

pool							
2000 m^2 production plant with a continuous demand for 70kW of thermal energy,	AG 4	2 × MCHP XRGI 20	95.096,-	3,9	2,7	2,8	1,9
2000 m^2 production plant with a continuous demand for 140 kW of thermal energy	AG 4	4 × MCHP XRGI 20	187.827,-	4,0	2,8	2,9	2,0

For residential buildings, guest houses or hotels, where there is no swimming pool, the only heat demand in the summer is the warming of water for domestic use. In the case of residential buildings, even where there is a swimming pool, operating savings are not able to provide return at a rate justifying the investment. For guest houses and hotels with fewer than 80 guests (hotel 2000 m^2), investment return periods are longer than 7 years. The situation will improve considerably if the hotel (even a small one) has a swimming pool. At this point, the time of return on investment is clearly reduced.

In the case of the terraced houses, significant savings are only visible in housing communities of ca 40 houses. Investment return time without subsidy is ca 5.7 years, and when a 30% subsidy can be included, this period is reduced to 4.8 years.

On the other hand, hotels with a number of guests around 140 (hotel 4000 m^2) reach an investment return time of 4.5 years, even if there is no swimming pool and even if no subsidy is provided for equipment and installations. Very significant operational savings and short investment return times are found as well for the public swimming pools and production facilities with constant year-round demand for thermal energy.

The following reference facilities show simple return on investment (total cost of equipment, project and installation) to be less than 5 years without any additional funding:

- hotel 2000 m^2 with a year-round swimming pool of 80 m^2,
- hotel 4000 m^2,
- hotel 4000 m^2 with a year-round swimming pool of 120 m^2,
- 3000 m^2 urban swimming pool, with 312 m^2 of a year-round swimming pool,
- 2000 m^2 production plant with a continuous demand for 70 kW of thermal energy,
- 2000 m^2 production plant with a continuous demand for 140 kW of thermal energy.

By comparing the changes in the return times of investment outlays for different levels of co-financing, it can be stated that direct subsidies to micro-cogeneration plants are able to expand the segment of economically-justified customers, for it to encompass the housing communities and small hotels (with or without a swimming pool).

Conclusion

High-performance gas cogeneration is a cost-effective technology for achieving cost savings in buildings with varying electrical and heat demands. It is important, however, that the energy performance of the building should be analyzed, which would then allow for selection of the technologies, which are the best from the purely technological, as well as economic and environmental point of view. Any given technology is not economically justified for use everywhere, as shown by the analysis of reference facilities. Application of economic instruments may extend the scope of use of MCHP XRGI technology, but this is conditional upon the introduction of comprehensive solutions through national legislation.

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OPŁACALNOŚĆ STOSOWANIA MIKROKOGENERACJI GAZOWEJ DLA BUDYNKÓW Z PRZYGOTOWANĄ CHARAKTERYSTYKĄ ENERGETYCZNĄ

Streszczenie: W artykule przedstawione zostały uwarunkowania dotyczące zastosowania wysokosprawnej mikrokogeneracji gazowej w obiektach o zróżnicowanym zapotrzebowaniu na energię elektryczną i ciepło. Opisano zmiany w prawodawstwie europejskim i krajowym dotyczące metodologii wyznaczania charakterystyki energetycznej. Dla wybranych obiektów referencyjnych wyznaczono opłacalność zastosowania mikrokogeneracji gazowej MCHP XRGI poprzez określenie czasu zwrotu nakładów inwestycyjnych.

Słowa kluczowe: Mikrokogeneracja gazowa, charakterystyka energetyczna, czas zwrotu nakładów inwestycyjnych

ASSESSMENT OF THE REALIZATION QUALITY OF INITIAL TRAININGS (GENERAL INSTRUCTION) IN A COAL MINE

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Abstract: The paper presents the assessment of general initial trainings (general instruction) organized in two coal mines (KWK), accepting as the assessment criteria the quality of the delivered subjects structured within the training framework presented in the Annex 1 to the Regulation of the Minister of Economy, Labor and Social Policy on OHS trainings dated 27 July 2004. For calculation purposes, we applied one of the multicriteria assessment method – Promethee II in which the assessed criteria are maximized and the entities are compared pairwise with respect to the *i-th* criterion. To determine so called net flows, the function type No.4 was applied, which necessitated the determination of the indifference threshold *q* and the strict preference threshold *p*.

Keywords: the assessment of general initial trainings (general instruction), the multicriteria assessment method, the Promethee II method.

1 Introduction

The article One of relevant factors involving modern management styles is the training of working staff. It is viewed as a process aiming to accommodate the knowledge and skills of the employees to the requirements imposed by the organizational objectives and the method of their implementation [1]. The Regulation of the Minister of Economy, Labor and Social Policy dated 27 July 2004 on OHS trainings [6] provides detailed principles involving trainings on occupational health and safety, the scope of such trainings, requirements involving the subjects and the realization of training programs, the method of documenting the carried out trainings and the cases when the employers or employees can be exempted from the participation in certain specific trainings. Thus, the provisions of the said Regulation are implementing the regulations of the Directive 89/391/EWG dated 12 June 1989 [4].

Annex 1 to the above Regulation defines the framework of training programs, including also the initial, general trainings (general instruction) which have been designed for all people, who start their employment in a given company and which should be carried out before the employee starts work in a given company. The following subjects should be included in such trainings [6]:

- basic principles of the occupational health and safety,

- scope of the responsibilities and rights of the employer, employees and specific organizational units or social organizations of the company in terms of binding occupational health and safety regulations,
- liability for breaching the regulations or principles of the occupational health and safety,
- rules of safe movement within the premises of the company,
- accident hazards or health hazards which can occur within the premises of the company and general preventive measures,
- general OHS principles involving the use of technical tools and equipment as well as internal transport in the company,
- rules regulating the allotment of work clothes and boots as well as the means of individual protection, including also the workstation of the employees being trained,
- issues pertaining to order and cleanliness at workstations and their impact on the employees' health and safety,
- principles of the preventive medical care and its realization with respect to the workstation occupied by the trained employee,
- general principles of fire hazard protection and appropriate reaction in the event of fire,
- principles involving the reaction in the event of accidents, including the arrangement and administration of first aid.

The above scope of subjects should be covered within three school hours (3 x 45min).

2 Application of assessment subjectivity (individual preferences of the decision maker) in the decision-making process on the example of the method Promethee II

In the classical decision theory, the decision-making process is understood as a set of thinking or calculation processes logically related with one another, aiming to solve a decision-making problem by selecting one of possible variants of action to follow (decision), the best in the opinion of the decision maker. In a great majority of cases the selection of a solution usually comes down either to the choice of "the best" decision (in the opinion of the decision maker), or to the decisions are divided into classes of "equally good" decisions (satisfactory ones). The application of the assessment subjectivity (individual preferences of the decision maker) is illustrated by the method Promethee II which is one of so called discrete multicriteria methods. In this method the decision maker defines a finite set of decision-making variants (entities) from among which they want to extract a variant (entity) which in the best way corresponds with their preferences [2], [5]. In this method we investigate maximized criteria, and the particular decision-making variants (*a,b,c,…n*) are compared pairwise in terms of the *i-th* criterion. The preferences of the decision maker are determined on the basis of the obtained differences, i.e. preference functions are created, defined as the generalized criterion related to the *i-th* criterion. The values of the preferences are contained within the interval [0;1], with the value 1 (or close to 1) denoting strong preference of one variant as compared to the other, and the value 0 (or values close to 0) denoting very small preference. There are six types of generalized criteria commonly applied in practice [3], [5]:

- the criterion of function type No. 1 $$\mathrm{P}_1(\delta)=\begin{cases}0 & \delta \le 0\\ 1 & \delta > 0\end{cases} \qquad (1)$$

- the criterion of function type No. 2

$$P_2(\delta)=\begin{cases}0 & \delta \le q\\ 1 & \delta > q\end{cases} \qquad (2)$$

- the criterion of function type No. 3

$$P_3(\delta)=\begin{cases}0 & \delta \le 0\\ \delta/p & 0<\delta \le p\\ 1 & \delta > p\end{cases} \qquad (3)$$

- the criterion of function type No. 4

$$P_4(\delta)=\begin{cases}0 & \delta \le q\\ 1/2 & q<\delta \le p\\ 1 & \delta > p\end{cases} \qquad (4)$$

- the criterion of function type No. 5

$$P_5(\delta)=\begin{cases}0 & \delta \le q\\ \dfrac{\delta-q}{p-q} & q<\delta \le p\\ 1 & \delta > p\end{cases} \qquad (5)$$

- the criterion of function type No. 6

$$P_6(\delta)=\begin{cases}0 & \delta \le 0\\ 1-\exp\left(-\dfrac{\delta^2}{2s^2}\right) & \delta > 0\end{cases} \qquad (6)$$

which are applied to define the following parameters [3], [5]:

- indifference threshold q – when for the investigated criterion the said threshold will be defined and δ_i *(a, b)* $\le q_i$, it means that the difference of assessments in terms of this criterion is too small for the decision maker to give preference to the higher value,
- strict preference threshold p - when for the investigated criterion the said threshold will be defined and δ_i *(a, b)* $> q_i$, it means that the difference of assessments in terms of this criterion is important enough for the decision maker to give preference to the variant *a* over the variant *b*,
- accepted (declared) parameter of the value between q and p *(s)*,
- difference between two decision-making variants with respect to the investigated criterion (δ).

  Additionally, significance indexes w_i ($\sum_{i=1}^{n} w_i = 1$) are allocated to particular criteria.

3 Application of the method Promethee II in the assessment process – case study

To provide an example, the results of research studies carried out in the coal mines KWK1 and KWK2 were applied. The assessment process with the application of the accepted criteria involved the realization quality of the subjects delivered during the initial training (general instruction) defined in the training program, i.e.:

- criterion No.1 (f_1) *involved the assessment of the quality of the instruction provided on basic OHS principles*,
- criterion No.2 (f_2) *involved the assessment of the quality of the instruction provided on the scope of the responsibilities and rights of the employer, employees and specific organizational units or social organizations of the company in terms of binding occupational health and safety regulations*,

- criterion No.3 (f_3) *involved the assessment of the quality of the instruction provided on the liability for breaching the regulations or principles of the occupational health and safety*, etc.

The accepted range of assessment scale was from 1 (negative assessment, the lowest achievable one) to 10 (ideal assessment, the highest achievable one).

The significance of particular criteria was not being differentiated $(\boldsymbol{w_{1-11}} = \mathbf{0.091})$.

The collected assessments of the entities in terms of the successive criteria are presented in Table 1.

Table 1. Collected assessments of the entities in terms of the accepted criteria

		Assessment criterion										
		f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}
Entity	KWK 1	6	4	8	6	9	8	8	5	5	8	4
	KWK 2	6	6	8	8	6	4	8	8	5	5	8

Using the criterion of function type No. 4 in the calculation part, the values of preference functions were determined assuming p=3 and q=1 (Table 2).

Table 2. Collected values of preference functions

δ_1	KWK 1	KWK 2
KWK 1	0	0
KWK 2	0	0
δ_2	KWK 1	KWK 2
KWK 1	0	0
KWK 2	0,5	0
δ_3	KWK 1	KWK 2
KWK 1	0	0
KWK 2	0	0
δ_4	KWK 1	KWK 2
KWK 1	0	0
KWK 2	0,5	0
δ_5	KWK 1	KWK 2
KWK 1	0	0,5
KWK 2	0	0
δ_6	KWK 1	KWK 2
KWK 1	0	1
KWK 2	0	0
δ_7	KWK 1	KWK 2
KWK 1	0	0
KWK 2	0	0
δ_8	KWK 1	KWK 2
KWK 1	0	0
KWK 2	0,5	0
δ_9	KWK 1	KWK 2
KWK 1	0	0
KWK 2	0	0
δ_{10}	KWK 1	KWK 2
KWK 1	0	0,5
KWK 2	0	0
δ_{11}	KWK 1	KWK 2
KWK 1	0	0
KWK 2	1	0

For both coal mines aggregated preference indexes were calculated:

$$\Pi(KWK1, KWK2) = \sum_{j=1}^{11} w_j P_j (KWK1, KWK2) \tag{7}$$

$$\Pi(KWK2, KWK1) = \sum_{j=1}^{11} w_j P_j (KWK2, KWK1) \tag{8}$$

and then positive and negative preference flows:

$$\Phi^+(KWK1) = \frac{1}{n-1} \sum_{y \in A} \prod (KWK1, KWK2) \tag{9}$$

$$\Phi^-(KWK1) = \frac{1}{n-1} \sum_{y \in A} \prod (KWK2, KWK1) \tag{10}$$

$$\Phi^+(KWK2) = \frac{1}{n-1} \sum_{y \in A} \prod (KWK2, KWK1) \tag{11}$$

$$\Phi^-(KWK2) = \frac{1}{n-1} \sum_{y \in A} \prod (KWK1, KWK2) \tag{12}$$

where n – number of entities subjected to the assessment.

In order to determine the final assessments (to build the final ranking), net flows were determined:

$$\Phi(KWK) = \Phi^+(KWK) - \Phi^-(KWK) \tag{13}$$

which had the following values respectively: -0.046 for KWK 1 and 0.046 for KWK 2.

Conclusion

The assessment results involving the realization quality of training subjects within the framework of general instruction can be treated in two ways: as final assessments (for each assessed entity), we obtain 11 so called single-criterion assessments, and as partial assessments, which allow us to determine so called net flows (aggregated final assessments).

Interpreting the research results as the results of single-criterion assessments we can state that:

- in the case of criteria No.1 (*the assessment of the quality of the instruction provided on basic OHS principles*), No.3 (*the assessment of the quality of the instruction provided on the liability for breaching the regulations or principles of the occupational health and safety*), No.7 (*the assessment of the quality of the instruction provided on the allotment of work clothes and boots as well as the means of individual protection, including also the workstation of the employees being trained*) and No.9 (*the assessment of the quality of the instruction provided on the principles of the preventive medical care and its realization with respect to the workstation occupied by the trained employee*) we found no differences in the realization quality of training subjects in the coal mines KWK1 and KWK2,
- in the case of criteria No. 5 (*the assessment of the quality of the instruction provided on accident hazards or health hazards which can occur within the premises of the company and general preventive measures*), No.6 (*the assessment of the quality of the instruction provided on general OHS principles involving the use of technical tools and equipment as well as internal transport in the company*) and No.10 (*the assessment of the quality of the instruction provided on general principles of fire hazard protection and appropriate reaction in the event of fire*) the realization quality of training subjects in the coal mine KWK1 was assessed higher,

- in the case of criterion No.2 (*the assessment of the quality of the instruction provided on the scope of the responsibilities and rights of the employer, employees and specific organizational units or social organizations of the company in terms of binding occupational health and safety regulations*), No.4 (*the assessment of the quality of the instruction provided on the rules of safe movement within the premises of the company*), No.8 (*the assessment of the quality of the instruction provided on the issues pertaining to order and cleanliness at workstations and their impact on the employees' health and safety*) and No.11 (*the assessment of the quality of the instruction provided on the principles involving the reaction in the event of accidents, including the arrangement and administration of first aid*) the realization quality of training subjects in the coal mine KWK2 was assessed higher.

To determine the aggregated assessment, we applied one of discrete multicriteria methods – PROMETHEE II. Basing on the determined with this method so called net flow values, we can state that with the accepted assessment criteria (preference function type No. 4, q=1, p=3) higher realization quality of general initial trainings (general instruction) was found in the case of KWK2 $\left(\Phi(KWK\,2) = 0.046\right)$.

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OCENA JAKOŚCI REALIZACJI SZKOLEŃ WSTĘPNYCH OGÓLNYCH (INSTRUKTAŻU OGÓLNEGO) W KWK

Abstrakt (Streszczenie): W artykule poddano ocenie szkolenia wstępne ogólne (instruktaż ogólny) realizowane w dwóch kopalniach węgla kamiennego (KWK), przyjmując jako kryteria oceny jakość realizacji tematów zdefiniowanych w ramowym programie szkoleń zawartym w zał. nr 1 do Rozporządzenia Ministra Gospodarki i Pracy z dnia 27 lipca 2004 r. w sprawie szkolenia w dziedzinie bhp. W warstwie obliczeniowej wykorzystana została jedna z metod oceny wielokryterialnej – metoda Promethee II, w której oceniane kryteria są maksymalizowane, a obiekty porównywane parami ze względu na *i* – te kryterium. W celu wyznaczenia tzw. przepływów netto wykorzystany został typ 4 funkcji, co wymagało zdefiniowania wartości progu indyferencji *q* oraz progu ścisłej preferencji *p*.

Klíčová slova (Słowa kluczowe): szkolenia wstępne ogólne (instruktaż ogólny), ocena wielokryterialna, metoda Promethee II.

Dense Distance Magic Graphs

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Abstract: Let $G = (V, E)$ be a graph with n vertices. A bijection f from V to the set of integers $\{1, 2, \ldots, n\}$ is called a *distance magic labeling* of G if for every vertex in G the sum of labels of all adjacent vertices equas the same number k. A graph that allows such a labeling is a *distance magic graph*. For graphs with an even number of vertices there is an elegant construction of r-regular distance magic graphs for all feasible values of r. For graphs with an odd number of vertices some necessary and several sufficient conditions are known for a graph to have a distance magic labeling. In this paper we focus on distance magic graphs of high regularity: we provide constructions of $(n-5)$-regular distance magic graphs with n vertices. Magic labelings are used in tournament scheduling.

Keywords: labeling, distance magic, regular graph.

1 Introduction and definitions

All graphs in this paper are finite, undirected without loops and multiple edges. A *distance magic labeling* of a graph G with n vertices is a bijection $f : V(G) \to \{1, 2, \ldots, n\}$ with the property that there exists an integer k such that for every vertex x is

$$w(x) = \sum_{y \in N(x)} f(y) = k,$$

where $N(x)$ is the set of all vertices adjacent to x. The constant k is the *magic constant* and $w(x)$ is the *weight* of vertex x. We say a graph is *distance magic* if it admits a distance magic labeling.

For convenience we identify vertices with their labels, e.g. vertex labeled 1 we call vertex 1. An example of a distance magic graph are in Figures 1 and 4.

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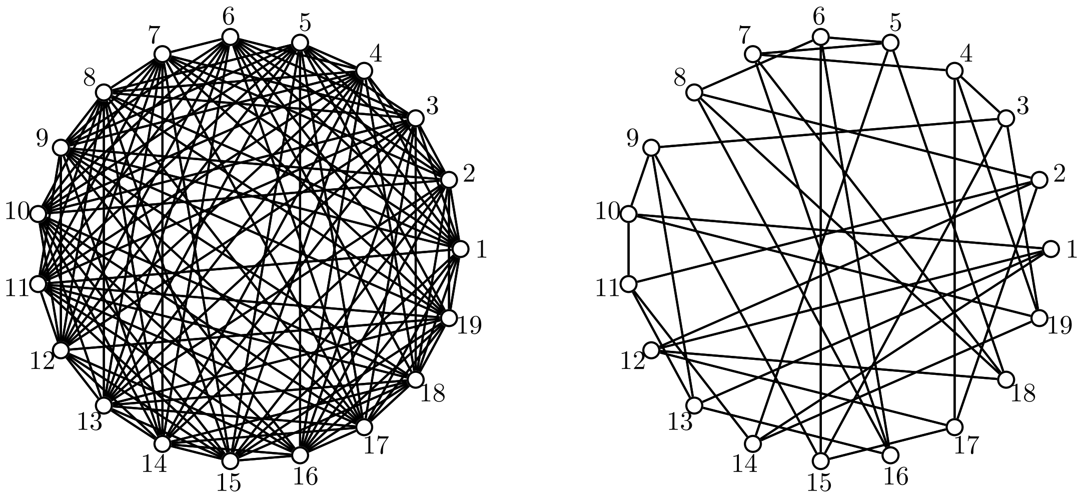

Figure 1: A 14-regular distance magic graph with 19 vertices and its complement.

2 Known results

The concept of distance magic labelings was introduced independently by several authors. Formerly, it was called 1-vertex magic vertex labelings [9] or Σ-labelling [5].

There is a survey on distance magic labelings by Arumugam, Fronček, and Kamatchi [1]. Here, we summarize results relevant to this paper, we focus on regular graphs. Most necessary conditions for a distance magic labeling of a given graph to exist are based on counting arguments and parity. Sufficient conditions involve constructions. For a distance magic labeling f of G with n vertices holds the following [9]:

$$nk = \sum_{x \in V(G)} \deg(x) f(x), \tag{1}$$

where $\deg(x)$ is the degree of vertex x. Thus, the magic constant of regular graphs is determined uniquely. In any r-regular distance magic graph G is

$$k = r(n+1)/2. \tag{2}$$

In [1], it was shown that the magic constant is determined uniquely for every distance magic graph.

From (2) it follows that a regular distance magic graphs can exist only if r is even or n is odd. The existence of distance magic labelings with an even number of vertices was completely settled by Fronček, Kovářová and the first author [3,4] who investigated the relation of regular distance magic graphs to scheduling of incomplete tournaments. The following proposition from [3] gives a necessary and sufficient condition for a distance magic graph with an even number of vertices to exist.

Proposition 2.1 *For n even a* $\mathrm{FIT}(n,k)$ *exists if and only if* $1 \le k \le n-1$, $k \equiv 1 \pmod 2$ *and either* $n \equiv 0 \pmod 4$ *or* $n \equiv k+1 \equiv 2 \pmod 4$.

An analogous simple condition for regular distance magic graphs with an odd number of vertices does not exist, because there are several exceptions for small orders. In [6] all orders of 4-regular distance magic graphs with an odd number of vertices were characterized; the smallest example has 17 vertices.

In [7] all $(n-3)$-regular distance magic graphs were characterized: they exists if and only if $n \equiv 3 \pmod 6$. Moreover, they are isomorphic to a balanced complete $n/3$-partite graph. In this paper we extend the results of Fronček [2] and of Silber and the first author [7].

In the next section we provide a construction of $(n-5)$-regular handicap graphs. For convenience, the construction describes the graph complement, which is a 4-regular graph with an *auxiliary* labeling. In an $(n-5)$-regular distance magic graph the weight of every vertex is $k = r(n+1)/2 = (n-5)(n+1)/2$. In a complete graph with vertices labeled 1 through n, the weight (obtained as the sum of adjacent vertices) is

$$w(i) = \sum_{i=1}^{n} i - i = n(n+1)/2 - i.$$

Thus, in the 4-regular complement of a $(n-5)$-regular distance magic graph the weight of every vertex is

$$w(i) = n(n+1)/2 - i - (n-5)(n+1)/2 = 5(n+1)/2 - i. \tag{3}$$

The constructive proof deals with several cases modulo 12, for each case a different distance magic graph is constructed. We provide details for one of the cases and only a basic idea for the remaining cases. In the conclusion we summarize the result in a single statement for all feasible orders.

The motivation for distance magic labeling comes from scheduling incomplete tournaments [1].

3 Kotzig array

Kotzig arrays are a generalization of magic rectangles [8]. In this paper we use the following array for $n = 2t+1$.

$$a_{i,j} = \begin{bmatrix} 1 & 2 & \cdots & t & t+1 & t+2 & t+3 & \cdots & 2t & 2t+1 \\ 4t+2 & 4t & \cdots & 2t+4 & 2t+2 & 4t+1 & 4t-1 & \cdots & 2t+5 & 2t+3 \\ 5t+3 & 5t+4 & \cdots & 6t+2 & 6t+3 & 4t+3 & 4t+4 & \cdots & 5t+1 & 5t+2 \end{bmatrix} \tag{4}$$

Notice, every integer $1, 2, \ldots, 3n = 6t+3$ appears in the array precisely once: $1, 2, \ldots, 2t+1$ in the first row, $2t+2, 2t+3, \ldots, 4t+2$ in the second row and $4t+3, 4t+4, \ldots, 6t+3$ in the third row. Moreover, the sum of every column is $9t+6$.

Lemma 3.1 *The sum of the first and the last entry of column of the Kotzig array (a_{ij}) containing $3t+2$ is $6t+4$. All remaining columns of the Kotzig array can be paired so that for the pair x, y of columns is $a_{1x} + a_{3y} = a_{2x} + a_{2y} = a_{3x} + a_{1y} = 6t+4$.*

Proof. The first part of the claim is obvious: since the sum of each column is $9t+6$, the sum of the remaining two elements in the column besides $3t+2$ is $6t+4$.

The proof of the second part follows. Wlog let $x < y$. For $x \in \{1, 2, \ldots, \lfloor t/2 \rfloor\}$ take $y = t+2-x$. Then $y \in \{2 + \lceil t/2 \rceil, \ldots, t+1\}$. Now $a_{1x} + a_{3y} = x + (6t+4-x) = 6t+4$, $a_{2x} + a_{2y} = (4t+4-2x) + (2t+2x) = 6t+4$, and $a_{3x} + a_{1y} = (5t+2+x) + (t+2-x) = 6t+4$. Similarly, for $x \in \{t+2, t+3, \ldots, \lfloor 3t/2 \rfloor\}$ take $y = 3t+3-x$. Again, one can check that $a_{1x} + a_{3y} = a_{2x} + a_{2y} = a_{3x} + a_{1y} = 6t+4$. $\square$

4 Orders of all $(n-5)$-regular distance magic graphs

In this section we show for which orders there exists an $(n-5)$-regular distance magic graph. The proof is constructive. As stated before, it suffices to give for the 4-regular complement $\overline{G}$ an auxiliary labeling f, where the weight of every vertex i is $w(i) = 2(n+1) - i$, following the convention $f(i) = i$.

Lemma 4.1 *An $(n-5)$-regular distance magic graph G with n vertices, where $n \equiv 5 \pmod{12}$, exists for all $n \geq 17$.*

Proof. Let $n = 12k + 5$ for $n \geq 17$, denote $t = 2k$. Let G be a 4-regular graph consisting of $k+1$ components: one component K_5 (see Figure 2) and k components $C_4[C_3]$, which is a composition of C_4 and C_3, see Figure 3.

Let $(a_{i,j})$ be a Kotzig array with 3 rows and $2t+1$ columns, let j be the column of the array containing $3t+2$. Let $u_1, u_2, \ldots, u_5$ be the vertices of K_5. We define auxiliary labeling f as follows.

$$f(u_1) = 1, \quad f(u_2) = 1 + a_{1,j}, \quad f(u_3) = 1 + a_{2,j}, \quad f(u_4) = 1 + a_{3,j}, \quad f(u_5) = 6t + 5$$

The weight of each vertex in K_5 is the sum of labels of all remaining vertices.

$$\begin{aligned}
w(u_1) &= 15t + 14 = 5(n+1)/2 - 1, \\
w(u_2) &= 15t + 14 - a_{1,j} = 5(n+1)/2 - f(u_2), \\
w(u_3) &= 15t + 14 - a_{2,j} = 5(n+1)/2 - f(u_3), \\
w(u_4) &= 15t + 14 - a_{3,j} = 5(n+1)/2 - f(u_4), \\
w(u_5) &= 9t + 10 = 5(n+1)/2 - f(u_5)
\end{aligned}$$

since $a_{1,j} + a_{2,j} + a_{3,j} = 9t + 6$. Notice, the weights follow (3).

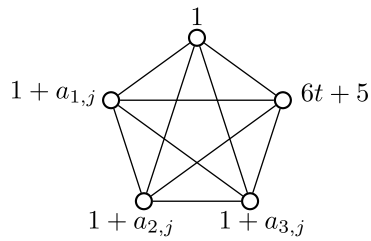

Figure 2: An auxiliary labeling of K_5.

Next, we extend labeling f to vertices of the k-th component $C_4[C_3]$. We take two pairs (x, y) and (w, z) of columns of the Kotzig array guaranteed by Lemma 3.1. We label the vertices in $C_4[C_3]$ as shown in Figure 3. Now, the weight of $1 + a_{1,x}$ (vertex labeled $1 + a_{1,x}$) is

$$\begin{aligned}
w(1 + a_{1,x}) &= ((1 + a_{2,x}) + (1 + a_{3,x})) + ((1 + a_{1,w}) + (1 + a_{3,z})) \\
&= (9t + 8 - a_{1,x}) + (6t + 6) = 15t + 14 - a_{1,x} = 5(n+1)/2 - f(a_{1,x}).
\end{aligned}$$

Again, the weight follows (3). In the same fashion we label all $C_3[C_4]$ components always with numbers from a different pair of columns from the Kotzig array. Thus, labeling f is an auxiliary labeling of G with $12k + 5$ vertices, where G has one component K_5 and k components $C_3[C_4]$.

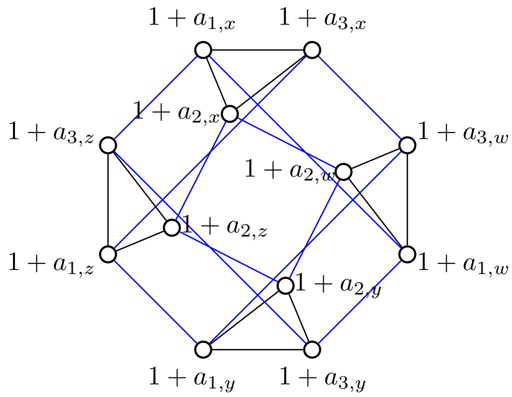

Figure 3: An auxiliary labeling of a component $C_4[C_3]$.

Complement $\overline{G}$ is a connected $12k$-regular graph with $12k+5$ vertices. Using the same labels for every vertex as in f, by (3) we obtain a labeling that has the same weight for every vertex. Clearly, the labeling is a 1-to-1 mapping to the set of $\{1,2,\ldots,12k+5\}$, thus f is a distance magic labeling of an $(n-5)$-regular graph with n vertices. $\square$

A similar construction was found for $n \equiv 1,3,7,9,11 \pmod{12}$. Each construction uses heavily Lemma 3.1, just the first component differs for each case. We provide the full constructions in the journal paper published after the conference.

5 Conclusion

The main objective of this paper is to study the existence of regular distance magic graphs with an odd number of vertices. For graphs with an even number of vertices a single construction always works. The odd case requires several different approaches since many exceptional cases do not allow for a distance magic graph, unlike the even case.

A brute force computer search reveals, no $(n-5)$-regular distance magic graph with less than 15 vertices exists. An example of an $(n-5)$-regular distance magic graph with 15 vertices is in Figure 4.

Based on Lemma 4.1 and analogous lemmas for $n \equiv 1,3,7,9,11 \pmod{12}$ we coclude the following.

Theorem 5.1 *An $(n-5)$-regular distance magic graph G with n vertices exists for all odd feasible values $n \geq 15$.*

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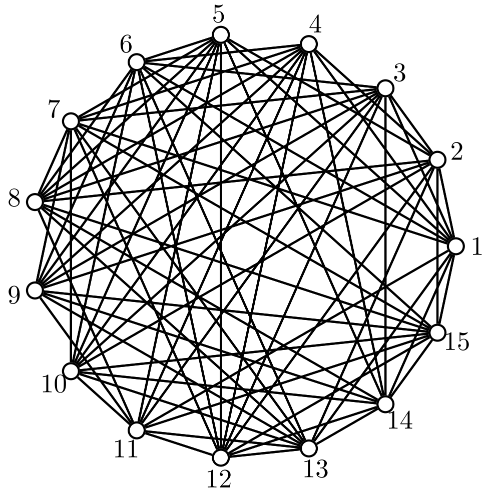

Figure 4: A distance magic 10-regular graph with 15 vertices.

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Husté distančně magické grafy

Abstrakt (Streszczenie): Mějme graf $G = (V, E)$ s n vrcholy. Bijektivní zobrazení f množiny V do $\{1, 2, \ldots, n\}$ se nazývá *distančně magické ohodnocení* grafu G, jestliže pro každý vrchol grafu G je součet ohodnocení sousedních vrcholů roven stejné hodnotě k. Graf, pro který takové ohodnocení existuje, nazýváme *distančně magický graf*. Pro grafy se sudým počtem vrcholů existuje jednoduchá konstrukce r-pravidelných distančně magických grafů pro všechny přípustné pravidelnosti r. Pro grafy s lichým počtem vrcholů jsou známy některé nutné podmínky existence a několik konstrukcí pro některé pravidelnosti. V tomto článku se věnujeme konstrukci distančně magických grafů s vysokou hodnotou r: konstruujeme $(n-5)$-pravidelné distančně magické grafy s n vrcholy. Magická ohodnocení se využívají při losování sportovních turnajů.
Klíčová slova (Słowa kluczowe): ohodnocení, distančně magický graf, pravidelný graf.

Digital Simulation of 16-QAM Modulation

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Abstract: In this work shows the rules for QAM modulation - Quadrature Amplitude Modulation. For 16-QAM modulation are given different constellation diagrams. Shows an example of a 16-QAM modulation. Then shows a simulation of the demodulation. Calculations in Matlab.

Keywords: modulation, simulation, constellation diagram, quadrature amplitude modulation.

1 Introduction to QAM modulation

QAM modulation (*Quadrature Amplitude Modulation*), is a quadrature modulation of amplitude-phase. Is used to send digital data over radio channel used in DVB broadcasts. Input to the modulation of the form string bits. QAM modulation is a combination of amplitude modulation and phase modulation. The data are formed into two, three, four, etc. that correspond to both amplitude and phase. Are created according to the diagram of the constellation (*Constellation diagram*). QAM signal is a linear combination of two orthogonal waveforms (shifted in phase by π/2): cosine and sine [8, 12, 14].

There are 2^n combinations created from n bits. Grouping data input n bits per symbol gives so 2^n points constellation, which are often referred to as phasors, or complex vectors. Phasors related to these points can have different amplitude and/or phase values and, therefore, this type of modulation is called multi-level modulation, where the number of levels is equal to the number of constellation points [2]. For n=4 bits are grouped after 4 and we have to deal with the modulation 16-QAM.

Data in digital form is divided into two streams. Then, each stream is converted to an analog signal in the digital-to-analog converter. Analog signal can pass through a low-pass filter. In the next stage of one signal is multiplied by the carrier, and the second by the carrier shifted in phase by π/2. At the end of both modulation signals are summed and sent as QAM signal. Channels through which pass signals to I (in-phase) and Q (in quadrature). Analog signal in the channel and is multiplied by the cosine function, and an analog signal on channel Q is multiplied by sine function [8, 12, 14].

The process of creating signal describe mathematical formulas [8, 12, 14]:

$$S_i(t) = A_i \cos(\omega_0 t + \phi_i) \tag{1}$$

$$S_i(t) = A_i(\cos\phi_i \cos\omega_0 t - \sin\phi_i \sin\omega_0 t) \quad (2)$$

$$S_i(t) = a_i \cos\omega_0 t - b_i \sin\omega_0 t \quad (3)$$

where:

$$a_i(t) = A_i \cos\phi_i \quad (4)$$

$$b_i(t) = A_i \sin\phi_i \quad (5)$$

These formulas describe the equivalence of modulation amplitude and phase with the sum of passes of the shifted in phase by $\pi/2$ [8, 12]. From the formula (3) shows that for i-this group of n consecutive bits, the signal on the channel I has the form $a_i\cos(\omega_0 t)$, and the channel Q $-b_i\sin(\omega_0 t)$.

2 16-QAM constellation and 16-QAM modulation

As before I was told a group of n bits shall be assigned a point in the constellation.

Constellation diagram is a graphical representation of the modulated signal digitally, for example. using the QAM or PSK. It shows the signal as a two-dimensional graph on the complex plane. If the signal is represented by a complex number can be selected in the complex plane. The real axis is often called the axis of the in-phase (I), and the imaginary axis the quadrature axis (Q). Points for all modulation signals, make up the constellation diagram are called constellation points [9]. Each constellation point is assigned the amplitude and phase. Amplitude, the distance of the constellation point from the origin. Phase, is the angle between the axis of the actual I (horizontal), and the vector between the origin and the point of the constellation. Modulation 16- QAM can also be different layouts of the constellation. There are three types of constellation. They are shown in figure 1.

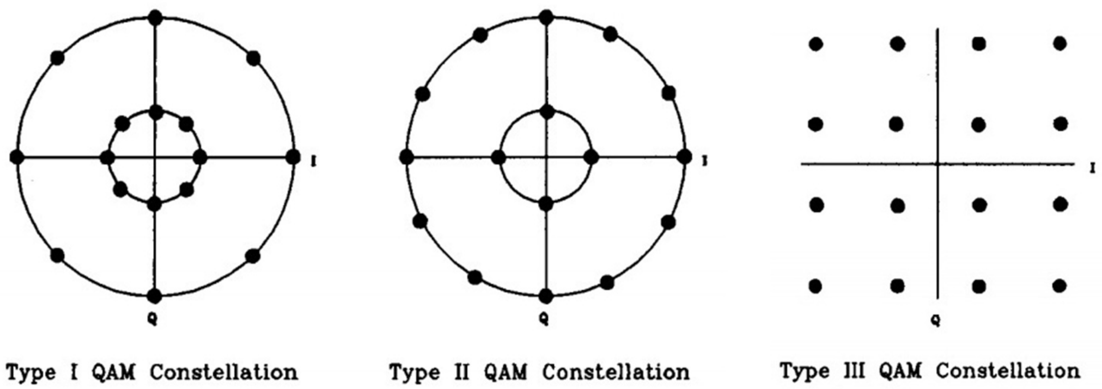

Fig. 1. Examples of the QAM constellation types I, II and III [2, 7]

There are also different variations of these types of constellation.

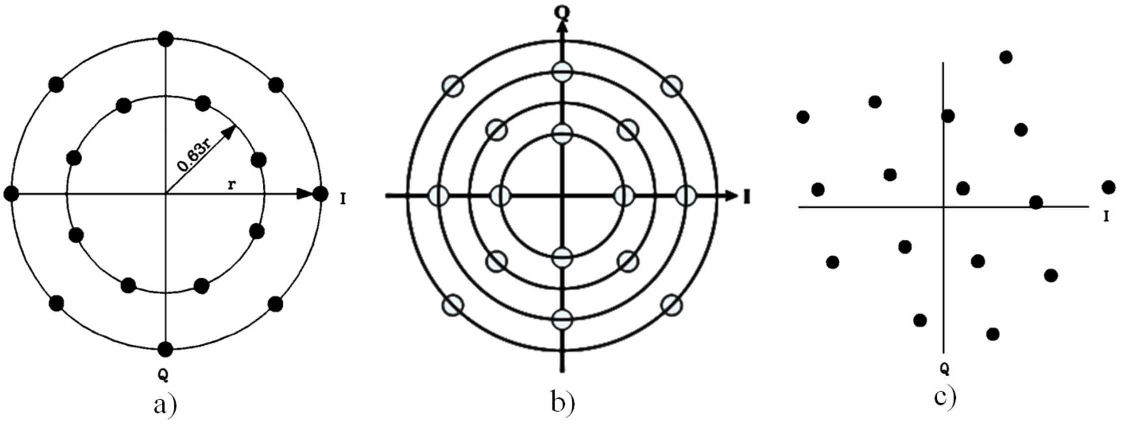

Fig. 2. a) optimum 16 levels constellation by Lucky [6]
b) not orthogonal 16-QAM modulation [10]
c) optimum 16 levels constellation by Foschini [1]

In practice, the greatest recognition met with constellations rectangular type III presented in figure 1. Other types of constellation does not apply due to the complexities associated with calculations.

You must assign the points of constellation 16 combinations of four bits. You can also do so in different ways. Important in the coding of the constellation points are to this encoding was using Gray code. Gray code is a binary code without the weight, not position, characterised by the fact that the next two code words differ only state one bit (in this case, two adjacent four bits to constellation differ only by a single bit). In other words, the Hamming Distance between adjacent four bits is 1.

Each point of the constellation (i.e. each four bits are assigned coordinates in the plane. Typically, it is assumed that the horizontal axis „I" and on the vertical axis „Q" the coordinates of the points are: -3, -1, 1 3. This is shown in the figure 3. It is important that distance in the horizontal and vertically between the points were the same.

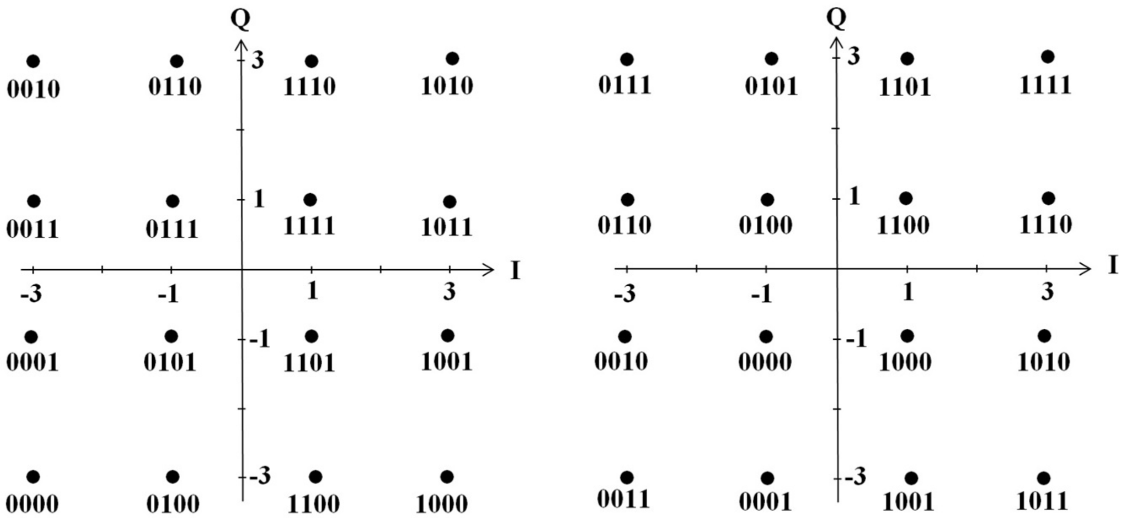

Fig. 3. Constellation diagrams for 16-QAM [13]

Other placement of four bits in the constellation can be seen in the work of [8, 9, 10, 11, 12, 15].

Constellation diagram can be represented as a matrix (6), a two-bit parts the coded numbers a_i (4), b_i (5), indicate the elements of the array that contains the amplitude I and Q component waves. The diagram assigns to the coded numbers amplitude I and Q waves. Exactly, it $|a_i|$ is the amplitude of the signal $\cos(\omega_0 t)$ in the channel I (3), and $|b_i|$ is the amplitude of the signal $-\sin(\omega_0 t)$ in channel Q (3).

$$[a_i, b_i] = \begin{bmatrix} (-3, 3) & (-1, 3) & (1, 3) & (3, 3) \\ (-3, 1) & (-1, 1) & (1, 1) & (3, 1) \\ (-3, -1) & (-1, -1) & (1, -1) & (3, -1) \\ (-3, -3) & (-1, -3) & (1, -3) & (3, -3) \end{bmatrix} \tag{6}$$

Source: [8, 12]

So in 16-QAM modulation with a string of bits input groups after the four bits. For each four bits, based on selected constellations, specifies the number of a_i and b_i. The number of the digital-to-analog signals are converted into constant a_i (t) (4) and b_i (t) (5). Then in the channel and the signal $a_i(t)$ is multiplied by $\cos(\omega_0 t)$, and in the channel Q signal b_i(t) is multiplied by the $-\sin(\omega_0 t)$. The resulting QAM signal is obtained by adding both the signals of the channels I and Q (3). This is shown in figure 4.

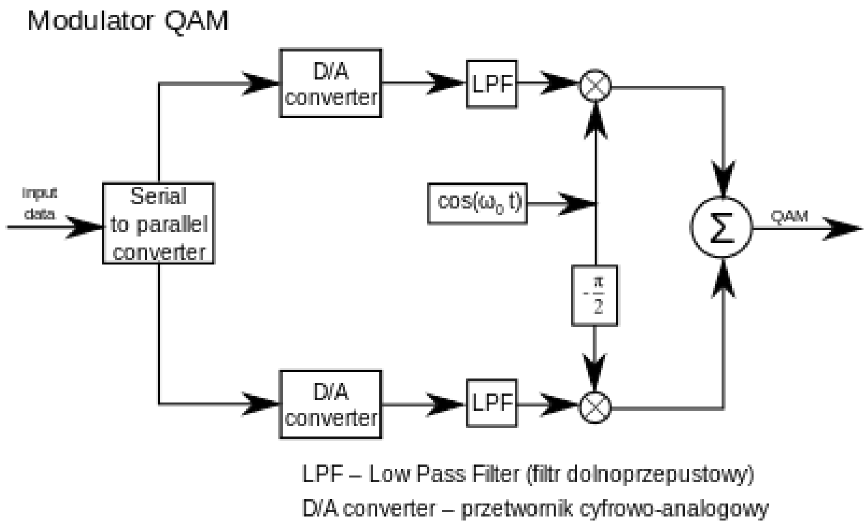

Fig. 4. Flowchart QAM modulation [8]

The above considerations we will illustrate the example.

3 Example 16-QAM modulation

Example 1

We have the following string input: $\{q\}$=10100100111100011000. These data are grouped four bits: 1010 0100 1111 0001 1000. Select the constellation, for example. the first figure 3. Then we have (a_i, b_i) = (3, 3), (-1, -3), (1, 1), (-3, -1), (3, -3).

Signals on the channel I will have the form (3):

$a_i\cos(\omega_0 t) = 3\cos(\omega_0 t), -\cos(\omega_0 t), \cos(\omega_0 t), -3\cos(\omega_0 t), 3\cos(\omega_0 t)$.

Signals on the channel Q will have the form (3):

$-b_i\sin(\omega_0 t)$ = $-3\sin(\omega_0 t)$, $3\sin(\omega_0 t)$, $-\sin(\omega_0 t)$, $\sin(\omega_0 t)$, $3\sin(\omega_0 t)$.

For calculations, the also:

dT_S=10 – this is the duration (length) of the continuous signal $a_i(t)$ or $b_i(t)$. It is the duration of the cosine or sine wave assigned ago continuous signal;

ilp=100 – this is the number of points in the period T_S needed to plot the cosine or sine wave (the number of values of these functions in the period T_S);

n – the number of bits in the $\{q\}$, n=20;

*n*1 – the number of fours of bits in the $\{q\}$, $n1=n/4=5$;

tmax – this is the simulation time, $tmax=dT_S \cdot n1=10\cdot 5=50$;

nmax – this is the number of points chart signal $S(t)$;

$nmax = ilp\cdot n1=100\cdot 5=500$;

delta – this is the distance between two adjacent points on the graph, $delta=dT_S/ilp=10/100=0.1$;

ω_0 – is the frequency of the cosine and sine wave. ω_0 is chosen so that the total number of times the cosine or sine occurred during the T_S. The result is that in subsequent periods of time T_S, the values of the cosine and sine are repeated, the same. In this example, it is assumed that during the T_S cosine or sine occurs exactly 3 times, $\omega_0=(2\cdot\pi/dT_S)\cdot 3=1.88495559$. For the purposes of computing, the created table, in which bits of the constellation are assigned the amplitude in the channels of the I and Q.

Table 1. Tabular record of the constellation

The number of the decimal	Bits of the constellation	The amplitude (a_i, b_i)
0	0 0 0 0	(-3, -3)
1	0 0 0 1	(-3, -1)
2	0 0 1 0	(-3, 3)
3	0 0 1 1	(-3, 1)
4	0 1 0 0	(-1, -3)
5	0 1 0 1	(-1, -1)
6	0 1 1 0	(-1, 3)
7	0 1 1 1	(-1, 1)
8	1 0 0 0	(3, -3)
9	1 0 0 1	(3, -1)
10	1 0 1 0	(3, 3)
11	1 0 1 1	(3, 1)
12	1 1 0 0	(1, -3)
13	1 1 0 1	(1, -1)
14	1 1 1 0	(1, 3)
15	1 1 1 1	(1, 1)

On the basis of four bits determine the amplitude of a_i and b_i in the following way: four bits change to a decimal number. The pointer *i*=decimal number+1 (note: the numbering of the vectors in Matlab starts at 1). Then we read (from table 2) the amplitude of the a_i and b_i. Based on the calculated amplitudes of a_i and b_i, we create constant functions for consecutive fours bits. It is shown in figure 5.

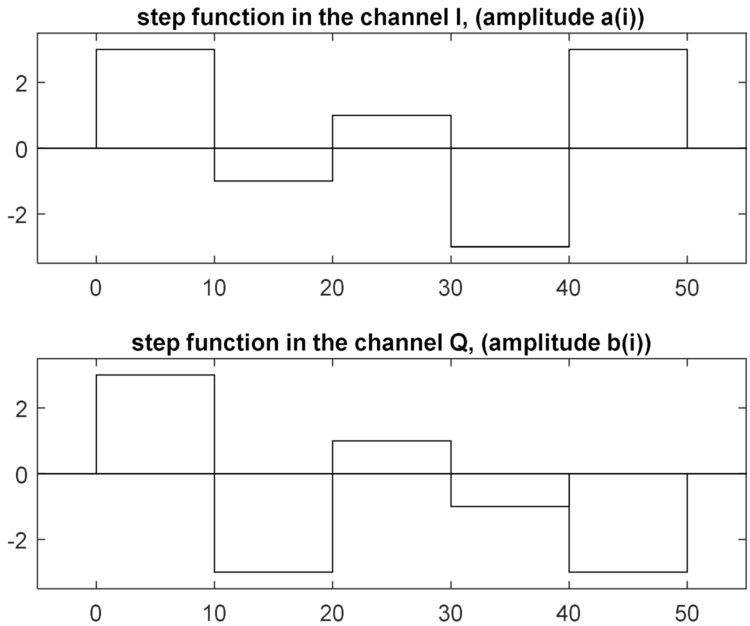

Fig. 5. Output signals from digital to analog converter

Then the first of these signals is multiplied by cos($\omega_0 t$), and the second signal is multiplied by -sin($\omega_0 t$). This is shown in figure 6.

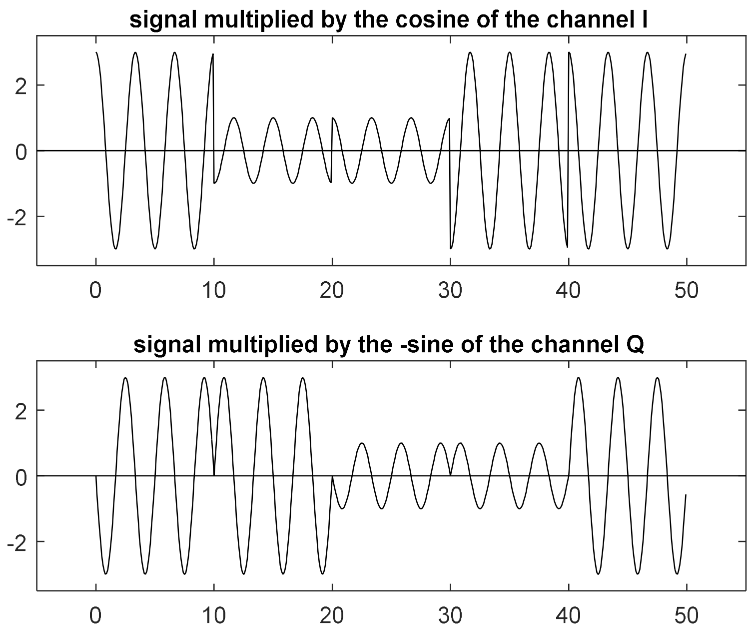

Fig. 6. Carrier signals in the channels I and Q

These two signals are summed to form the resulting signal QAM. It is shown in figure 7.

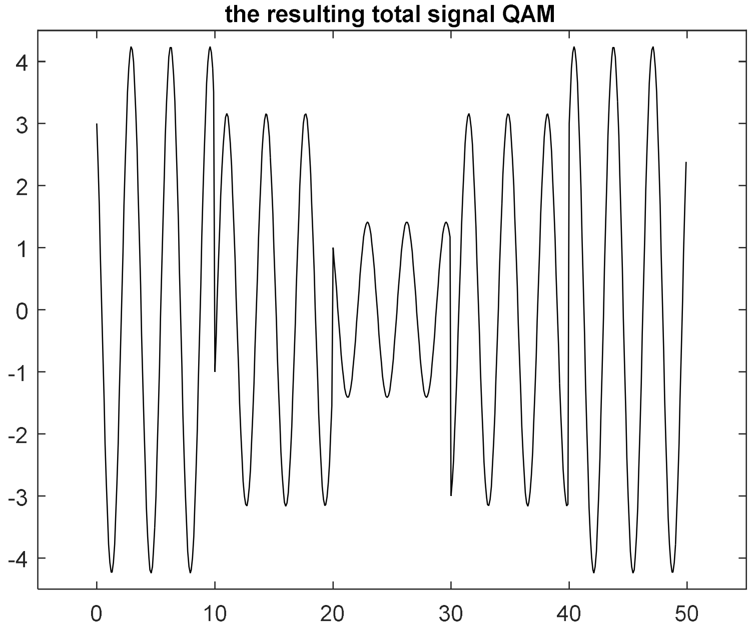

Fig. 7. The resulting signal QAM

It should be noted that in digital simulation, QAM signalM for the example is a vector consisting of 500 values $\{w_j\}$.

4 Simulation of demodulating

Simulation of demodulating QAM signal we will shown in figure 7 and for the constellation in the first figure 3. QAM signal in the digital version is a vector $w=\{w_j\}$, (j=1, …, $nmax$=500).

For $S_i(t)$ of the specified formula (3) calculate the values of $S_i(0)$

$$S_i(0) = a_i \cos(\omega_0 \cdot 0) - b_i \sin(\omega_0 \cdot 0) = a_i$$

For each time interval T_S so we have

$$a_i = w_{(i-1)ilp+1}$$

To calculate b_i we use equation (3) and the second element the $\{w_j\}$ in each time period T_S. Knowing the previously calculated a_i we get

$$w_{(i-1)ilp+2} = a_i \cos(\omega_0 \cdot delta) - b_i \sin(\omega_0 \cdot delta)$$

$$b_i = \frac{a_i \cos(\omega_0 \cdot delta) - w_{(i-1)ilp+2}}{\sin(\omega_0 \cdot delta)}$$

For the data from example 1 formula takes the form

$$b_i = \frac{a_i \cdot 0.98228725 - w_{(i-1)ilp+2}}{0.18738131}$$

So based on the first two numbers in the compartment of T_S we get two numbers a_i and b_i. Then based on the constellation and a pair of numbers (a_i, b_i), you must create a string four bits r_1, r_2, r_3, r_4. Bits of the constellation of save as decimal numbers. On the basis of the first constellation (figure 3), we can create the array d decimal values for groups of four bits

$$d=\begin{bmatrix}2 & 6 & 14 & 10\\ 3 & 7 & 15 & 11\\ 1 & 5 & 13 & 9\\ 0 & 4 & 12 & 8\end{bmatrix}$$

On the basis of a pair of numbers (a_i, b_i) we calculate indicators: k – number of line and l – number of column , for decimal numbers of matrix d.

$$k=-0.5b_i+2.5$$
$$l=0.5a_i+2.5$$

The number of $d(k, l)$ is a decimal number for binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions

r=dec2bin($d(k, l)$)

Then we get a string of four bits $r=\{r_1, r_2, r_3, r_4\}$. Repeat this sequence for each time interval T_S.

Example 2

We trace the demodulation for the QAM obtained in example 1.

For the first time interval T_S ($0\le t<10$) we w_1=3, w_2=2.38471781. We calculate

$a_1=w_1=3$

$$b_1=\frac{3\cdot 0.98228725-2.38471781}{0.18738131}=\frac{0.56214394}{0.18738131}=3$$

For a two of numbers (3, 3) we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d.

$$k=-0.5b_i+2.5=-0.5\cdot 3+2.5=-1.5+2.5=1$$
$$l=0.5a_i+2.5=0.5\cdot 3+2.5=1.5+2.5=4$$

The number of $d(1, 4)$=10 is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions r=dec2bin(10). We get $r=\{1\ 0\ 1\ 0\}$.

For the second time interval T_S ($10\le t<20$) we w_{101}=-1, w_{102}=-0.42014331. We calculate

$a_2=w_{101}=-1$

$$b_2=\frac{-0.98228725+0.42014331}{0.18738131}=\frac{-0.56214394}{0.18738131}=-3$$

For a two of numbers (-1, -3) we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d.

$$k=-0.5b_i+2.5=-0.5\cdot(-3)+2.5=1.5+2.5=4$$
$$l=0.5a_i+2.5=0.5\cdot(-1)+2.5=-0.5+2.5=2$$

The number of $d(4, 2)$=4 is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions r=dec2bin(4). We get $r=\{0\ 1\ 0\ 0\}$.

For the third time interval T_S ($20\le t<30$) we w_{201}=1, w_{202}= 0.79490594. We calculate

$a_3=w_{201}=1$

$$b_3=\frac{0.98228725-0.79490594}{0.18738131}=\frac{0.18738131}{0.18738131}=1$$

For a two of numbers (1, 1) we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d.

$$k=-0.5b_i+2.5=-0.5\cdot 1+2.5=-0.5+2.5=2$$

$$l = 0.5a_i + 2.5 = 0.5 \cdot 1 + 2.5 = 0.5 + 2.5 = 3$$

The number of $d(2, 3)$=15 is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions r=dec2bin(15). We get r={1 1 1 1}.

For the fourth time interval T_S ($30 \leq t < 40$) we w_{301}=-3, w_{302}=-2.75948044. We calculate

$$a_4 = w_{301} = -3$$

$$b_3 = \frac{-3 \cdot 0.98228725 + 2.75948044}{0.18738131} = \frac{-2.94686175 + 2.75948044}{0.18738131} + \frac{-0.18738131}{0.18738131} = -1$$

For a two of numbers (-3, -1) we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d.

$$k = -0.5b_i + 2.5 = -0.5 \cdot (-1) + 2.5 = 0.5 + 2.5 = 3$$

$$l = 0.5a_i + 2.5 = 0.5 \cdot (-3) + 2.5 = -1.5 + 2.5 = 1$$

The number of $d(3, 1)$=1 is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions r=dec2bin(1). We get r={0 0 0 1}.

For the fifth time interval T_S ($40 \leq t < 50$) we w_{401}=3, w_{402}=3.50900570. We calculate

$$a_5 = w_{401} = 3$$

$$b_3 = \frac{3 \cdot 0.98228725 - 3.50900570}{0.18738131} = \frac{2.94686175 - 3.50900570}{0.18738131} + \frac{-0.56214395}{0.18738131} = -3$$

For a two of numbers (3, -3) we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d.

$$k = -0.5b_i + 2.5 = -0.5 \cdot (-3) + 2.5 = 1.5 + 2.5 = 4$$

$$l = 0.5a_i + 2.5 = 0.5 \cdot 3 + 2.5 = 1.5 + 2.5 = 4$$

The number of $d(4, 4)$=8 is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions r=dec2bin(8). We get r={1 0 0 0}.

By connecting together five strings of four bits we get the resulting string decoded {1010 0100 1111 0001 1000}, which is the same as the input string $q=\{q_j\}$.

Conclusion

QAM modulation involves the use of a single frequency bands to transmit two different signals. For a clear separation, in the receiver both signals of information, carrier signals must be shifted against each other in a phase of about 90^0 [3, 4]. The work illustrates the general principle of QAM. Shows the different diagrams the constellation for 16-QAM modulation. Also made example simulation of digital modulation 16-QAM for 20-bits. Shows also the demodulation method based on the first two QAM signal in each time period T_S. They must, however, be met assumptions about the frequency ω0, that early were presented. In this case, the simulation of demodulating QAM signal must be set in the form of a numeric string.

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SYMULACJA CYFROWA MODULACJI 16-QAM

Streszczenie: W tej pracy przedstawiono zasady modulacji QAM - Quadrature Amplitude Modulation. Dla modulacji 16-QAM przestawiono różne diagramy konstelacji. Pokazano przykład modulacji 16-QAM. Następnie przedstawiono symulację demodulacji. Obliczenia wykonano w Matlabie.

Słowa kluczowe: modulacja, symulacja, diagram konstelacji, kwadraturowa modulacja amplitudowo-fazowa.

Mathematical modelling of periodic diffraction problems in optics

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Abstract: The optical diffraction on a periodical interface belongs to relatively less exploited applications of the boundary integral equations method. This contribution presents a less frequent formulation of the diffraction problem based on vector tangential fields. There are discussed properties of obtained boundary operators with singular kernel and several problems related to a numerical implementation.

Keywords: optical diffraction, tangential fields, boundary elements method.

1 Introduction

The diffraction of an optical wave on a periodical interface between two media belongs to frequently solved problems, especially, when the grating period Λ is comparable with the wavelength λ of the incident beam. Among other, these phenomena are studied and exploited for nanostructured optical elements design. Naturally, the theoretical modelling is of great importance in such cases. In the last two decades, there were published numerous works treating of the optical diffraction in periodical structures - see [1] and references therein. One of the relatively new approaches is based on the Boundary Integral Equations (BIE). In this article, we present one special integral formulation of the boundary problem for the system of the Maxwell equations based on the tangential vector fields and propose a numerical implementation. Unlike the usually used rigorous coupled waves algorithm (RCWA) advantageous in the far fields analysis [1], the BIE models enable effective modelling of near fields in the spatially modulated region.

2 Formulation of problem

Let $S\,:\,x_3 = f(x_1)$ in $\mathbb{R}^3$ be a smooth surface that is periodically modulated in the coordinate x_1 with the period Λ and uniform in the x_2 direction. The interface S with the normal vector $\boldsymbol{\nu}$ divides the space into two semi-infinite homogeneous domains

$$\Omega^{(1)} = \{(x_1, x_2, x_3) \in \mathbb{R}^3,\ x_3 > f(x_1)\}, \quad \Omega^{(2)} = \{(x_1, x_2, x_3) \in \mathbb{R}^3,\ x_3 < f(x_1)\}$$

with the constant relative permittivities $\varepsilon^{(1)} \neq \varepsilon^{(2)}$, $\varepsilon^{(1)} \in \mathbb{R}$ and $\varepsilon^{(2)} \in \mathbb{C}$, $\mathrm{Re}\,(\varepsilon^{(2)}) > 0$, $\mathrm{Im}\,(\varepsilon^{(2)}) \geq 0$, and, the relative permeabilities $\mu^{(1)} = \mu^{(2)} = 1$ (both the materials are magnetically neutral), see Fig.1.

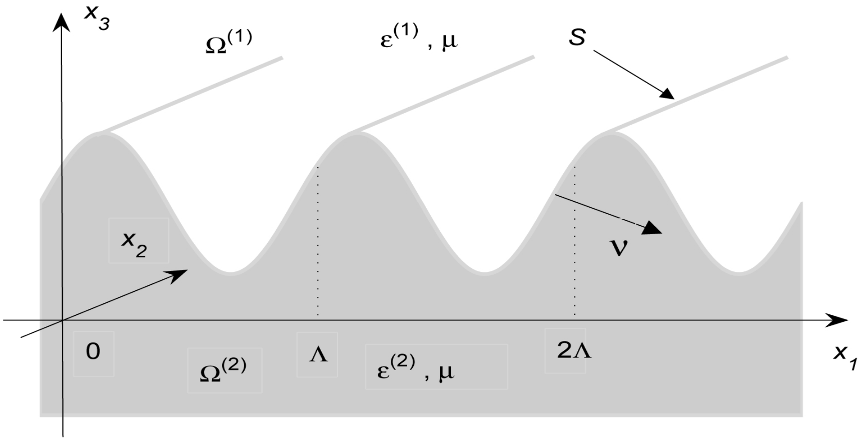

Figure 1: Semi-infinite domains with common periodical boundary

We aim to solve the optical diffraction problem for a monochromatic plane wave with the wavelength λ, i.e. with the wave number $k_0 = 2\pi/\lambda$, that is incoming from the domain $\Omega^{(1)}$ under the angle of incidence θ measured from the x_3 direction. We seek for the space-dependent amplitudes $\boldsymbol{E}^{(j)} = \boldsymbol{E}|_{\Omega^{(j)}}$, $\boldsymbol{H}^{(j)} = \boldsymbol{H}|_{\Omega^{(j)}}$ of the electromagnetic field intensity vectors $\boldsymbol{E}(x_1, x_2, x_3)\mathrm{e}^{-\mathrm{i}\omega t}$, $\boldsymbol{H}(x_1, x_2, x_3)\mathrm{e}^{-\mathrm{i}\omega t}$, where $\omega = c/\lambda$ and c represents the light velocity in the free space. The unknown intensities can be written as (the subscript 0 denotes the incident field)

$$\boldsymbol{E} = \begin{cases} \boldsymbol{E}_0^{(1)} + \boldsymbol{E}^{(1)} & \text{in} \quad \Omega^{(1)}, \\ \boldsymbol{E}^{(2)} \quad \text{in} \quad \Omega^{(2)}, \end{cases} \qquad \boldsymbol{H} = \begin{cases} \boldsymbol{H}_0^{(1)} + \boldsymbol{H}^{(1)} & \text{in} \quad \Omega^{(1)}, \\ \boldsymbol{H}^{(2)} \quad \text{in} \quad \Omega^{(2)}. \end{cases} \tag{1}$$

In the media without free charges, the vectors $\boldsymbol{E}^{(j)}$, $\boldsymbol{H}^{(j)}$, $j = 1, 2$ fulfil the Maxwell equations

$$\nabla \times \boldsymbol{E}^{(j)} = \mathrm{i}k_0\mu\boldsymbol{H}^{(j)}, \qquad \nabla \times \boldsymbol{H}^{(j)} = -\mathrm{i}k_0\varepsilon^{(j)}\boldsymbol{E}^{(j)} \quad \text{in} \quad \Omega^{(j)}, \tag{2}$$

$$\nabla \cdot \boldsymbol{E}^{(j)} = 0, \qquad \nabla \cdot \boldsymbol{H}^{(j)} = 0 \quad \text{in} \quad \Omega^{(j)}, \tag{3}$$

and their tangential components are continuous on the boundary

$$\boldsymbol{\nu} \times (\boldsymbol{E}^{(1)} - \boldsymbol{E}^{(2)}) = \boldsymbol{o}, \qquad \boldsymbol{\nu} \times (\boldsymbol{H}^{(1)} - \boldsymbol{H}^{(2)}) = \boldsymbol{o} \qquad \text{on } S. \tag{4}$$

For the far fields, the well-known Sommerfeld's radiation convergence conditions at infinity hold that enable to consider the problem on the common interface S only [3].

In the following we solve the problem (2)–(4) for the transverse magnetic (TM) polarization of the incident wave for which $\boldsymbol{E}^{(j)} = (E_1^{(j)}, 0, E_3^{(j)})$, $\boldsymbol{H}^{(j)} = (0, H_2^{(j)}, 0)$. The Maxwell equations (2),(3) lead to the Helmholtz equations for the scalar components $H_2^{(j)}$,

$$\Delta H_2^{(j)} + k_0^2\varepsilon^{(j)}H_2^{(j)} = 0 \qquad \text{on} \quad \Omega^{(j)}, \quad j = 1, 2. \tag{5}$$

Denoting $\boldsymbol{x} = (x_1, x_3)$, $\boldsymbol{y} = (y_1, y_3)$, the periodical fundamental solution of the Helmholtz equation in $\Omega^{(j)}$ can be written as [7]

$$\Psi^{(j)}(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{2\mathrm{i}\Lambda}\sum_{m=-\infty}^{\infty}\Psi_m^{(j)}(\boldsymbol{x},\boldsymbol{y})\,, \qquad \Psi_m^{(j)}(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{\beta_m^{(j)}}\;\mathrm{e}^{\mathrm{i}(\alpha_m(x_1-y_1)+\beta_m^{(j)}|x_3-y_3|)}\;, \tag{6}$$

where α_m, $\beta_m^{(j)}$ are the propagation constants defined as

$$\alpha_m = \alpha + (2\pi m)/\Lambda\;, \quad \alpha = k_0\sqrt{\varepsilon^{(1)}}\sin\theta\;, \quad \alpha_m^2 + \left(\beta_m^{(j)}\right)^2 = k_0^2\varepsilon^{(j)}\;. \tag{7}$$

In further considerations we exploit the following property of the functions $\Psi^{(j)}$.

Theorem 1. *For both of the function $\Psi^{(j)}(\boldsymbol{x},\boldsymbol{y})$ defined by (6) and for an arbitrary but fixed $\boldsymbol{x} \in \mathbb{R}^2$ the difference*

$$\tilde{\Psi}^{(j)}(\boldsymbol{y}) = \Psi^{(j)}(\boldsymbol{x},\boldsymbol{y}) - \frac{1}{2\pi}\ln\frac{1}{\|\boldsymbol{x}-\boldsymbol{y}\|} \tag{8}$$

is continuous in $\mathbb{R}^2$.

The proof of this theorem was presented in [9].

3 Mathematical model

We formulate the problem (2)–(4) as the boundary integral equations for the tangential fields

$$\boldsymbol{J} = \boldsymbol{\nu}\times\boldsymbol{E}^{(1)} = \boldsymbol{\nu}\times\boldsymbol{E}^{(2)}, \qquad \boldsymbol{I} = -\boldsymbol{\nu}\times\boldsymbol{H}^{(1)} = -\boldsymbol{\nu}\times\boldsymbol{H}^{(2)}\;, \tag{9}$$

where $\boldsymbol{\nu}$ is an unit normal vector of the boundary S oriented as shown in Fig.1. Similarly, $\boldsymbol{\tau}$ represents an unit tangential vector of S. On the boundary we can write $\boldsymbol{J} = -J_2\boldsymbol{e}_2$, where $J_2 = \boldsymbol{\tau}\cdot\boldsymbol{E}^{(1)} = \boldsymbol{\tau}\cdot\boldsymbol{E}^{(2)}$; and, $\boldsymbol{I} = I_\tau\boldsymbol{\tau}$, where $I_\tau = -H_2^{(1)} = -H_2^{(2)}$.

For the boundary points $\boldsymbol{\xi} = (\xi_1,\xi_3)$, $\boldsymbol{\eta} = (\eta_1,\eta_3)$ on the interface $S_\Lambda : \eta_3 = f(\eta_1)$, $\eta_1 \in \langle 0,\Lambda\rangle$ we obtain the following system of the boundary integral equations [4]

$$J_2(\boldsymbol{\xi}) = -J_0(\boldsymbol{\xi}) - \mathrm{i}k_0\boldsymbol{\tau}_\xi\cdot\int\limits_{S_\Lambda} I_\tau\boldsymbol{\tau}_\eta(\Psi^{(1)}-\Psi^{(2)})\,dl_\eta$$

$$-\frac{1}{\mathrm{i}k_0}\boldsymbol{\tau}_\xi\cdot\int\limits_{S_\Lambda}\frac{1}{\sigma}\frac{dI_\tau}{d\eta_1}\,\nabla_\eta\left(\frac{1}{\varepsilon^{(1)}}\Psi^{(1)} - \frac{1}{\varepsilon^{(2)}}\Psi^{(2)}\right)dl_\eta\; + \boldsymbol{\nu}_\xi\cdot\int\limits_{S_\Lambda} J_2\nabla_\eta(\Psi^{(1)}-\Psi^{(2)})\,dl_\eta\;, \tag{10}$$

$$I_\tau(\boldsymbol{\xi}) = -I_0(\boldsymbol{\xi}) - \mathrm{i}k_0\int\limits_{S_\Lambda} J_2(\varepsilon^{(1)}\Psi^{(1)} - \varepsilon^{(2)}\Psi^{(2)})\,dl_\eta + \int\limits_{S_\Lambda} I_\tau\;\boldsymbol{\nu}_\eta\cdot\nabla_\eta\left(\Psi^{(1)}-\Psi^{(2)}\right)\,dl_\eta\;, \tag{11}$$

where the terms $J_0(\boldsymbol{\xi})$ and $I_0(\boldsymbol{\xi})$ represent the incident wave in $\Omega^{(1)}$.

To derive these equations it was necessary to study properties of the integral operators

$$\boldsymbol{\mathcal{V}}^{(j)}v(\boldsymbol{x}) = \int\limits_{S_\Lambda} v(\boldsymbol{\eta})\Psi^{(j)}(\boldsymbol{x},\boldsymbol{\eta})\;dl_\eta\;, \qquad \boldsymbol{\mathcal{W}}^{(j)}v(\boldsymbol{x}) = \int\limits_{S_\Lambda} v(\boldsymbol{\eta})\frac{\partial\Psi^{(j)}(\boldsymbol{x},\boldsymbol{\eta})}{\partial\boldsymbol{\nu}_\eta}\;dl_\eta\;,$$

$$\boldsymbol{\mathcal{L}}^{(j)} v(\boldsymbol{x}) = \int\limits_{S_\Lambda} v(\boldsymbol{\eta}) \nabla_\eta \Psi^{(j)}(\boldsymbol{x}, \boldsymbol{\eta})\ dl_\eta\ , \qquad j = 1, 2 \tag{12}$$

when the inner point $\boldsymbol{x}$ tends to the boundary point $\boldsymbol{\xi}$ in the normal direction.

Whereas the first and the second of them are the well-known single and double layer potentials, the third one is worth to mention.

Theorem 2. *If S is the smooth boundary of the domain $\Omega \subset \mathbb{R}^2$ with the unit outward normal $\boldsymbol{\nu}$ and $v \in \mathrm{C}(S)$, then*

$$\lim_{\boldsymbol{x} \to \boldsymbol{\xi}} \boldsymbol{\mathcal{L}}^{(j)} v(\boldsymbol{x}) = \int\limits_{S_\Lambda} v(\boldsymbol{\eta}) \nabla_\eta \Psi^{(j)}(\boldsymbol{\xi}, \boldsymbol{\eta})\ dl_\eta \pm \frac{1}{2} v(\boldsymbol{\xi}) \boldsymbol{\nu}_\xi\ , \tag{13}$$

where $\boldsymbol{\xi} \in S$, minus holds for $\boldsymbol{x} \in \Omega$ and plus for $\boldsymbol{x} \in \mathbb{R}^2 \setminus \bar{\Omega}$.

This theorem is the vector generalization of the well-known statements for scalar integral operators, see e.g. [8] , Chapter 6.

To simplify calculations we introduce the parametrization

$$\boldsymbol{\pi} : \langle 0, 2\pi \rangle \to \mathbb{R}^2\ , \quad \boldsymbol{\pi}(t) = (p(t), q(t)) \tag{14}$$

of the curve $x_3 = f(x_1)$ having the unit normal vector $\boldsymbol{\nu}(t)$ with the norm $\nu(t) = \sqrt{p'(t)^2 + q'(t)^2}$.

The resulting system of the boundary integral equations for the scalar components I_τ and J_2 derived in [4] is of the following form:

$$\begin{aligned}
&-\mathrm{i}k_0 \mu \boldsymbol{\tau}(s) \cdot \int\limits_0^{2\pi} I_\tau(t) \boldsymbol{\tau}(t) \left(\Psi^{(1)}(s,t) - \Psi^{(2)}(s,t) \right)\ \nu(t)\ dt \\
&- \frac{1}{\mathrm{i}k_0} \boldsymbol{\tau}(s) \cdot \int\limits_0^{2\pi} \rho I_\tau(t) \nabla_t \left[\frac{1}{\varepsilon^{(1)}} \Psi^{(1)}(s,t) - \frac{1}{\varepsilon^{(2)}} \Psi^{(2)}(s,t) \right] \nu(t)\ dt \\
&+ J_2(s) - \boldsymbol{\nu}(s) \cdot \int\limits_0^{2\pi} J_2(t) \nabla_t \left[\Psi^{(1)}(s,t) - \Psi^{(2)}(s,t) \right]\ \nu(t)\ dt \ = \ -J_{2,0}(s)\ ,
\end{aligned} \tag{15}$$

$$\begin{aligned}
&I_\tau(s) + \mathrm{i}k_0 \int\limits_0^{2\pi} J_2(t) \left(\varepsilon^{(1)} \Psi^{(1)}(s,t) - \varepsilon^{(2)} \Psi^{(2)}(s,t) \right] \nu(t)\ dt \\
&- \int\limits_0^{2\pi} I_\tau(t) \boldsymbol{\nu}(t) \cdot \nabla_t \left[\Psi^{(1)}(s,t) - \Psi^{(2)}(s,t) \right]\ \nu(t)\ dt \ = \ -I_{\tau,0}(s)\ ,
\end{aligned} \tag{16}$$

where the functions $\Psi^{(j)}(s,t)$ in the operators kernels are the parametrized periodical fundamental solutions (6) of the Helmholtz equation (5) in $\Omega^{(j)}$.

Note, that the singularity of the logarithmic type is of the key importance, because it enables to split the operators into the compact ones with the continuous kernels and the other with the logarithmic singularity:

$$\Psi^{(j)}(s,t) = \Psi^{(j)}_r(s,t) + \psi(s,t) \tag{17}$$

with the regular part

$$\Psi_r^{(j)}(s,t) = \Psi_0^{(j)}(s,t) + \sum_{\substack{m\,\in\,\mathbb{Z}\\ m\,\neq\,0}} \left(\Psi_m^{(j)}(s,t) - \frac{1}{2\pi} \frac{\mathrm{e}^{-\mathrm{i}m(s-t)}}{2|m|} \right) , \tag{18}$$

and, the sigular one

$$\psi(s,t) = \frac{1}{2\pi} \ln \left| 2 \sin \frac{s-t}{2} \right| = \frac{1}{2\pi} \sum_{\substack{m\,\in\,\mathbb{Z}\\ m\,\neq\,0}} \frac{\mathrm{e}^{-\mathrm{i}m(s-t)}}{2|m|} . \tag{19}$$

4 Numerical implementation

To solve the system of the boundary integral equations (15),(16) we use the collocation method with $2N+1$ equidistant collocation points $s_j = \frac{2\pi j}{2N}$, $j = 0, \ldots, 2N$.

We seek for the discrete solutions

$$I_\tau(s) = \sum_{k=0}^{2N} c_k \phi_k(s) , \qquad J_2(s) = \sum_{k=0}^{2N} d_k \phi_k(s) \tag{20}$$

with an interpolation basis $\{\phi_k\}_{k=0}^{2N}$. The choice of the best basis functions system appears to be very important. The system of trigonometric polynomials, linear splines (piecewise linear functions) or cubic splines are the usual choices of basis functions. After experiments with mentioned basis functions we prefer the system of trigonometric polynomials with the nodes identical with the collocation points ($\phi_k(s_j) = \delta_{kj}$), i.e.

$$\phi_k(t) = \frac{1}{2N+1} \sum_{\ell=-N}^{N} \mathrm{e}^{-\frac{2\pi \mathrm{i}\ell k}{2N+1}} \mathrm{e}^{\mathrm{i}\ell t} , \qquad k = 0, 1, 2 \ldots, 2N . \tag{21}$$

Furthermore, we find advantageous to take the order N of the boundary discretization equal to the order of the diffraction modes truncation in the Green function (6), so that

$$\Psi^{(j)}(s,t) \approx \frac{1}{2\mathrm{i}\Lambda} \sum_{m=-N}^{N} \Psi_m^{(j)}(s,t) , \quad j = 1, 2 . \tag{22}$$

Since the integral operators in the solved system are splitted by (17), we evaluate numerically the compact operators with the continuous kernels – the trapezoidal rule with the nodes in the collocation points (i.e. $t_j = s_j$) gives sufficiently accurate results. The logarithmic-type singular operators can be evaluated analytically [6].

5 Numerical results

As an example, we consider the smooth sine boundary

$$S: \; x_3 = \frac{h}{2} \left(1 + \cos \frac{2\pi x_1}{\Lambda} \right) , \; x_1 \in \langle 0, \Lambda \rangle , \quad \Lambda = 500 \; nm, \; h = 50 \; nm$$

between two regions with the indices of refraction $n_1 = 1$ (air) and $n_2 = 1.5$ (glass), $n_j = \sqrt{\varepsilon^{(j)}}$. The incident beam of the wavelength $\lambda = 632.8$ nm propagates under the given angle of incidence θ. The Fig. 2 illustrates the increasing accuracy of approximation with growing discretization order. As analytical solution of the problem is not available we compare numerical solutions for various values of N.

Obtained results are demonstrated by the absolute value of the complex tangential component of the field $\boldsymbol{H}$ at one period of the common boundary. The low discretization orders enable more perspicuous view because the data calculated at collocation points are nearly equal (in the graph) roughly for $N \geq 30$. Note that we aimed to functionality verification of presented model as well as of proposed algorithm.

The distribution of reflected field $|H_2^{(1)}|$ in the superstrate is demonstrated at the Fig. 3 near to the boundary for several incidence angles.

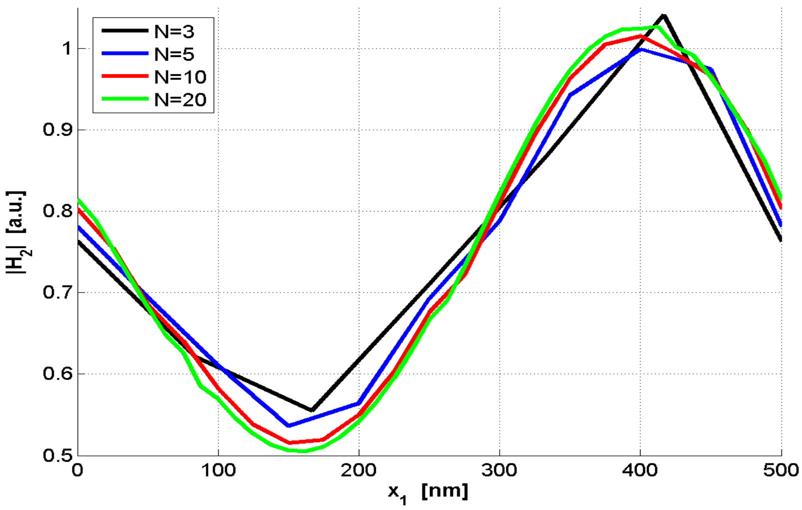

Figure 2: The convergence of the used BEM algorithm (incidence angle $\theta = 40°$).

Conclusion

The result obtained using the presented BEM algorithm shows possible applicability of the approach based on the tangential fields to the problems, in which the detailed analysis of the diffracted optical field at an interface and/or in the near region is studied. We suppose to exploit this method in future to the surface plasmon modelling.

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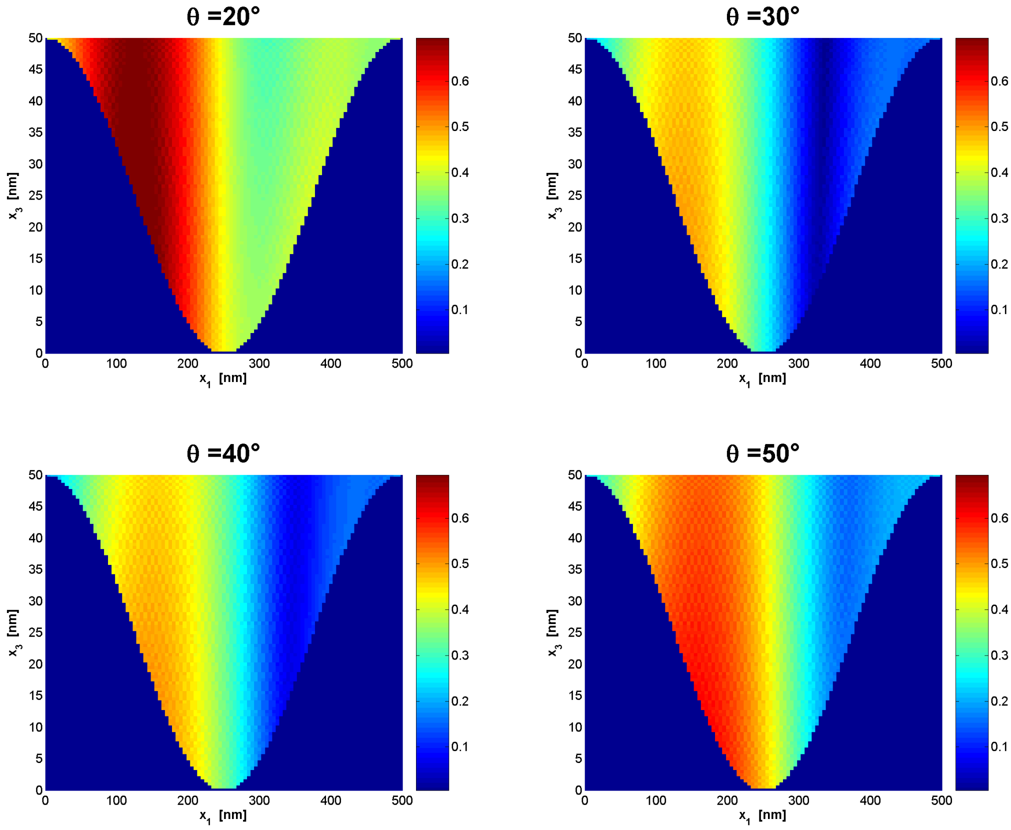

Figure 3: Reflected field $|H_2^{(1)}|$ for chosen incidence angle θ ($N = 50$).

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Matematické modelování periodických difrakčních úloh v optice

Abstrakt: Optická difrakce na periodckém rozhraní patří k relativně málo prozkoumaným aplikacím metody hraničních integrálních rovnic. V příspěvku je popsána méně obvyklá formulace difrakční úlohy pomocí vektorových tečných polí. Dále jsou diskutovány vlastnosti odvozených integrálních operátorů se singulárním jádrem stejně jako některé problémy související s numerickou implementací.

Klíčová slova: optická difrakce, tečná pole, metoda hraničních prvků.

The semi-smooth Newton method for Solving the Stokes flow under the leak boundary condition

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Abstract: We consider the Stokes equations under the leak boundary condition. Using the P1-bubble/P1 finite element approximation we get the algebraic optimization problem. Its optimality conditions are the starting point for the algorithm. We use an active set implementation of the semi-smooth Newton method to find the solution. Numerical experiments demonstrate the computational efficiency of an adaptive diagonal preconditioner.

Keywords: Stokes flow, leak boundary condition, semi-smooth Newton method, conjugate gradient method, preconditioning.

1 Introduction

Let Ω be a bounded domain in $\mathbb{R}^2$ with a sufficiently smooth boundary $\partial\Omega$ that is split into three nonempty disjoint parts: $\partial\Omega = \overline{\gamma}_D \cup \overline{\gamma}_N \cup \overline{\gamma}_C$. We consider the model of a viscous incompressible Newtonian fluid modelled by the Stokes system with the Dirichlet and Neumann boundary conditions on γ_D and γ_N, respectively, and with the leak boundary condition of the Navier-Tresca type on γ_C. We are searching for a vector function representing the flow velocity field $\mathbf{u} : \bar{\Omega} \to \mathbb{R}^2$, $\mathbf{u} = (u_1, u_2)$ and a scalar function representing the pressure field $p : \bar{\Omega} \to \mathbb{R}$ so that:

$$\left.\begin{aligned} -\nu\Delta\mathbf{u} + \nabla p = \mathbf{f} &\quad \text{in } \Omega,\\ \nabla\cdot\mathbf{u} = 0 &\quad \text{in } \Omega,\\ \mathbf{u} = \mathbf{u}_D &\quad \text{on } \gamma_D,\\ \boldsymbol{\sigma} = \boldsymbol{\sigma}_N &\quad \text{on } \gamma_N,\\ u_t = 0 &\quad \text{on } \gamma_C,\\ u_n = 0 \Rightarrow |\sigma_n| \le g &\quad \text{on } \gamma_C,\\ \sigma_n u_n + g|u_n| + \kappa u_n^2 = 0 &\quad \text{on } \gamma_C, \end{aligned}\right\} \tag{1}$$

where $\nu > 0$ is the viscosity, $\mathbf{f} : \bar{\Omega} \to \mathbb{R}^2$ describes the forces acting on the fluid, $\mathbf{u}_D : \gamma_D \to \mathbb{R}^2$ and $\boldsymbol{\sigma}_N : \gamma_N \to \mathbb{R}^2$ are the Dirichlet and Neumann boundary data, respectively. Further, $\mathbf{n}$ and

$\mathbf{t}$ are the unit outward normal and tangential vectors on $\partial\Omega$, for which we define the normal and tangential components of the velocity and the stress $u_n = \mathbf{u}\cdot\mathbf{n}$, $u_t = \mathbf{u}\cdot\mathbf{t}$, $\sigma_n = \nu\boldsymbol{\sigma}\cdot\mathbf{n}$, $\sigma_t = \boldsymbol{\sigma}\cdot\mathbf{t}$, respectively, where $\boldsymbol{\sigma} = \nu\partial\mathbf{u}/\partial\mathbf{n} - p\mathbf{n}$ is the stress vector on $\partial\Omega$ in the normal direction corresponding to a non-symmetric tensor. On γ_C we consider the given leak bound $g : \gamma_C \to \mathbb{R}_+$ and the adhesive coefficient $\kappa : \gamma_C \to \mathbb{R}_+$ defining the leak boundary condition. We get the classical Navier law for $g = 0$, while $\kappa = 0$ leads to the Tresca law. We assume that γ_D, γ_N, and γ_C are always non-empty sets.

2 Algebraic formulation

After the mixed finite element approximation based on the P1-bubble/P1 finite elements [8] we arrive at the minimization formulation with the following optimality conditions:

$$\left.\begin{array}{l}
\textit{Find } (\mathbf{u},\mathbf{p},\boldsymbol{s}_n,\boldsymbol{\lambda}_t) \in \mathbb{R}^{n_u}\times\mathbb{R}^{n}\times\mathbb{R}^{n_c}\times\mathbb{R}^{n_c} \textit{ such that}\\
\mathbf{Au} - \mathbf{l} + \mathbf{N}^T\mathbf{s}_n + \mathbf{T}^T\boldsymbol{\lambda}_t + \mathbf{B}^T\mathbf{p} = \mathbf{0},\\
\mathbf{Bu} - \mathbf{Ep} - \mathbf{c} = \mathbf{0},\\
\mathbf{Tu} = \mathbf{0},\\
\left.\begin{array}{l}
(\mathbf{Nu})_i = 0 \Rightarrow |s_{ni}| \le g_i,\\
(\mathbf{Nu})_i > 0 \Rightarrow s_{ni} = g_i + \kappa_i(\mathbf{Nu})_i,\\
(\mathbf{Nu})_i < 0 \Rightarrow s_{ni} = -g_i + \kappa_i(\mathbf{Nu})_i,
\end{array}\right\} \; i \in \mathcal{N},
\end{array}\right\} \tag{2}$$

where $\mathbf{s}_n = \boldsymbol{\lambda}_n + \mathbf{D}(\boldsymbol{\kappa})\mathbf{Nu}$ and $\mathcal{N} = \{1,...,n_c\}$. Here, $\mathbf{A} \in \mathbb{R}^{n_u\times n_u}$ is the symmetric, positive definite stiffness matrix for the Laplace operator, $\mathbf{l} \in \mathbb{R}^{n_u}$, $\mathbf{B} \in \mathbb{R}^{n\times n_u}$ is the full row rank stiffness matrix for the divergence operator, $\mathbf{T},\mathbf{N} \in \mathbb{R}^{n_c\times n_u}$ are the full row rank matrices given by the normal and tangential vectors at nodes $\mathbf{x}_i \in \bar{\gamma}_C\backslash\bar{\gamma}_D$, respectively, $\mathbf{D}(\boldsymbol{\kappa}) = \mathrm{diag}(\boldsymbol{\kappa}) \in \mathbb{R}^{n_c\times n_c}$, $\boldsymbol{\kappa} = (\kappa_1,...,\kappa_{n_c})^T \in \mathbb{R}^{n_c}$, $\kappa_i = h_i\kappa(\mathbf{x}_i)$, $g_i = h_i g(\mathbf{x}_i)$, and h_i is the length of the segment corresponding to $\mathbf{x}_i$, $i \in \mathcal{N}$; n_u is the number of the velocity components, n is the number of the finite elements nodes, and n_c is the number of the leak nodes. The symmetric, positive semidefinite matrix $\mathbf{E} \in \mathbb{R}^{n_c\times n_c}$ and the vector $\mathbf{c} \in \mathbb{R}^{n_c}$ arise from the elimination of the bubble components. While the unknowns $\mathbf{u}$, $\mathbf{p}$ are the vectors of the velocity and pressure components, respectively, $\boldsymbol{\lambda}_t$, $\boldsymbol{\lambda}_n$ are the Lagrange multipliers and $\mathbf{s}_n$ approximates the (negative) shear stress σ_n.

3 Semi-smooth Newton method

Its convenient to use the semi-smooth Newton method to find the solution of (2). Firstly, we reformulate the leak boundary condition in (2) as a nonsmooth equation. We introduce the projection on the interval $[a,b]$,

$$P_{[a,b]}(x) = x - \max\{0, x-b\} + \max\{0, a-x\}, \quad x \in \mathbb{R}$$

and represent the leak boundary condition from (2), see Lemma 1 in Appendix such that

$$\left.\begin{array}{rcll}
(\boldsymbol{N}\mathbf{u})_i &=& \max\{0, \kappa^{-1}(s_{ni}-g_i)\} - \max\{0, -\kappa^{-1}(s_{ni}+g_i)\} & \text{for} \quad \rho_i = \kappa_i > 0,\\
\rho_i(\boldsymbol{N}\mathbf{u})_i &=& \max\{0, s_{ni}-g_i+\rho_i(\boldsymbol{N}\mathbf{u})_i\} - \max\{0, -s_{ni}-g_i-\rho_i(\boldsymbol{N}\mathbf{u})_i\} & \text{for} \quad \kappa_i = 0,
\end{array}\right\}.$$

Firstly, we divide the index set $\mathcal{N}$ into two sets $\mathcal{N}_0$ and $\mathcal{N}_+$ so that $\mathcal{N} = \mathcal{N}_0 \cup \mathcal{N}_+$ as follows:

$$\mathcal{N}_0 = \{i \in \mathcal{N} : \kappa_i = 0\}, \quad \mathcal{N}_+ = \{i \in \mathcal{N} : \kappa_i > 0\}.$$

Let us write the problem (2) as one equation:

$$\boldsymbol{G}(\mathbf{y}) = 0, \tag{3}$$

where $\boldsymbol{G}(\mathbf{y}) = (\boldsymbol{G}_1^\top(\mathbf{y}), \boldsymbol{G}_2^\top(\mathbf{y}), \boldsymbol{G}_3^\top(\mathbf{y}), \boldsymbol{G}_4^\top(\mathbf{y}))^\top$ and $\mathbf{y} = (\mathbf{u}^\top, \mathbf{s}_n^\top, \lambda_t^\top, \boldsymbol{p}^\top)^\top$, where $\boldsymbol{G}_1(\mathbf{y}) = \mathbf{Au} - \mathbf{l} + \mathbf{N}^T\mathbf{s}_n + \mathbf{T}^T\boldsymbol{\lambda}_t + \mathbf{B}^T\mathbf{p}$,

$$\begin{aligned} \boldsymbol{G}_2(\mathbf{y}) &= \boldsymbol{N}_+\mathbf{u} - \max\{0, \boldsymbol{D}_{\kappa_+}^{-1}(s_{n_+} - g_+)\} + \max\{0, -\boldsymbol{D}_{\kappa_+}^{-1}(s_{n_+} + g_+)\}, \\ \boldsymbol{G}_3(\mathbf{y}) &= \rho\boldsymbol{N}_0\mathbf{u} - \max\{0, s_{n_0} - g_0 + \rho\boldsymbol{N}_0\mathbf{u}\} + \max\{0, -s_{n_0} - g_0 - \rho\boldsymbol{N}_0\mathbf{u}\}, \end{aligned}$$

$\boldsymbol{G}_4(\mathbf{y}) = \mathbf{Tu}$ and $\boldsymbol{G}_5(\mathbf{y}) = \mathbf{Bu} - \mathbf{Ep} - \mathbf{c} = \mathbf{0}$.

The equation (3) can be solved by the semi-smooth Newton method, because $\boldsymbol{G}$ is semimooth in the sense of [5].

4 Algorithm

According to the division of $\mathcal{N}$, we define for each $\mathbf{y}^k$ two types of the inactive sets

$$\mathcal{I}_+^+ = \{i \in \mathcal{N}_+ : s_{ni}^k \geq g_i\}, \quad \mathcal{I}_+^- = \{i \in \mathcal{N}_+ : s_{ni}^k \leq -g_i\}$$

and

$$\mathcal{I}_0^+ = \{i \in \mathcal{N}_0 : s_{ni}^k + \rho(Nu)_i \geq g_i\}, \quad \mathcal{I}_0^- = \{i \in \mathcal{N}_0 : s_{ni}^k + \rho(Nu)_i \leq -g_i\}$$

or

$$\mathcal{I}_0^+ = \{i \in \mathcal{N}_0 : s_{ni}^k - \rho r_i \geq g_i\}, \quad \mathcal{I}_0^- = \{i \in \mathcal{N}_0 : s_{ni}^k - \rho r_i \leq -g_i\}$$

and the active sets as their complements $\mathcal{A}_+ = \mathcal{N}_+ \setminus (\mathcal{I}_+^+ \cup \mathcal{I}_+^-)$ and $\mathcal{A}_0 = \mathcal{N}_0 \setminus (\mathcal{I}_0^+ \cup \mathcal{I}_0^-)$.

Further, we define the indicator matrices $\mathbf{D}(\mathcal{I}_+^-)$, $\mathbf{D}(\mathcal{I}_+^+)$ and $\mathbf{D}(\mathcal{I}_0^-)$, $\mathbf{D}(\mathcal{I}_0^+)$, respectively. Note that the indicator matrix to $\mathscr{S}_+ \subseteq \mathcal{N}_+$ is given by $\mathbf{D}(\mathscr{S}_+) = \operatorname{diag}(s_1, ..., s_{n_{c+}}) \in \mathbb{R}^{n_{c+} \times n_{c+}}$, where $n_{c+} := |\mathscr{S}_+| \leq n_c$, with $s_i = 1$ for $i \in \mathscr{S}_+$ and $s_i = 0$ if $i \notin \mathscr{S}_+$. The new iterate $\mathbf{y}^{k+1}$ is computed by solving the following linear system. Moreover, in the case $\mathcal{N}_0 \neq \emptyset$ we get $\mathbf{s}_{n,\mathcal{A}_0} = \boldsymbol{\lambda}_{n,\mathcal{A}_0}$ and set:

$$\mathbf{s}_{n,\mathcal{I}_0^+} = \mathbf{g}_{\mathcal{I}_0^+}, \qquad \mathbf{s}_{n,\mathcal{I}_0^-} = -\mathbf{g}_{\mathcal{I}_0^-}.$$

$$\left(\begin{array}{c|cccc} \mathbf{A} & \mathbf{N}_+^T & \mathbf{N}_{\mathcal{A}_0}^T & \mathbf{T}^T & \mathbf{B}^T \\ \hline \mathbf{N}_+ & -\mathbf{D}(\boldsymbol{\kappa}_+)^{-1}\mathbf{D}(\mathcal{I}_+^+ \cup \mathcal{I}_+^-) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{N}_{\mathcal{A}_0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{T} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{E} \end{array}\right) \begin{pmatrix} \mathbf{u}^{k+1} \\ \mathbf{s}_{n+}^{k+1} \\ \boldsymbol{\lambda}_{n\mathcal{A}_0}^{k+1} \\ \boldsymbol{\lambda}_t^{k+1} \\ \mathbf{p}^{k+1} \end{pmatrix}$$
$$= \begin{pmatrix} \mathbf{l} - \mathbf{N}_{\mathcal{I}_0^+}^T\mathbf{g}_{\mathcal{I}_0^+} + \mathbf{N}_{\mathcal{I}_0^-}^T\mathbf{g}_{\mathcal{I}_0^-} \\ \mathbf{D}(\boldsymbol{\kappa}_+)^{-1}(\mathbf{D}(\mathcal{I}_+^-) - \mathbf{D}(\mathcal{I}_+^+))\mathbf{g}_+ \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{c} \end{pmatrix}. \tag{4}$$

The conjugate gradient method with adaptive precision control is used to find the solution. As you can see in the section Numerical experiments, it is convenient to use a preconditioner.

5 Preconditioning

To solve bigger linear systems and more complex meshes we use the Schur complement to solve (4):

$$\boldsymbol{S}^k\mathbf{r}^{k+1} = \boldsymbol{C}\boldsymbol{A}^{-1}\mathbf{l} - \mathbf{h}^k,$$

where $\boldsymbol{S}^k = \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{C}^\top + \bar{\boldsymbol{E}}^k$, where

$$\boldsymbol{C} = \begin{pmatrix} \boldsymbol{N}_+ \\ \boldsymbol{N}_{\mathcal{A}_0} \\ \boldsymbol{T} \\ \boldsymbol{B} \end{pmatrix}, \quad \bar{\boldsymbol{E}}^k \begin{pmatrix} \mathbf{D}(\boldsymbol{\kappa}_+)^{-1}\mathbf{D}(\mathcal{I}_+^+ \cup \mathcal{I}_+^-) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{E} \end{pmatrix}$$

and

$$\mathbf{r}^{k+1} = \begin{pmatrix} \mathbf{s}_{n_+}^{k+1} \\ \lambda_{n\mathcal{A}_n}^{k+1} \\ \lambda_t^{k+1} \\ \boldsymbol{p}^{k+1} \end{pmatrix}, \mathbf{h}^k = \begin{pmatrix} \mathbf{D}(\boldsymbol{\kappa}_+)^{-1}(\mathbf{D}(\mathcal{I}_+^-) - \mathbf{D}(\mathcal{I}_+^+))\mathbf{g}_+ \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{c} \end{pmatrix}.$$

We use the diagonal preconditioner

$$\boldsymbol{P}^k = diag\boldsymbol{S}^k.$$

6 Numerical experiments

We consider the L-shaped domain $\Omega = (0,5) \times (0,2) \setminus \overline{\mathcal{S}}$, $\mathcal{S} = (0,1) \times (0,1)$ with $\nu = 1$ and $\mathbf{f} = \mathbf{0}$; $\gamma_D = \gamma_{top} \cup \gamma_{left}$ with $\gamma_{top} = (0,5) \times \{2\}$, $\gamma_{left} = \{0\} \times (1,2)$, $\mathbf{u}_{D|\gamma_{top}} = \mathbf{0}$, and $\mathbf{u}_{D|\gamma_{left}} = (4(x_2-2)(1-x_2), 0)$; $\gamma_N = \{5\} \times (0,2)$ with $\boldsymbol{\sigma}_N = \mathbf{0}$; $\gamma_C = \partial\Omega \setminus \overline{(\gamma_D \cup \gamma_N)}$ with $g = 10$ for $x < 1$ and else $g = 1$. In tables below we report $iter/n_S$, where $iter$ is the number of the outer (Newton) iterations, while n_S denotes the total number of the matrix-vector multiplications by the Schur complements. Note that n_S determines the computational efficiency. The computational efficiency without preconditioning for different adhesive coefficients κ is shown in Table 1. One can see that n_S increases considerable for finer meshes and smaller κ. This unacceptable effect is eliminated by preconditioning, as it is seen from Table 2.

Table 1: The computational complexity for different κ without preconditioning.

$n_u/n/n_c$	$\kappa = 1$	$\kappa = 0.5$	$\kappa = 0.1$	$\kappa = 0.01$	$\kappa = 0.001$	$\kappa = 0$
344/206/32	15/1358	15/1139	20/1614	23/1984	23/1919	10/865
1352/744/64	18/3143	18/2666	19/2888	19/3817	24/4925	15/2047
5366/2819/128	16/4248	18/5440	23/7591	20/8558	23/12066	18/3889
21386/10965/256	17/6526	21/11170	17/14308	17/14207	26/27023	17/5468
85394/43241/512	21/17776	22/20523	24/24900	21/29581	24/34877	18/8640

Table 2: The computational complexity for different κ with preconditioning.

$n_u/n/n_c$	$\kappa=1$	$\kappa=0.5$	$\kappa=0.1$	$\kappa=0.01$	$\kappa=0.001$	$\kappa=0$
344/206/32	8/162	10/179	10/163	16/186	7/185	7/152
1352/744/64	9/191	9/180	10/179	10/140	6/82	9/205
5366/2819/128	9/226	10/218	13/234	19/303	23/294	9/233
21386/10965/256	8/256	10/291	12/250	17/351	24/435	7/213
85394/43241/512	11/407	10/331	14/352	8/166	9/177	10/388

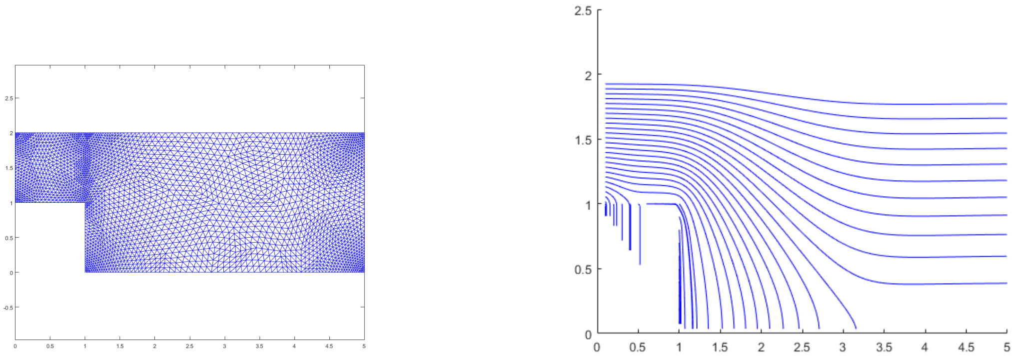

Figure 1: Finite element approximation and velocity field

Appendix

Lemma 1 *Let* $\lambda, u \in \mathbb{R}^1,\ g \geq 0,\ \kappa \geq 0$. *The relations*

$$\left.\begin{array}{rcrcl} |\lambda| & \leq & g & \Rightarrow & u = 0 \\ \lambda & > & g & \Rightarrow & \lambda = g + \kappa u \\ \lambda & < & -g & \Rightarrow & \lambda = -g + \kappa u \end{array}\right\} \tag{5}$$

hold iff

$$\psi(\lambda, u) = 0,$$

where $\psi(\lambda, u) := \rho u - \max\{0, \lambda - g + (\rho - \kappa)u\} + \max\{0, -\lambda - g + (\kappa - \rho)u\}$, *see Figure 2.*

Proof: We assume $g > 0$, as $g = 0$ is trivial. First we prove the implication '$\Rightarrow$'. The relations (5) are satisfied. In the first case $|\lambda| \leq g$ and $u = 0$ we get $\psi(\lambda, 0) = 0 - \max\{0, \lambda - g\} + \max\{0, -\lambda - g\} = 0$. In the second case $\lambda > g$ and $\lambda = g + \kappa u$ we get $\psi(\lambda, u) := \rho u - \max\{0, \rho u\} + \max\{0, -2g - \rho u\} = \rho u - \rho u + 0 = 0$. In the third case $\lambda < -g$ and $\lambda = -g + \kappa u$ we get $\psi(\lambda, u) := \rho u - \max\{0, -2g + \rho u\} + \max\{0, -\rho u\} = \rho u - 0 - \rho u = 0$. To prove the opposite implication, we start from $\rho u = \max\{0, \lambda - g + (\rho - \kappa)u\} - \max\{0, -\lambda - g + (\kappa - \rho)u\}$. If $|\lambda| \leq g$ then $\lambda - g + (\rho - \kappa)u > 0$ and $-\lambda - g + (\kappa - \rho)u < 0$ or $-\lambda - g + (\kappa - \rho)u > 0$ and $\lambda - g + (\rho - \kappa)u < 0$. In the first case we suppose $\lambda - g + (\rho - \kappa)u = \rho u$ and $-\lambda - g + (\kappa - \rho)u < 0$, in the second case $-\lambda - g + (\kappa - \rho)u = -\rho u$ and $\lambda - g + (\rho - \kappa)u < 0$, $\lambda - g \leq 0$ as well as $-\lambda - g \leq 0$, then $u = 0$. Analogously, in the case $\lambda > g$ we get $\lambda = g + \kappa u$ and $\lambda = -g + \kappa u$ the last case. $\square$

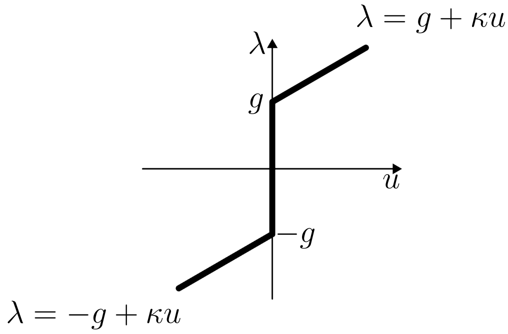

Figure 2: Projection

Conclusion

We have analysed the numerical solution of the Stokes flow with the leak boundary condition of friction type. The analysis is analogical to the analysis of the Stokes flow with stick-slip boundary condition. The numerical experiments have shown that it is necessary to use the preconditioner in the conjugate gradient method to implement a computationaly efficient solver.

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Nehladká Newtonova metoda pro řešení Stokesova proudění s průsakem

Abstrakt (Streszczenie): Uvažujeme Stokesovu úlohu s průsakem. Po aproximaci pomocí konečných prvků P1-bubble/P1 dostáváme algebraický optimalizační problém. Jeho podmínky optimality jsou výchozím bodem pro algoritmus. Pro nalezení řešení problému použijeme nehladkou Newtonovu metodu implementovanou pomocí aktivních množin. Numerické experimenty demonstrují výpočetní efektivitu adaptivního diagonálního předpodmíňovače.

Klíčová slova (Słowa kluczowe): Stokesovo proudění, nehladká Newtonova metoda, průsak, metoda konjugovanych gradientu, předpodmínění.

Semi-smooth Newton method for solving the Stokes problem with the stick-slip boundary condition

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Abstract: The paper deals with the Stokes flow with the monotonously increasing threshold slip boundary condition. Using the P1-bubble/P1 finite element approximation of the velocity-pressure formulation we arrive at an algebraic variational inequality. This inequality is equivalent to a saddle-point problem whose optimality conditions are the starting point for the algorithm. The semi-smooth Newton method implementation of the algorithm is based on active/inactive sets. We discuss two possible options on how to create them. The algorithm is then tested in MATLAB environment. Experiments are done on the "L-shaped" domain, where we study the effects of the adhesive coefficient and preconditioning on the efficiency of computations.

Keywords: semi-smooth Newton method, stick-slip condition, Stokes problem, preconditioning

1 Introduction

Let Ω be a bounded domain in $\mathbb{R}^2$ with a sufficiently smooth boundary $\partial\Omega$ that is split into three nonempty disjoint parts: $\partial\Omega = \overline{\gamma}_{\mathrm{D}} \cup \overline{\gamma}_{\mathrm{N}} \cup \overline{\gamma}_{\mathrm{C}}$. We consider the model of a viscous incompressible Newtonian fluid modelled by the Stokes system with the Dirichlet and Neumann boundary conditions on γ_{D} and γ_{N}, respectively, and with the impermeability and the stick-slip boundary condition of the Navier-Tresca type on γ_{C}:

$$\left.\begin{aligned} -\nu\Delta\mathbf{u} + \nabla p &= \mathbf{f} && \text{in } \Omega,\\ \nabla\cdot\mathbf{u} &= 0 && \text{in } \Omega,\\ \mathbf{u} &= \mathbf{u}_{\mathrm{D}} && \text{on } \gamma_{\mathrm{D}},\\ \boldsymbol{\sigma} &= \boldsymbol{\sigma}_{\mathrm{N}} && \text{on } \gamma_{\mathrm{N}},\\ u_n &= 0 && \text{on } \gamma_{\mathrm{C}},\\ u_t = 0 \Rightarrow |\sigma_t| &\leq \ \mathrm{g} && \text{on } \gamma_{\mathrm{C}},\\ \sigma_t u_t + \mathrm{g}|u_t| + \kappa u_t^2 &= 0 && \text{on } \gamma_{\mathrm{C}}. \end{aligned}\right\} \tag{1}$$

We are searching for a vector function representing the flow velocity field $\mathbf{u} : \bar{\Omega} \to \mathbb{R}^2$, $\mathbf{u} = (u_1, u_2)$ and a scalar function representing the pressure field $p : \bar{\Omega} \to \mathbb{R}$, where $\nu > 0$ is the dynamic viscosity, $\mathbf{f} : \bar{\Omega} \to \mathbb{R}^2$ describes the forces acting on the fluid, $\mathbf{u}_{\mathrm{D}} : \gamma_{\mathrm{D}} \to \mathbb{R}^2$ and $\boldsymbol{\sigma}_{\mathrm{N}} : \gamma_{\mathrm{N}} \to \mathbb{R}^2$ are the Dirichlet and Neumann boundary data, respectively. Further $\mathbf{n}$ and $\mathbf{t}$ are the unit outward normal and tangential vectors on $\partial\Omega$ and we define the normal and tangential component of the velocity and the stress as

$$u_n = \mathbf{u} \cdot \mathbf{n},\ u_t = \mathbf{u} \cdot \mathbf{t},\ \sigma_n = \boldsymbol{\sigma} \cdot \mathbf{n},\ \sigma_t = \boldsymbol{\sigma} \cdot \mathbf{t},$$

where

$$\boldsymbol{\sigma} = \nu \frac{\partial \mathbf{u}}{\partial \mathbf{n}} - \mathrm{p}\mathbf{n},\ \frac{\partial \mathbf{u}}{\partial \mathbf{n}} = (\frac{\partial u_1}{\partial \mathbf{n}}, \frac{\partial u_2}{\partial \mathbf{n}}) \quad \text{on } \partial\Omega$$

is the stress vector on $\partial\Omega$. On γ_{C} we consider the given slip bound $g : \gamma_C \to \mathbb{R}_+$ and the adhesive coefficient $\kappa : \gamma_C \to \mathbb{R}_+$ defining the stick-slip boundary condition. We get the classical Navier law for $g = 0$, while $\kappa = 0$ leads to the Tresca law.

The solution of (1) for $\kappa = 0$ has been discussed and solved in [1, 3, 10]. In texts [2] and [9] we can find solutions for $\kappa \neq 0$. Furthermore, in [2] and [7] the interior point method is introduced as one of the methods that may be used in either cases. The other method used to find solution of (1) for $\kappa \neq 0$ is the semi-smooth Newton method [5]. It has been described in [9] and in this text, we will introduce the given algorithm and modify it so that it accepts even $\kappa = 0$.

2 Weak formulation and algebraic problems

The weak formulation of (1) leads to the following variational inequality problem:

$$\left.\begin{array}{l} \text{Find } (\mathbf{u}, p) \in V_{\mathbf{u}_D} \times L^2(\Omega) \text{ so that} \\ a(\mathbf{u}, \mathbf{v} - \mathbf{u}) + b(p, \mathbf{v} - \mathbf{u}) + j(\mathbf{v}, \mathbf{u}) - j(\mathbf{u}, \mathbf{u}) \geq l(\mathbf{v} - \mathbf{u}) \quad \forall \mathbf{v} \in V_{\mathbf{u}_D} \\ b(q, \mathbf{u}) = 0 \quad \forall q \in L^2(\Omega), \end{array}\right\} \tag{2}$$

where $V_{\mathbf{u}_D} = \{\mathbf{v} \in (H^1(\Omega))^2 : \mathbf{v} = \mathbf{u}_D \text{ on } \gamma_D,\ \nu_n = 0 \text{ on } \gamma_C\}$ is the velocity set and

$$a(\mathbf{w}, \mathbf{v}) = \nu \int_\Omega \nabla \mathbf{w} : \nabla \mathbf{v} d\mathbf{x}, \quad b(q, \mathbf{v}) = -\int_\Omega q(\nabla \cdot \mathbf{v}) d\mathbf{x},$$
$$l(\mathbf{v}) = \int_\Omega \mathbf{f} \cdot \mathbf{v} d\mathbf{x} + \int_{\gamma_N} \boldsymbol{\sigma}_N \cdot \mathbf{v} ds, \quad j(\mathbf{v}, \mathbf{w}) = \int_{\gamma_C} g|v_t| + \kappa w_t v_t ds$$

for $\mathbf{v}, \mathbf{w} \in (H^1(\Omega))^2$ and $q \in L^2(\Omega)$. The solvability of (2) can be proven in a similar way as in [3] for $\kappa = 0$. The symbol $\mathbf{u}$ denoting here the vector function will be used also in the subsequent parts for the algebraic vector representing its finite element counterpart.

We approximate (2) by the mixed finite element method based on the P1-bubble/P1 finite elements [8]. The resulting algebraic problem leads to the saddle-point formulation which is equivalent to the following optimality conditions:

$$\left.\begin{array}{l}
\text{Find } (\mathbf{u},\mathbf{p},\mathbf{s}_t,\boldsymbol{\lambda}_n) \in \mathbb{R}^{n_u} \times \mathbb{R}^{n} \times \mathbb{R}^{n_c} \times \mathbb{R}^{n_c} \text{ such that} \\
\mathbf{Au} - \mathbf{l} + \mathbf{T}^T\mathbf{s}_t + \mathbf{N}^T\boldsymbol{\lambda}_n + \mathbf{B}^T\mathbf{p} = \mathbf{0}, \\
\mathbf{Bu} - \mathbf{Ep} - \mathbf{c} = \mathbf{0}, \\
\mathbf{Nu} = \mathbf{0} \\
\left.\begin{array}{l}
(\mathbf{Tu})_i = 0 \Rightarrow |s_{ti}| \le g_i, \\
(\mathbf{Tu})_i > 0 \Rightarrow s_{ti} = g_i + \kappa_i(\mathbf{Tu})_i, \\
(\mathbf{Tu})_i < 0 \Rightarrow s_{ti} = -g_i + \kappa_i(\mathbf{Tu})_i,
\end{array}\right\} \; i \in \mathcal{N},
\end{array}\right\} \qquad (3)$$

where $\mathbf{s}_t = \boldsymbol{\lambda}_t + \mathbf{D}(\boldsymbol{\kappa})\mathbf{Tu}$ and $\mathcal{N} = \{1,\dots,n_c\}$. Here, $\mathbf{A} \in \mathbb{R}^{n_u \times n_u}$ is the symmetric, positive definite stiffness matrix for the Laplace operator, $\mathbf{l} \in \mathbb{R}^{n_u}$, $\mathbf{B} \in \mathbb{R}^{n \times n_u}$ is the full row rank stiffness matrix for the divergence operator. $\mathbf{T}, \mathbf{N} \in \mathbb{R}^{n_c \times n_u}$ are full row rank matrices given by the normal and tangential vectors at the nodes $\mathbf{x}_i \in \bar{\gamma}_C \backslash \bar{\gamma}_D$, respectively. Further, $\mathbf{D}(\boldsymbol{\kappa}) = \operatorname{diag}(\boldsymbol{\kappa}) \in \mathbb{R}^{n_c \times n_c}$, where $\boldsymbol{\kappa} = (\kappa_1,\dots,\kappa_{n_c})^T \in \mathbb{R}^{n_c}$, $\kappa_i = h_i\kappa(\mathbf{x}_i)$, $g_i = h_i g(\mathbf{x}_i)$, and h_i is the length of the segment connecting $\mathbf{x}_i$ and $\mathbf{x}_{i+1}$, $i \in \mathcal{N}$; n_u is the number of velocity components, n is the number of the finite elements nodes, and n_c is the number of the stick-slip nodes. The symmetric, positive semidefinite matrix $\mathbf{E} \in \mathbb{R}^{n_c \times n_c}$ and the vector $\mathbf{c} \in \mathbb{R}^{n_c}$ arise from the elimination of the bubble components. The unknowns $\mathbf{u}$, $\mathbf{p}$ are the vectors of velocity and pressure components, respectively. Finally, $\boldsymbol{\lambda}_t$, $\boldsymbol{\lambda}_n$ are Lagrange multipliers regularizing the non-differentiability of j from (2) and releasing the impermeability condition, respectively. Note that $\mathbf{s}_t, \boldsymbol{\lambda}_n$ approximate $-\sigma_t, -\sigma_n$ on γ_C, respectively.

3 Semi-smooth Newton method for $\kappa > 0$

The stick-slip law in (3) is given by the relation between $(\mathbf{Tu})_i$ and s_{ti}; see Figure 1. It can be represented by the piecewise linear function with the independent variable s_{ti} and the dependent variable $(\mathbf{Tu})_i$ as follows:

$$(\mathbf{Tu})_i = \max\{0, \kappa_i^{-1}(s_{ti} - g_i)\} - \max\{0, -\kappa_i^{-1}(s_{ti} + g_i)\}. \qquad (4)$$

Problem (3), with the stick-slip condition expressed by (4), can be written as one equation:

$$\mathbf{G}(\mathbf{y}) = \mathbf{0} \qquad (5)$$

for $\mathbf{G}(\mathbf{y}) = (\mathbf{G}_1^T(\mathbf{y}), \mathbf{G}_2^T(\mathbf{y}), \mathbf{G}_3^T(\mathbf{y}), \mathbf{G}_4^T(\mathbf{y}))^T$ and $\mathbf{y} = (\mathbf{u}^T, \mathbf{s}_t^T, \boldsymbol{\lambda}_n^T, \mathbf{p}^T)^T$, where $\mathbf{G}_1(\mathbf{y}) = \mathbf{Au} - \mathbf{l} + \mathbf{T}^T\mathbf{s}_t + \mathbf{N}^T\boldsymbol{\lambda}_n + \mathbf{B}^T\mathbf{p}$, $\mathbf{G}_2(\mathbf{y}) = \mathbf{Tu} - \boldsymbol{\phi}(\mathbf{D}(\boldsymbol{\kappa})^{-1}(\mathbf{s}_t - \mathbf{g})) + \boldsymbol{\phi}(-\mathbf{D}(\boldsymbol{\kappa})^{-1}(\mathbf{s}_t + \mathbf{g}))$, $\mathbf{G}_3(\mathbf{y}) = \mathbf{Nu}$, $\mathbf{G}_4(\mathbf{y}) = \mathbf{Bu} - \mathbf{Ep} - \mathbf{c}$, and $\boldsymbol{\phi}(\mathbf{z}) = (\phi(z_1),\dots,\phi(z_{n_c}))^T$, $\mathbf{z} \in \mathbb{R}^{n_c}$, $\phi(z) = \max\{0, z\}$, $z \in \mathbb{R}$. Equation (5) can be solved by a Newton-type method. Due to the presence of the max-functions, the Jacobian matrix to $\mathbf{G}$ does not exist at all points so that the classical Newton method can not be used. On the other hand $\mathbf{G}$ is semi-smooth in the sense of [5] and, therefore, the semi-smooth Newton method can be used as a solver.

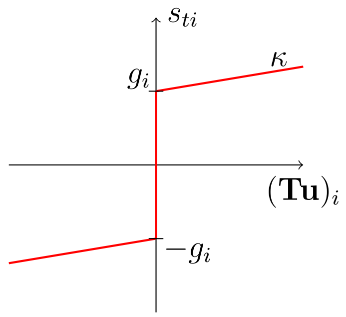

Figure 1: Stick-slip law in the node $\mathbf{x}_i$, $i \in \mathcal{N}$.

The algorithm generates the sequence $\{\mathbf{y}^k\}$. For each $\mathbf{y}^k$ we define the inactive sets

$$\mathcal{I}_t^+ = \{i \in \mathcal{N} : s_{ti}^k \geq g_i\}, \quad \mathcal{I}_t^- = \{i \in \mathcal{N} : s_{ti}^k \leq -g_i\}$$

and the indicator matrices $\mathbf{D}(\mathcal{I}_t^-)$, $\mathbf{D}(\mathcal{I}_t^+)$, respectively. The indicator matrix to $\mathscr{S} \subset \mathcal{N}$ is given by $\mathbf{D}(\mathscr{S}) = \operatorname{diag}(s_1, \ldots, s_{n_c}) \in \mathbb{R}^{n_c \times n_c}$ with $s_i = 1$ for $i \in \mathscr{S}$ and $s_i = 0$ if $\notin \mathscr{S}$. The new iterate $\mathbf{y}^{k+1}$ is computed by solving the following linear system:

$$\left(\begin{array}{c|ccc} \mathbf{A} & \mathbf{T}^T & \mathbf{N}^T & \mathbf{B}^T \\ \hline \mathbf{T} & -\mathbf{D}(\boldsymbol{\kappa})^{-1}\mathbf{D}(\mathcal{I}_t^+ \cup \mathcal{I}_t^-) & \mathbf{0} & \mathbf{0} \\ \mathbf{N} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B} & \mathbf{0} & \mathbf{0} & -\mathbf{E} \end{array}\right) \begin{pmatrix} \mathbf{u}^{k+1} \\ \mathbf{s}_t^{k+1} \\ \boldsymbol{\lambda}_n^{k+1} \\ \mathbf{p}^{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{l} \\ \mathbf{D}(\boldsymbol{\kappa})^{-1}(\mathbf{D}(\mathcal{I}_t^-) - \mathbf{D}(\mathcal{I}_t^+))\mathbf{g} \\ \mathbf{0} \\ \mathbf{c} \end{pmatrix}. \quad (6)$$

As it is unrealistic to find the exact solution of large linear algebraic systems, we use the Schur complement system to solve (6):

$$\mathbf{S}^k \mathbf{r}^{k+1} = \mathbf{C}\mathbf{A}^{-1}\mathbf{l} - \mathbf{h}^k,$$

with $\mathbf{S}^k = \mathbf{C}\mathbf{A}^{-1}\mathbf{C}^T + \bar{\mathbf{E}}^k$, where

$$\mathbf{C} = \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix}, \quad \bar{\mathbf{E}}^k = \begin{pmatrix} \mathbf{D}(\boldsymbol{\kappa})^{-1}\mathbf{D}(\mathcal{I}_t^+ \cup \mathcal{I}_t^-) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \end{pmatrix}$$

and

$$\mathbf{r}^{k+1} = \begin{pmatrix} \mathbf{s}_t^{k+1} \\ \boldsymbol{\lambda}_n^{k+1} \\ \mathbf{p}^{k+1} \end{pmatrix}, \quad \mathbf{h}^k = \begin{pmatrix} \mathbf{D}(\boldsymbol{\kappa})^{-1}(\mathbf{D}(\mathcal{I}_t^-) - \mathbf{D}(\mathcal{I}_t^+))\mathbf{g} \\ \mathbf{0} \\ \mathbf{c} \end{pmatrix}.$$

We use the conjugate gradient method with an adaptive precision control and the diagonal preconditioner

$$\mathbf{P}^k = \operatorname{diag} \mathbf{S}^k.$$

One can prove [6, 9] that the spectral condition number of $(\mathbf{P}^k)^{-1}\mathbf{S}^k$ does not depend on the block $\mathbf{D}(\boldsymbol{\kappa})^{-1}$, which is ill-conditioned if κ or h_i are too small.

4 Semi-smooth Newton method for $\kappa \geq 0$

Considering the same stick-slip law in (3), we can represent it by another piecewise linear function that will allow us to work even if $\kappa = 0$, which was not possible in (4). Using the projection on the interval $[a, b]$

$$P_{[a,b]}(x) = x - \max\{0, x - b\} + \max\{0, a - x\}, \quad x \in \mathbb{R}$$

the stick-slip law in (3) can be represented by

$$\left.\begin{aligned} (\mathbf{Tu})_i &= \max\{0, \kappa_i^{-1}(s_{ti} - g_i)\} - \max\{0, -\kappa_i^{-1}(s_{ti} + g_i)\} && \text{for } \rho_i = \kappa_i > 0, \\ \rho(\mathbf{Tu})_i &= \max\{0, s_{ti} - g_i + \rho(\mathbf{Tu})_i\} - \max\{0, -s_{ti} - g_i - \rho(\mathbf{Tu})_i\} && \text{for } \kappa_i = 0, \rho > 0. \end{aligned}\right\} \tag{7}$$

The problem (3), with the stick-slip condition represented by (7), can again be written as one equation (5), but with modified $\mathbf{G}(\mathbf{y})$. Now $\mathbf{G}(\mathbf{y}) = (\mathbf{G}_1^T(\mathbf{y}), \mathbf{G}_2^T(\mathbf{y}), \mathbf{G}_3^T(\mathbf{y}), \mathbf{G}_4^T(\mathbf{y}), \mathbf{G}_5^T(\mathbf{y}))^T$ and $\mathbf{y} = (\mathbf{u}^T, \mathbf{s}_t^T, \boldsymbol{\lambda}_n^T, \mathbf{p}^T)^T$, where $\mathbf{G}_1(\mathbf{y}) = \mathbf{Au} - \mathbf{l} + \mathbf{T}_+^T\mathbf{s}_{t+} + \mathbf{T}_0^T\mathbf{s}_{t0} + \mathbf{N}^T\boldsymbol{\lambda}_n + \mathbf{B}^T\mathbf{p}$, $\mathbf{G}_2(\mathbf{y}) = \mathbf{T}_+\mathbf{u} - \max\{\mathbf{0}, \mathbf{D}_{\kappa+}^{-1}(\mathbf{s}_{t+} - \mathbf{g}_+)\} + \max\{\mathbf{0}, -\mathbf{D}_{\kappa+}^{-1}(\mathbf{s}_{t+} + \mathbf{g}_+)\}$, $\mathbf{G}_3(\mathbf{y}) = \rho\mathbf{T}_0\mathbf{u} - \max\{\mathbf{0}, \mathbf{s}_{t0} - \mathbf{g}_0 + \rho\mathbf{T}_0\mathbf{u}\} + \max\{\mathbf{0}, -\mathbf{s}_{t0} - \mathbf{g}_0 - \rho\mathbf{T}_0\mathbf{u}\}$, $\mathbf{G}_4(\mathbf{y}) = \mathbf{Nu}$, $\mathbf{G}_5(\mathbf{y}) = \mathbf{Bu} - \mathbf{Ep} - \mathbf{c}$, and $\mathcal{N}_+ = \{i \in \mathcal{N} : \kappa_i > 0\}$, $\mathcal{N}_0 = \{i \in \mathcal{N} : \kappa_i = 0\}$, $\mathbf{T}_+ := \mathbf{T}_{\mathcal{N}_+}$, $\mathbf{T}_0 := \mathbf{T}_{\mathcal{N}_0}$, $\mathbf{s}_{t+} := \mathbf{s}_{t\mathcal{N}_+}$, $\mathbf{s}_{t0} := \mathbf{s}_{t\mathcal{N}_0}$, $\mathbf{g}_+ := \mathbf{g}_{t\mathcal{N}_+}$, $\mathbf{g}_0 := \mathbf{g}_{t\mathcal{N}_0}$. Again we obtain equation that can be solved by a Newton-type method. Since max-functions appear, the Jacobian matrix to $\mathbf{G}$ does not exist. We use the semi-smooth Newton method as a solver again. The algorithm generates the sequence $\{\mathbf{y}^k\}$. For each $\mathbf{y}^k$ we define the active/inactive sets:

$$\begin{aligned} &\mathcal{I}_{t+}^+ = \{i \in \mathcal{N}_+ : s_{ti}^k - g_i \geq 0\}, && \mathcal{I}_{t+}^- = \{i \in \mathcal{N}_+ : s_{ti}^k + g_i \leq 0\}, \\ &\mathcal{I}_{t0}^+ = \{i \in \mathcal{N}_0 : s_{ti}^k - g_i + \rho(\mathbf{Tu})_i \geq 0\}, && \mathcal{I}_{t0}^- = \{i \in \mathcal{N}_+ : s_{ti}^k + g_i + \rho(\mathbf{Tu})_i \leq 0\}, \\ &\mathcal{A}_0 = \mathcal{N}_0 \backslash (\mathcal{I}_{t0}^+ \cup \mathcal{I}_{t0}^-), \end{aligned}$$

and the indicator matrices in the same way as in Section 3. The new iterate $\mathbf{y}^{k+1}$ is computed by solving the following linear system:

$$\left(\begin{array}{c|cccc} \mathbf{A} & \mathbf{T}_+^T & \mathbf{T}_{\mathcal{A}_0}^T & \mathbf{N}^T & \mathbf{B}^T \\ \hline \mathbf{T}_0 & -\mathbf{D}_{\kappa+}^{-1}\mathbf{D}(\mathcal{I}_{t+}^+ \cup \mathcal{I}_{t+}^-) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{T}_+ & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{N} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{E} \end{array}\right) \begin{pmatrix} \mathbf{u}^{k+1} \\ \mathbf{s}_{t+}^{k+1} \\ \mathbf{s}_{t\mathcal{A}_0}^{k+1} \\ \boldsymbol{\lambda}_n^{k+1} \\ \mathbf{p}^{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{b} - \mathbf{T}_{\mathcal{I}_{t0}^+}^T\mathbf{g}_{\mathcal{I}_{t0}^+} + \mathbf{T}_{\mathcal{I}_{t0}^+}^T\mathbf{g}_{\mathcal{I}_{t0}^-} \\ \mathbf{D}_{\kappa+}^{-1}(\mathbf{D}(\mathcal{I}_{t+}^-) - \mathbf{D}(\mathcal{I}_{t+}^+))\mathbf{g}_+ \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{c} \end{pmatrix}. \tag{8}$$

To solve (8), we use again the Schur complement $\mathbf{S}^k = \mathbf{CA}^{-1}\mathbf{C}^T + \mathbf{D}^k$ with

$$\mathbf{C} = (\mathbf{T}_+^T, \mathbf{T}_{\mathcal{A}_0}^T, \mathbf{N}^T, \mathbf{B}^T)^T, \quad \mathbf{D}^k = \operatorname{diag}(\mathbf{D}_{\kappa+}^{-1}\mathbf{D}(\mathcal{I}_{t+}^+ \cup \mathcal{I}_{t+}^-), \mathbf{0}, \mathbf{0}, \mathbf{E}).$$

The conjugate gradient method with the adaptive precision control is used again in computations as well as the preconditioner described in Section 3.

5 Numerical experiments

We consider the L-shaped domain $\Omega = (0,5) \times (0,2) \backslash \bar{\mathcal{S}}$, $\mathcal{S} = (0,1) \times (0,1)$ with $\nu = 1$ and $\mathbf{f} = \mathbf{0}$; $\gamma_D = \gamma_{top} \cup \gamma_{left}$ with $\gamma_{top} = (0,5) \times \{2\}$, $\gamma_{left} = \{0\} \times (1,2)$, $\mathbf{u}_{D|\gamma_{top}} = \mathbf{0}$, and $\mathbf{u}_{D|\gamma_{left}} = (4(x_2-2)(1-x_2), 0)$; $\gamma_N = \{5\} \times (0,2)$ with $\boldsymbol{\sigma}_N = \mathbf{0}$; $\gamma_C = \partial\Omega \backslash (\gamma_D \cup \gamma_N)$ with $g = 1$. In tables below we present $iter/n_S$, where $iter$ is the number of the Newton iterations and n_S is the total number of the matrix-vector multiplications by the Schur complements. In other words, n_S determines the computational efficiency. We illustrate the efficiency of both algorithms introduced in Sections 3 and 4. Table 1 and Table 4 show the computational efficiency without preconditioning, while Table 2 and Table 5 with preconditioning for each algorithm, respectively. It is easy to observe that n_S increases considerably for finer meshes and smaller κ in both cases and that this effect is eliminated by preconditioning. Table 3 and Table 6 display the computational efficiency of the interior point method. In comparison with the semi-smooth Newton method we see that the interior point method requires more matrix-vector multiplications.

$n_u/n_p/n_c$	$\kappa = 1$	$\kappa = 0.5$	$\kappa = 0.1$	$\kappa = 0.01$	$\kappa = 0.001$
344/206/32	8/1313	8/1197	9/1755	10/2375	*
1352/744/64	8/2727	9/3776	9/4684	10/6559	11/8858
5366/2819/128	9/6539	9/7314	10/11801	11/19484	11/17000
21386/10965/256	9/13064	10/16384	10/23394	11/28977	*
85394/43241/512	10/23324	10/29940	10/36387	13/62109	*

Table 1: The computational complexity for different $\kappa > 0$ without preconditioning.

$n_u/n_p/n_c$	$\kappa = 1$	$\kappa = 0.5$	$\kappa = 0.1$	$\kappa = 0.01$	$\kappa = 0.001$
344/206/32	7/108	8/140	8/113	8/113	10/143
1352/744/64	7/139	8/182	8/163	9/171	10/189
5366/2819/128	8/249	8/262	9/231	10/268	12/340
21386/10965/256	8/286	8/258	9/271	11/339	10/262
85394/43241/512	8/378	8/322	8/248	11/444	11/434

Table 2: The computational complexity for different $\kappa > 0$ with preconditioning

$n_u/n_p/n_c$	$\kappa = 1$	$\kappa = 0.5$	$\kappa = 0.1$	$\kappa = 0.01$	$\kappa = 0.001$
344/206/32	18/347	18/351	19/397	18/381	18/387
1352/744/64	20/494	20/486	20/503	20/491	20/483
5366/2819/128	18/500	18/445	19/521	19/494	19/476
21386/10965/256	19/605	19/565	19/608	19/557	19/608
85394/43241/512	19/687	19/690	19/742	19/705	19/671

Table 3: The computational complexity for different $\kappa > 0$ for interior point method

$n_u/n_p/n_c$	$\kappa = 1$	$\kappa = 0.5$	$\kappa = 0.1$	$\kappa = 0.01$	$\kappa = 0.001$	$\kappa = 0$
344/206/32	9/1377	9/1467	10/2001	11/2770	11/2837	12/1054
1352/744/64	10/3698	10/4210	11/6850	11/7429	12/9523	12/1686
5366/2819/128	9/6305	10/9021	10/1161	11/16596	13/27768	15/3942
21386/10965/256	10/14848	10/16680	11/29878	12/38700	*	10/5386
85394/43241/512	10/21789	10/27256	11/41416	*	*	*

Table 4: The computational complexity for different $\kappa \geq 0$ without preconditioning.

$n_u/n_p/n_c$	$\kappa = 1$	$\kappa = 0.5$	$\kappa = 0.1$	$\kappa = 0.01$	$\kappa = 0.001$	$\kappa = 0$
344/206/32	8/146	8/153	9/159	10/164	10/149	8/150
1352/744/64	8/199	9/222	10/256	10/205	11/223	8/168
5366/2819/128	9/269	10/319	10/298	11/304	12/326	8/207
21386/10965/256	9/342	8/247	10/340	11/398	12/404	8/290
85394/43241/512	9/353	9/346	9/349	11/522	12/509	9/477

Table 5: The computational complexity for different $\kappa \geq 0$ with preconditioning

$n_u/n_p/n_c$	$\kappa = 1$	$\kappa = 0.5$	$\kappa = 0.1$	$\kappa = 0.01$	$\kappa = 0.001$	$\kappa = 0$
344/206/32	18/344	18/357	19/367	18/382	19/381	19/410
1352/744/64	20/491	20/455	20/480	20/492	20/471	20/483
5366/2819/128	18/454	19/505	19/473	19/487	19/483	19/507
21386/10965/256	19/581	20/625	19/561	19/579	19/560	19/561
85394/43241/512	19/733	19/733	19/769	19/743	19/691	19/677

Table 6: The computational complexity for different $\kappa \geq 0$ for interior point method

Conclusion

Summarizing and using the results from [9], we succesfully modified the algorithm to accept $\kappa = 0$ as well, increasing its potential use. Furthermore, we have shown that the preconditioning has an important impact on the computational efficiency and that without it, it is impossible to solve larger linear systems.

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Nehladká Newtonova metoda pro řešení Stokesových rovnic se skluzovou okrajovou podmínkou

Abstrakt (Streszczenie): Tento text se zabývá řešením Stokesových rovnic s monotónně rostoucí skluzovou podmínkou. Použitím P1-bubble/P1 aproximace konečných prvků dostaneme algebraickou variační nerovnici, která je ekvivalentní jisté minimalizační úloze, jejíž podmínky optimality jsou výchozím bodem pro návrh algoritmu. Použitým algoritmem je implementace nehladké Newtonovy metody, založená na použití aktivních a neaktivních množin, kde zmíníme 2 možné postupy konstrukce. Algoritmus je poté testován v prostředí MATLAB. Experimenty jsou provedeny na čtvercové a „L-shaped“ oblasti, přičemž studujeme vlivy koeficientu přilnavosti a předpodmínění na efektivitu výpočtů.

Klíčová slova (Słowa kluczowe): nehladká Newtonova metoda, skluzová podmínka, Stokesův problém, předpodmínění

The Way of Use Scenario Methods in Maintenance Management of Selected Network Technical Systems

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Abstract: The paper presents the results of research on the possibility of use scenario methods in the planning and implementing of the maintenance work in the selected network technical system. The first part discusses the specific organizational and technical factors of maintenance work into wastewater system collection. Based on the conclusions formulated, resulting from the exploitation specificity of network systems, the proposal for use of scenario methods in modelling exploitation processes has been discussed. It is assumed that scenario models can be the basis of construction of an autonomous maintenance strategy for these network technical systems. In this regard, it has been presented the guidelines for the construction of exploitation scenarios and key aspects of their assessment in relation to specific organizational and technical conditions.

Keywords: network technical system, scenario methods, sewer system, maintenance strategy.

1 Introduction

Network technical systems (NTS) are included in the technical infrastructure, which is the basis of operation of municipal engineering sectors. Through the NTS, there are supplied various types of media complying with the required specifications to multiply groups of customers, territorially dispersed and belonging to different categories, such as households, industrial plants, utilities, service facilities and other [9]. The most common NTS include the:

- water supply system - which functions is water supply to customers in an organized and constant manner, with the required level of pressure and of appropriate quality,
- sewer system - allowing the discharge of domestic, industrial, rain and snowmelt sewage to the wastewater treatment plants, and then to the final receiver after appropriate cleaning,
- gas supply system - the main task is to meet the needs of customers in the supply of gas, which should retain sufficient amount and pressure and meet all quality requirements,
- heat supply system - main task is to transfer heat from the heat source (power plant, heating plant, boiler) to heat consumers, which are residential buildings, public buildings, industrial sites,

- electric transmission system - the main task is to ensure the supply of electricity to customers on the appropriate quality parameters (frequency, voltage).

2 Structure and characteristics of a typical sewer system

A typical example of a technical system is a sewer system, which should be understood as a set of interrelated technical elements which are used for drainage and disposal of all types of sewage from a particular area. In other words, sewer system task is to establish such a system of sewer lines and other equipment, which in a economically reasonable manner will enable the collection, sewage disposal and treatment caused by human life and activity and runoff of rainwater [18]. Sewer system has several specific features, which include first of all.

- territorial dispersion of the system components, requiring a special approach to maintenance tasks,
- large number and variety of types of objects within the system,
- powerful links and relationships between system components,
- highly dynamic of the system, which requires continuous control and monitoring of the processes performed,
- uninterrupted operation for most installations, equipment and buildings belonging to the system.

Due to the fact that the NTS belong to very expensive components of the technical infrastructure, and the period of their operation is often several years, the basic elements of such a system should perform their functions as long as possible.

Bibliography [3, 4, 11, 18, 19, 20], comprises methods for the classification of sewer systems, which arise from different needs (organizational, legal, or technical). In one aspect which is presented in this article, the most important is layout associated with the identification, physical connections between elements, and mutual location of each object constituting sewer network discussed here. The most important and most common is the external sewer system, which can take one of four forms (Fig. 1).

- combined sewer system (Fig. 1a), in the form of single lead network through which they flow together domestic, industrial and rain wastes,
- separate sewer system (Fig. 1b), in the form of two-wire network, where in one pipe domestic and industrial wastes flow, and in the second pipe runoff wastewater flow,
- semi-separate sewer system (Fig. 1c), as similar as the separate sewer, where pipes are connected for the purpose of common action system,
- mixed sewer system (Fig. 1d), as a territorially separated units of separate sewer system and combined sewer system.

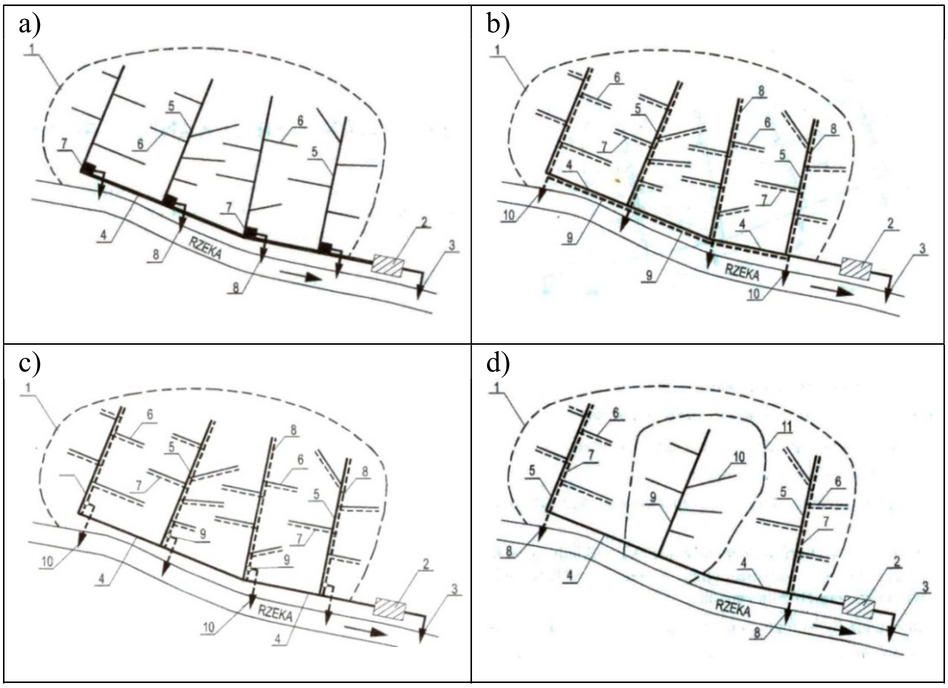

Fig. 1. Schemes of sewage systems [20]: a) combined sewage system, b) separate sewer system, c) semi-separate sewer system, d) mixed sewer system

3 Exploitation specificity of sewer system

From a technical point of view, the proper functioning of the sewer system requires to ensure continuity and quality of facilities within an extensive technical infrastructure geographically dispersed over a large area. In practice this means the need to provide an adequate level of reliability. Sewer system consists of logically interrelated subsystems, in particular:

- network subsystem,
- pumping subsystem,
- wastewater treatment subsystem,
- outlet subsystem.

Each of these subsystems has a different organizational and technical approach to the ways and ranges of maintenance work.

Supply and outlet subsystems are designed to keep the possibility of continuous transport of sewage through the pipe system and collectors to the outlet and receiver. Maintenance work, having the character of both prevention and intervention, comes down to maintain their efficiency and tightness. These works include:

1. routine network inspections including visual and instrumented checks of tightness and capacity of pipes and collectors, prevention of pollution of canals, laterals and network equipment,
2. maintain and corrective works, including cleaning and flushing of pipes and ducts, cleaning street gullies, removal of pipe system blockage,
3. repair works, including removal of damage, and replacement of worn parts of sewage system.

The nature of maintenance work of network subsystem depends largely on the availability (often limited) to specific sections.

Pumping subsystem is designed to maintain reliable and uninterrupted operation of sewerage system. To achieve such an explicit objective, maintenance works are carried out, including:

1. Supervision over pump unit, which is aimed at a constant evaluation the selected parameters.

2. Routine maintenance work, performed on the so-called “move” and including visual inspection of the pump during operation with a special focus on the vibration and the indications of measuring instruments. In addition, the exchange is made of grease or oil in the bearings and sealing glands. This type of work concerns the sewage tanks or lattices and rely on their routine cleaning.
3. Overhauls, carried on so-called downtime, include the corrective and regenerating work that depend on the pump operation time or the level of wear. For example, centrifugal pump major overhaul is carried out every 2-4 years (12000-18000 work hours) and consists of the pump disassemble and replacement of worn parts of the: rotors, steering shafts, bearings, couplings, rotor rings, exchange of measuring instruments, repair of installation of the cooling of bearings, etc.
4. Emergency repairs, involve the removal of faults of the pump units and it shall be carried out on an ongoing basis as detecting irregularities in the operation of individual technical objects. The necessity of these works is the result of such: different types of leakage, lack of water intake, reduced productivity, increased power consumption, increased vibration or heating of the bearings. A detailed overview of the most characteristic events with possible unintended symptoms, causes, and the procedures are summarized in [18].

Wastewater treatment subsystem is designed to maintain continuous operation of technical facilities belonging to other subsystems and to ensure the assumed effect of sewage. Maintenance work in this case include:

1. Boot of treatment plant, performed when you first start and after every long break in the work. Boot is a gradual and multi-stage (hydraulic start-up, technological start-up), during operational work is carried out preventive and corrective.
2. control works, including:
   - measurements of parameters of technological processes (inflow and outflow intensity, energy, air, steam, hot water and reagents), and analysis of water quality, sediments and other factors separately for each element, and also for sewage treatment as a whole,
   - assessment of the operational performance through the routine celebration and inspections of particular objects for identification of defects occurring.
3. Maintain and repair works (preventive), aimed at keeping components in the sewage treatment of full technical efficiency. This type of work are a consequence of the technical assessment as a result of routine inspection activities, as well as the effect of reliability research, reflected in the operating resources of particular objects contained in the maintenance documentation.
4. Repair work (corrective and emergency), including adjustment of mechanical equipment, power and automation, removing sediment from the channels, ducts, conduits and other places where there should not be accumulated, any failure or deficiencies removal of technical facilities.

The study of the functioning of selected Polish sewage systems and the organization of maintenance activities, allowed to distinguish several aspects pointing to the exploitation specificity of the area, which is discussed in this. In particular, these aspects include:

- structural reliable complexity of technical objects of sewer system,
- organizational and technical complexity of maintenance strategy and systems.

Reliability structure of sewage system can be represented by typical system models [11]. However, the vast majority of exploited sewage systems is based on complex models, which link serial and parallel structures [7]. In an extended sewer system, damage of a single element (such as clogging of the selected pipe) does not immediately stop the whole system, but only a particular section, the rest of the system can operate in the normal way. Sewerage system should be operational in general, regardless of events or performed maintenance work. This idea also translates into quantitative evaluation and control of exploitation efficiency, which should be referred not to the whole system but to the extracted parts or sub-systems.

4. Organizational and technical complexity of maintenance strategy and systems

In practice, construction and functionality of the sewer system determine classical forms of maintenance activities. From an organizational point of view, the activities may be enclosed in widely known and accepted forms of base maintenance strategies, especially: breakdown maintenance, preventive maintenance and predictive maintenance [10].

In order to determine the exploitation and maintenance specificity, the studies were performed, which consisted of identification and inventory of inventory events relating to selected Polish sewage systems. It was found that the items qualified for particular subsystems characterized by different organizational and technical forms for the possibility and the way of maintenance tasks performance. Based on these studies and based on the characteristics of the maintenance activities of most important elements of the sewerage system, there have been developed a hierarchy of the various strategies referenced to the particular subsystems (Tab. 1).

Table 1 Hierarchy of the various strategies referenced to the particular sewerage subsystems

Subsystem	**Base maintenance strategy**	**Interpretation**
Network and outlet subsystems	1. Breakdown maintenance strategy 2. Preventive maintenance strategy 3. Preventive maintenance strategy	On the layout of the maintenance work types influences difficulty in the direct access to selected objects and their high reliability (especially channels and pipes). Not without significance is the necessity to conduct excavation works making it difficult to carry out normal activities in the area. Therefore, in this case, highest percentage share characterizes intervention works, but preventive works are limited mainly to non-invasive activities.
Pumping subsystem	1. Preventive maintenance strategy 2. Breakdown maintenance strategy 3. Preventive maintenance strategy	Of all mentioned subsystems, pumping subsystem is characterized by the highest tool diagnosing. Pumps and their equipment are susceptible to simple and complex diagnostic procedures. With a relatively high availability of components of the subsystem, it allows you to make ongoing assessment of technical condition.
Wastewater treatment subsystem	1. Preventive maintenance strategy 2. Breakdown maintenance strategy 3. Preventive maintenance strategy	Wastewater treatment subsystem is different in the functioning and in the methodology of approach to the maintenance activities. It is assumed high direct access to technical objects and less diagnosing than pumping system. Therefore, the best in this case is a strategy based on an extended multi-level prevention from simple audit works through inspection, maintenance, repairs to complex overhauls. Other types of strategies are also important here, but they are complementary (such as diagnostics for the audit work or repair, fault, etc.).

The study and the information contained in the table (Tab. 1) shows that optimization of exploitation processes in the sewer system should be multifaceted. This results primarily from the specific structure and location of the sewerage system. On the one hand, a significant number of components of such system is located and operates under the ground, making it difficult or impossible to carry out such preventive activities, which are characteristic for typical manufacturing companies (the celebration, overviews). It is also important the dispersion of technical objects over a large area. On the other hand, here are objects such as pumping stations or sewage treatment plants, which can be seen as a typical manufacturing company (from the exploitation point of view). This allows the use of effective methods and supporting tools such as strategy TPM and CMMs/EAM system.

4. Proposal of use scenario methods to build an autonomous maintenance strategy

Presented variety and ambiguity of strategic possibilities in respect of individual components the sewerage system, cause difficulty in defining the optimal exploitation policy. However, data collected and structured allowed make assumptions for the purpose of development of autonomous computer aided exploitation strategy in the form of analytical and advisory system taking into account:

1. different nature of operational work in relation to individual subsystems of sewer system (intervention nature of the sewerage network, diagnostic character of pumping, the preventive nature of wastewater treatment plant),
2. exploitation „point" of needs of maintenance tasks realization (preventive action, removing the effects of emergencies),
3. typical aspects of the physical wear of the technical objects,
4. technical and geographic location of the technical objects,
5. information about terrain characteristics in the planning and optimization of maintenance work.

Implementation of the concept is possible on a consistent set of guidelines, which are the basis for decision-making. In this regard scenario technique can be helpful. They belong to the group of forecasting methods and they have been traditionally used in the field of economics and strategic management [1, 2, 5, 6, 12, 15, 16, 17]. In the area of technical sciences, it has still not have gained greater appreciation.

According to [8], scenario planning is based on descripting of events and indicating their logical and coherent consequences in order to determine the way of development of an object or situation. It is assumed defined reference point, which in the case of maintenance management can be, for example, past or current technical condition of the object. According to, we must clearly differentiate between the external scenario, relating to surrounding reality (eg, environmental or industrial) and internal scenario, the property of a single person.

Examples, described widely in literature are a large and diverse set of proven methods of scenarios creating, their practical use and evaluation of effectiveness. For example, according to [15], there are four basic types of scenarios: scenarios of possible events, simulation scenarios, environmental conditions scenarios, processes in the environment scenarios. These scenarios differ in the logic of their creation and the method and scope of data collection. However, according to [6], scenarios can be either inductive or deductive. The overall conclusion is that there are many forms and techniques of scenarios construction that result from the lack of explicit modelling principles developed in this task area.

Based on the results of diagnosis and identified analogy to other areas, it must be concluded that the area of exploitation of technical systems (exploitation of sewerage system) is very susceptible to the use of scenario techniques for modelling exploitation processes and building autonomous maintenance strategy.

The starting point could be the models of descriptions of activities under the exploitation processes that were proposed in [10]. It is about "passage" model of the object from the initial state (the transfer of the object to use), the final state (recall the object from the exploitation - such as scrapping), or to identify possible (typical) ways of use the “living” time of object. There are four typical ways (models) of such management, referred to the term of scenario, in particular:

1. Scenario of exploitation process of 1st type (Fig. 2), consists of use of the new object until lose the ability to perform the tasks, arising from the objective function. After the loss of suitability, the object is definitely withdrawn from exploitation. This scenario can be used to describe the exploitation processes of irreparable objects.

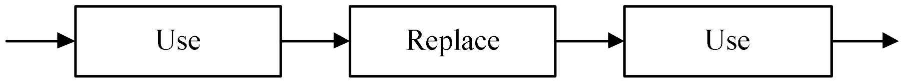

Fig. 2. Exemplary arrangement of structural links of exploitation tasks for type 1 scenario

2. Scenario of exploitation process of 2nd type (Fig. 3), consists of supplementing type 1 scenario with rehabilitation suitability activities. Therefore, this scenario can be presented in a sequence: use - loss of suitability - renovation of suitability - use...

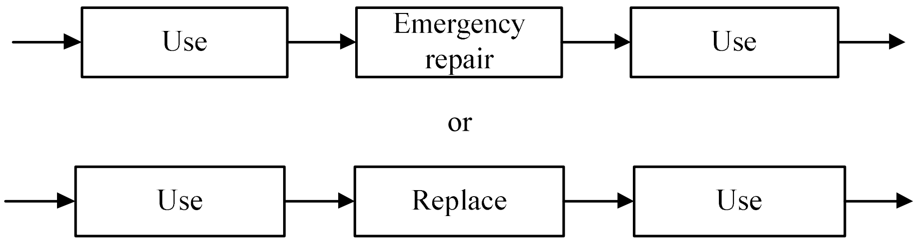

Fig. 3. Exemplary arrangement of structural links of exploitation tasks for type 2 scenario

3. Scenario of exploitation process of 3rd type (Fig. 4), consists of supplementing type 2 scenario with activities, which aiming at extending the periods of use, and thus reduce downtime periods. This can be achieved for example by introducing preventive activities.

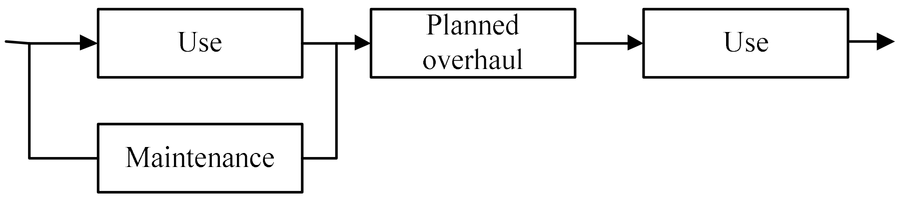

Fig. 4. Exemplary arrangement of structural links of exploitation tasks for type 3 scenario

4. Scenario of exploitation process of 4th type (Fig. 5), consists of supplementing type 2 scenario or type 3 scenario with activities, which aiming at identification of the technical condition of object (eg, inspections).

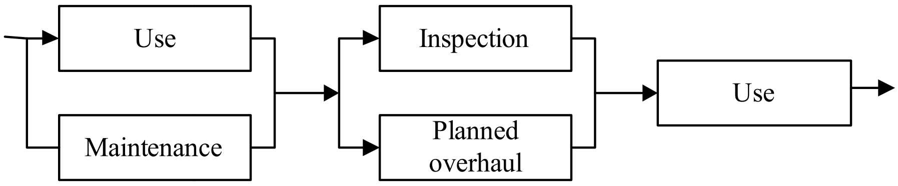

Fig. 5. Exemplary arrangement of structural links of exploitation tasks for type 4 scenario

Presented structural models are here an example. In a description real cases, the scope, as well as the layout of relationship may vary significantly and it can be more extensive.

Conclusion

Key elements of the scenario methodology and the specificity of technical systems exploitation determine a set of necessary activities (specific objectives), whose implementation will allow to develop an autonomous maintenance strategy in relation to the sewerage system [13, 14]. These activities include:

- defining the need and basis for scenario/collection of possible scenarios generation based on specific exploitation models, which may result from the reliability criteria, in this case (eg. a set of quantitative exploitation indicators),
- determining the internal and formal structure of the description of the scenario (identifying a set of parameters, the quantitative elements and features, defined as qualitative components of the situation/event),
- filling in scenarios for the object as such by mapping its environment (that is, by analogy, the author of the article proposes to describe the "scenery" in which scenario is “going on” - in addition to the same scenario),
- solution to the problem of practical use of scenarios in maintenance works, and optimization of decision-making processes relating to operating technical systems, taking into account multivariant issue of possible events and behavioral simulation of objects in shorter and longer term.

An important aspect is the use of terrain information, which in this case should be based on the study:

- possibilities and efficiency of use of information technologies (network model of open and closed channels, the atmospheric model, terrain model, NTS, soil zone model: the roughness and/or permeability of the area, density of buildings and other), to develop components of the hydrological and hydraulic water catchment model of studied area,
- opportunities for implementing and visualizing the simulation results using the developed hydrologic and hydraulic water catchment model of studied area, in the form of new thematic layers in GIS system,
- possibility of using information from the new thematic layers (spatial-statistical analysis) in the construction of model of autonomous exploitation strategies and in creating a dynamic, variant scenario of events.

The operation will be the subject of research, expressed in a further publication of the author.

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SPOSÓB ZASTOSOWANIA METOD SCENARIUSZOWYCH W ZARZĄDZANIU UTRZYMANIEM RUCHU WYBRANYCH SIECIOWYCH SYSTEMÓW TECHNICZNYCH

Abstrakt (Streszczenie): W artykule przedstawiono wyniki badań nad możliwością zastosowania scenariuszy w planowaniu i wdrażaniu prac konserwatorskich w wybranym sieciowym systemie technicznym. W pierwszej części omówiono specyficzne organizacyjne i techniczne cechy prac obsługowo-naprawczych systemów kanalizacyjnych. Na podstawie sformułowanych wniosków, wynikających ze specyfiki eksploatacyjnej systemów sieciowych, omówiono propozycję stosowania metod scenariuszy w modelowaniu procesów eksploatacyjnych. Przyjęto, że modele scenariuszy mogą stanowić podstawę budowy autonomicznej strategii konserwacji sieciowych systemów technicznych. W związku z tym przedstawiono wytyczne dotyczące budowy scenariuszy eksploatacyjnych i kluczowych aspektów ich oceny w odniesieniu do specyficznych warunków organizacyjnych i technicznych.

Klíčová slova (Słowa kluczowe): sieciowy system techniczny, metody scenariuszowe, system kanalizacyjny, strategia eksploatacyjna

Teaching of Calculus – Hidden Gems Students Has Never Spot

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Abstract: The author would like to draw the reader's attention to the several elementary calculus results which are not generally well known even though they posses an intrinsic mathematical beauty. The Universal chord theorem, for example, has been repeatedly proved in the last century but never found its way into the mainstream calculus textbooks.

Keywords: Teaching, calculus, Universal chord theorem.

1 Introduction

There are several so called *mean value theorems* that are very well known and are part of a usual calculus curriculum. The most prevalent is (see also Figure 1):

Theorem 1 ([8, Theorem 5.10])**.** *If f is a real continuous function on $[a, b]$ which is differentiable in (a, b), then there is a point $x \in (a, b)$ at which*

$$f(b) - f(a) = (b - a)f'(x).$$

On a picture you can see the geometric meaning of the mean value theorem. It says the function has a tangent line parallel to the straight line joining points $[a, f(a)]$ and $[b, f(b)]$.

The previous theorem is a special case of what is sometimes called a *generalized mean value theorem*:

Theorem 2 ([8, Theorem 5.9])**.** *If f and g are continuous real functions on $[a, b]$ which are differentiable in (a, b), then there is a point $x \in (a, b)$ at which*

$$\big(f(b) - f(a)\big)g'(x) = \big(g(b) - g(a)\big)f'(x).$$

Let us mention that the case of the mean value theorem, when $f(a) = f(b)$, is known as the Rolle's theorem. On the other hand, the mean value theorem can be considered as an easy consequence of the Rolle's theorem. It's sufficient to apply it on a function:

$$f^*(x) = f(x) - \big(\frac{f(b)-f(a)}{b-a}(x-a) + f(a)\big).$$

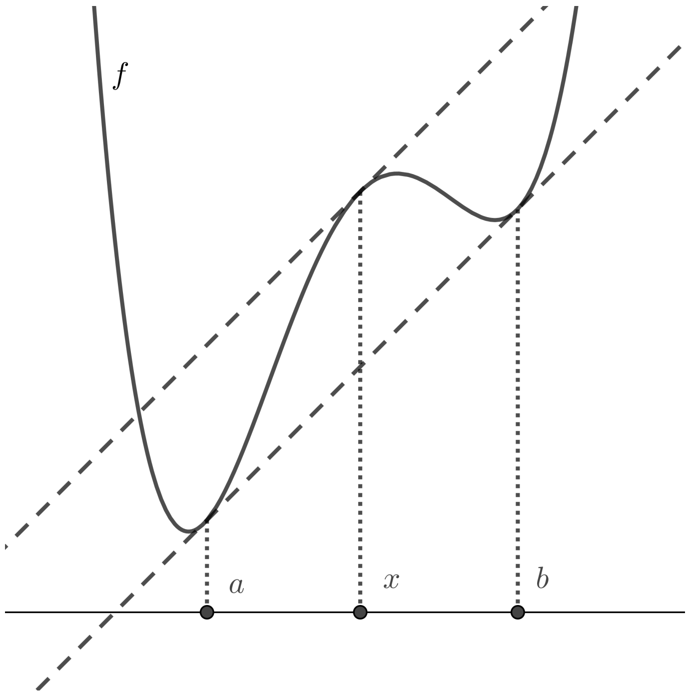

Figure 1: Mean value theorem

When differentiability is not assumed we are not allowed to talk about the tangent lines. Nevertheless, one fundamental result, called the *intermediate value theorem*, holds true:

Theorem 3 ([8, Theorem 4.23])**.** *Let f be a continuous real function on the interval $[a, b]$. If $f(a) < f(b)$ and if c is a number such that $f(a) < c < f(b)$, then there exists a point $x \in (a, b)$ such that $f(x) = c$.*

2 Universal chord theorem

Instead of the tangent lines, we can employ the *chords* (see Figure 2). For any real function defined on a bounded or unbounded interval we define the *chord set* as:

$$H(f) = \{h \in [0, \infty) : f(x) = f(x+h) \text{ for some } x\}.$$

The *Universal chord theorem* of P. Lévy ([6], see also [2, p. 98]) states:

Theorem 4 (Universal chord theorem)**.** *Let f be a continuous real function defined on an interval. Then*

(i) If $h \in H(f)$, then $h/n \in H(f)$ for every positive integer n.

(ii) If a and h are positive numbers and a is not a submultiple of h, then there exists a continuous function f with $h \in H(f)$ and $a \notin H(f)$.

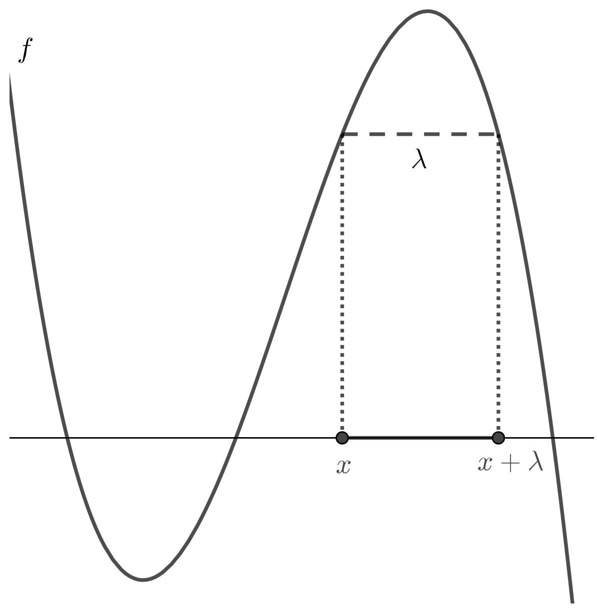

Figure 2: Chord

I furnish you with a proof and several consequences and applications. All of the results provided here are available in literature and I provide precise references too.

Proof (of the Universal chord theorem). (i) Suppose $h \in H(f)$ and $f(a) = f(a+h)$. Consider the continuous function g, defined by

$$g(x) = f\left(x + \frac{h}{n}\right) - f(x) \quad \text{for all } x \in [a, a + h - h/n].$$

If 0 is in a range of g the proof is finished. If not then g would be either positive for all x in its domain or negative for all x in the domain. That follows from the intermediate value theorem. So

$$g(a) + g\left(a + \frac{h}{n}\right) + g\left(a + \frac{2h}{n}\right) + \cdots + g\left(a + h - \frac{h}{n}\right)$$

would be either positive of negative. On the other hand:

$$\begin{aligned}
&g(a) + g\left(a + \frac{h}{n}\right) + g\left(a + \frac{2h}{n}\right) + \cdots + g\left(a + h - \frac{h}{n}\right) \\
&= (f(a + h/n) - f(a)) + (f(a + 2h/n) - f(a + h/n)) + \cdots + (f(a + h) - f(a + h - h/n)) \\
&= f(a + h) - f(a) = 0
\end{aligned}$$

which is a contradiction.

(ii) We define such a function as:

$$f(x) = \sin^2\left(\frac{\pi x}{a}\right) - \frac{x}{h}\sin^2\left(\frac{\pi h}{a}\right).$$

Then $h \in H(f)$ because

$$\begin{aligned} & f(0) = 0, \\ & f(h) = \sin^2\left(\frac{\pi h}{a}\right) - \frac{h}{h}\sin^2\left(\frac{\pi h}{a}\right) = 0. \end{aligned}$$

On the other hand, if $a \in H(f)$ there exists $x \in \mathbb{R}$ that $f(x+a) = f(x)$. Equivalently:

$$\begin{aligned} \sin^2\left(\frac{\pi(x+a)}{a}\right) - \frac{x+a}{h}\sin^2\left(\frac{\pi h}{a}\right) &= \sin^2\left(\frac{\pi x}{a}\right) - \frac{x}{h}\sin^2\left(\frac{\pi h}{a}\right), \\ \sin^2\left(\frac{\pi x}{a} + \pi\right) - \left(\frac{x}{h} + \frac{a}{h}\right)\sin^2\left(\frac{\pi h}{a}\right) &= \sin^2\left(\frac{\pi x}{a}\right) - \frac{x}{h}\sin^2\left(\frac{\pi h}{a}\right), \\ \frac{a}{h}\sin^2\left(\frac{\pi h}{a}\right) &= 0, \\ \sin^2\left(\frac{\pi h}{a}\right) &= 0, \end{aligned}$$

which is a contradiction since a is not a submultiple of h.

□

We may now discuss two questions:

1. If we strengthen the assumption will we obtain reasonably stronger results?
2. If we weaken the assumptions will we obtain reasonably interesting results?

I will try to answer these questions in the rest of the paper.

3 Strengthening the assumptions

Theorem 5 ([7, Exercise 4.7.9])**.** *Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous and periodic function. Then $H(f) = \mathbb{R}$.*

Proof. Let T be the period of f. Pick $\lambda > 0$. We want to prove that $\lambda \in H(f)$. Consider the continuous function

$$g(x) = f(x+\lambda) - f(x)$$

and choose $x_m, x_M \in [0,T]$ such that $f(x_m) = \min_{x\in[0,T]} f(x)$ and $f(x_M) = \max_{x\in[0,T]} f(x)$. Then $g(x_m) \geq 0$ and $g(x_M) \leq 0$. Since g is a continuous function, it sports the intermediate value property and there must exist $x_0 \in [0,T]$ such that $g(x_0) = 0$. It means:

$$0 = g(x_0) = f(x_0+\lambda) - f(x_0), \text{ hence } f(x_0+\lambda) = f(x_0) \text{ and } \lambda \in H(f).$$

□

The previous theorem can be slightly strengthen.

Theorem 6 ([7, Exercise 4.7.11]). *Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous and periodic function with period T. Then for every $\lambda \in H(f)$ there exist two different points x_1, x_2 such that $f(x_2+\lambda) - f(x_2) = f(x_1+\lambda) - f(x_1) = 0$.*

Proof. Let $\lambda \in H(f)$. If f is constant, the statement is clear. Assume f is non-constant. Consider the auxiliary function

$$g(x) = f(x+\lambda) - f(x).$$

The function g is T-periodic and so $g(0) = g(T)$. Also

$$\int_0^T g(x)\,dx = \int_0^T f(x+\lambda)\,dx - \int_0^T f(x)\,dx = 0.$$

and therefore g has both positive and negative values on $[0,T]$. Thus, g vanishes at least at two points in $[0,T)$, we denote these points as $x_1 \neq x_2$. It follows that

$$f(x_1+\lambda) = f(x_1) \text{ and } f(x_2+\lambda) = f(x_2)$$

which completes the proof. □

Theorem 7. *Let f be a continuous convex function defined on $[a,b]$ such that $f(a) = f(b)$. Then $[0, b-a] \subset H(f)$.*

Proof. If f is constant then the statement is trivially true. Assume otherwise. Let $m = \min_{x\in[a,b]} f(x)$. Then the set $\{x; f(x) = m\}$ is an preimage of a convex set by a convex function and therefore convex itself. As a preimage of a closed set it is closed as well. The closed convex sets in $\mathbb{R}$ are exactly the closed intervals. We denote $\{x; f(x) = m\} = [c_1, c_2]$.

The function f is then decreasing on $[a, c_1]$ (we denote $g_1 = f \restriction_{[a,c_1]}$) constant on $[c_1, c_2]$ and increasing on $[c_2, b]$ (we denote $g_2 = f \restriction_{[c_2,b]}$).

Clearly $[0, c_2 - c_1] \subset H(f)$. We define an auxiliary function

$$g(y) = g_2^{-1}(y) - g_1^{-1}(y), \text{ for } y \in [m, f(a)].$$

The function g is continuous and decreasing on its domain,

$$g(m) = g_2^{-1}(m) - g_1^{-1}(m) = c_2 - c_1 \text{ and } g(f(a)) = g_2^{-1}(f(b)) - g_1^{-1}(f(a)) = b - a.$$

Due to the intermediate value property $g([m, f(a)]) = [c_2 - c_1, b-a]$ and hence $[c_2 - c_1, b-a] \subset H(f)$. Employing the trivial observation $[0, c_2 - c_1] \subset H(f)$ we may conclude that $[0, b-a] \subset H(f)$.

□

Corollary 1. *The same conclusion as in Theorem 7 holds true for concave functions.*

Proof. If the function f is concave then $-f$ is convex. Then we may apply Theorem 7 on $-f$ and employ a trivial identity $H(f) = H(-f)$. □

Remark 1. The author did not find the previous theorem and its corollary anywhere in literature. These simple observations are thus probably original results of the paper in hand.

4 Weakening of assumptions

It is worth noting that the proof of the positive part of the Universal chord theorem does not actually require a continuity of f. We used the intermediate value property of g and hence we can generalize the statement a bit. Class of the functions enjoying the intermediate value property contains the continuous functions but is broader. For example, it contains also all the derivatives of the differentiable functions.

Lemma 1. *Let f be a real function defined on an interval, $h \in H(f)$ and $n \in \mathbb{N}$. If the function $x \to f(x + \frac{h}{n}) - f(x)$ enjoys the intermediate property, then $h/n \in H(f)$.*

Remark 2. It's not enough to assume the intermediate property of f, since the sum of two functions with this property does not have to enjoy it. The sum of two continuous functions is continuous as well and this fact is employed in the proof.

Another important generalization of the continuous functions are the Baire one functions.

Definition 1. Let $D \subset \mathbb{R}$. We say a function $f : D \to \mathbb{R}$ is a *Baire-one function* if it is a pointwise limit of a sequence of continuous functions, that is, if there is a sequence (f_n) of functions continuous on D such that for every $x \in D$, $f(x) = \lim_{n\to\infty} f_n(x)$.

There is an example of a function provided in [5, p. 468] given by the formula:

$$f(x) = \begin{cases} \cos\left(\frac{1}{\sin x}\right) + \frac{1}{2}(-1)^{[x/\pi]} & \text{if } x/\pi \notin \mathbb{Z} \\ \frac{1}{2}(-1)^{[x/\pi]} & \text{if } x/\pi \in \mathbb{Z}, \end{cases}$$

where $[x]$ denotes the largest integer less then or equal to x. It is a 2π-periodic Baire one function with the intermediate property which has no chord of length π. This shows that nor Theorem 4 nor Theorem 5 can be generalized dropping continuity of f in favour of the intermediate value property or being Baire one functions.

5 Possible applications

One satisfactory illustration of the Universal chord theorem's meaning is used as an initial motivation in [3]. Let me rephrase the problem. On November 16, 2013, Molly Huddle ran 37:49 for 12 kilometres and set a world record for this non-standard distance. However, Mary Keitany's world record for the half marathon (i.e., 21.1 kilometres) was 65:50 and her average pace was *higher* than in case of Molly Huddle. Does it mean that Mary Keitany must have run some 12 km subset of the race faster than 37:49? Surprisingly, because 12 is not a submultiple of 21.1 the answer is a conclusion of the Universal chord theorem and it is negative! Let us remark that in this case we use a (straightforward) generalization of the theorem for non-parallel chords. A construction of a hypothetical Mary Keitany's recordless run is provided in [3].

Conclusion

Investigation of chords of real function might be a source of many fruitful observations and statements. Many of them have elementary proofs and accessible even for the students of the first year of the university. It could be used in a special seminaries for students with deeper interest in calculus. The Universal chord theorem has also several 'popular' applications (see [3], [4]) which might engage students' attention.

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Výuka kalkulu – poklady, které nejspíš zůstanou studentům navždy skryty

Abstrakt: Rádi bychom čtenáře seznámili s některými výsledky kalkulu, které byť jsou dokazatelné elementárními prostředky a je jim vlastní matematická elegance, tak nejsou všeobecně známé. Zejména se zmíníme o všeobecné tětivové větě, která byla dokázána na začátku 20. století, ale nenašla si cestu do klasických učebnic kalkulu, tak byla několikrát nezávisle znovudokázána.

Klíčová slova: Výuka, kalkulus, všeobecná tětivová věta.

QUALITY IN A TRADITIONAL APPROACH TO PROJECT MANAGEMENT

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Abstract: Quality management is one of the basic components of the project, it refers to both the products and the actions taken which lead to the realization of the final product. The article presents the essence of quality management in projects in a traditional approach to a project management.

Keywords: quality management, project management, traditional approach to project management, quality

1 Introduction

The functioning of organizations in today's world is complex and variable, and the simple and routine operations do not bring the expected results. The importance of projects is constantly growing, and organizations adapt their approach from process to design to keep up with ever-changing environments [5]. Projects "(...) are unique projects of high complexity, indicated at the time of their execution - with spotlight beginning and end - requiring the involvement of significant but limited resources (material, human, financial, informational), carried out by a team of highly qualified contractors from different disciplines (interdisciplinary) in a relatively independent manner from repetitive activities associated with a high risk of technical, organizational and economic, and therefore require the use of special methods of their preparation and implementation" [14].

Quality management is one of the core components of the project, it refers both to the products and to the actions taken that lead to the completion of the final product [19]. Quality is a measure of the expected results formulated, delivered as a result of execution of the project. It increases the likelihood of successful completion of the project and customer satisfaction. Management through projects in organizations and the quality of processes and activities are elements that have a high value in the business environment and lead to a competitive advantage [8].

This article attempts to discuss the essence of quality in the traditional project management methodologies. The article aims to show how quality is perceived in the projects implemented on the basis of literature studies.

2 Quality management in the project - basic definitions

Quality in the project management can be productive, usable and productive. Product refers to the individual, key product properties, utility, determines the satisfaction of the subjective needs of the product, and the product refers to compliance with the specification of manufacturing requirements and minimization of deficiencies. The quality objective is to produce a specific end product that meets the key needs of its users while maintaining a certain efficiency [14]. Quality is one of the elementary objectives of project management and refers to the quality of the intended result obtained at the assumed cost and time [11].

One of them is the ISO standard granted by the International Organization for Standardization. Quality issue in the project was addressed in ISO 9000, ISO 9001, ISO 9004, ISO 10006 and IS0 21500 standards. The last ISO 21500 standard is dedicated to project management and sets its global standards [22]. Its guidelines can be applied to organizations of any type and to projects of varying lengths, complexity and size. The quality of the project should be considered in two perspectives, with regard to the quality of the final product and the quality of the project management processes [9]. Quality is also one of the basic parameters of the project. The rule for the basic parameters of the project as follows: "(...) to complete the project well (intended result at the appropriate level of quality), as a whole (range), cheap (costs) and fast (time)" [14]. In Figure 1 there are shown the basic parameters of the project and dependencies that arise between them.

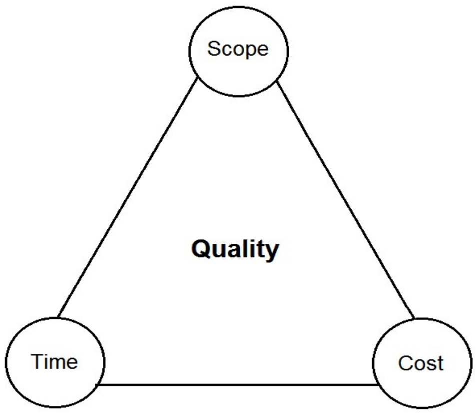

Fig. 1. Basic parameters of projects
Source: Own study based on [13]

3 Quality in the traditional approach to project management

Project management has evolved extensively over the years and therefore many industry standards have emerged that define how they are managed. The resulting methodology is a collection of good practices, standards, procedures, and processes that determine what needs to be done to achieve a successful project. Traditional project management methodologies are based on the life cycle of the project, which highlights the sequence of steps that should be taken to implement a particular project [16]. These methods are used in projects with clearly defined goals and the way to achieve them, low level of change of fixed range during the project [15]. Traditional project management methods include PRINCE2 and PMBOK and IPMA [12].

PRINCE2 is a standard in the UK where it was developed by the Office of Government Commerce, United Kingdom and is a project management method that is based on a process approach. As a part of this approach, the business justification of the project is created, the project products are precisely defined and the stages and tasks are defined. Risk management, quality, change and configuration techniques are well developed within the PRINCE2 methodology [20]. According to the PRINCE2 methodology, quality is all the properties and characteristics attributed to the product. The terminology used by PRINCE2 comes mostly from ISO 9000 standards [16]. The PRINCE2 methodology has identified seven topics that influence project management: business justification, organization, quality, plans, risk, change, progress. Specified quality, the main purpose of which is to guarantee the quality of the project at all stages and to comply with the applicable environment, along with its standards and procedures. The PRINCE2 methodology is distinguished by the quality planning and quality control. Planning takes place at the beginning of the project in the initiation phase, and identifies all project products that you want to control, product descriptions, quality criteria, how they are evaluated, methods used, and responsibilities [14]. Quality planning is to provide and communicate the basic arrangements to the Steering Committee about the quality expectations of the product and the criteria contained therein, and to determine the rules for controlling them. Quality control implements, monitors and records established quality parameters [2]. Figure 2 shows the path of quality audit which consists of the processes related to planning and quality control.

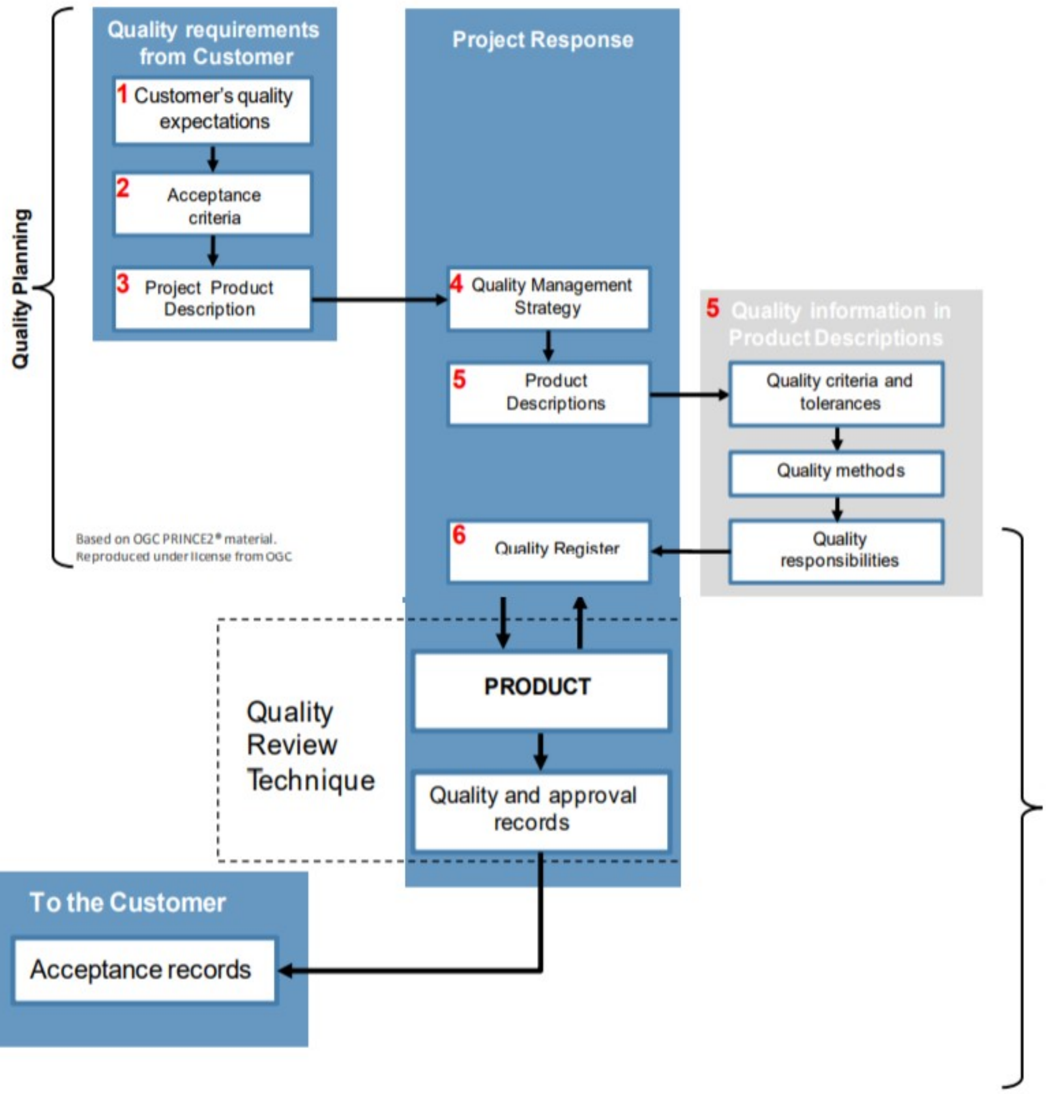

Fig. 2. Quality audit trail
Source: [2,3]

PMBOK is an American project management standard developed by the Project Management Institute, is a collection of processes, methods and techniques and covers all aspects of project management. It proposes the use of specific methods for individual processes. The methodology PMI quality management project is one of 10 areas of knowledge, in addition to management integration, scope, time, cost, human resources, communications, risk, procurement and stakeholders [1] "(...) to each of the areas there are assigned processes, which it is necessary to implement for the project management in this area to be effective" [10]. The model contained in the PMI methodology is universal, that is, you can freely choose the processes that you want to use in the implementation of projects of different types and sizes [4]. PMI project quality management processes include:

- Quality Planning - the process of identifying quality requirements, standards for the project and its achievements, and documenting how the project will demonstrate compliance with quality requirements or standards. The planning document is a quality management plan that outlines how the project will be implemented and the scope of the product.
- Quality Assurance - The process of transforming a quality management plan into quality activities takes into account the quality policy of the organization.
- Quality Control - the process of monitoring and recording the quality management results of activities to evaluate the results and ensure that the results of the project are complete, correct and meet customer expectations [1].

The area of the project quality management according to PMI methodology comprises incorporating the quality policy for the organization in planning, control and assurance requirements for design and product quality in order to meet the expectations of stakeholders [18]. Processes are intended to identify key quality standards for the design and methods of verification, use the quality of the planned activities, which will guarantee the fulfillment of the requirements, monitoring specific project results to determine whether they meet the quality standards and in case of unsatisfactory results remove or reduce the causes [1]. Figure 3 is a schematic diagram of major inputs, outputs and processes based on quality management in projects and informs about the fact that quality management processes involved in the management of quality throughout the project.

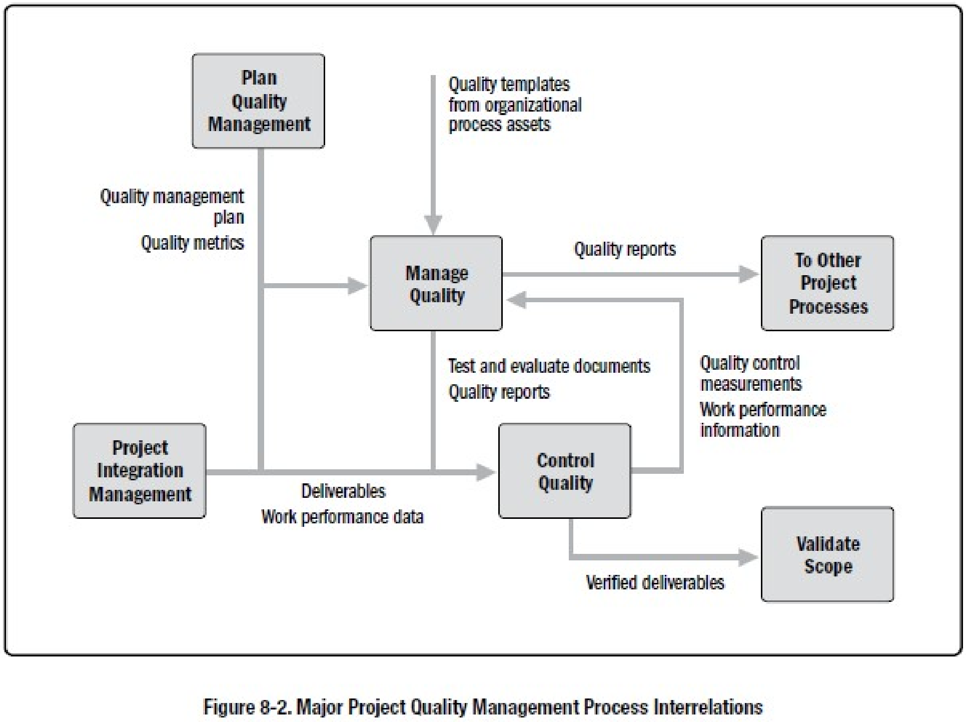

Fig. 3. Major Project Quality Management Process Interrelations
Source: [1]

The International Project Management Association is an international, not-for-profit international organization of Project Managers and promotes project management ideas. This approach does not define the processes and techniques of management, but rather the 46 competences that the Project Manager should have. Competences are technical, contextual and behavioral. Technically they refer to the creation of project products. Contextually, to the ability to function in a design environment. Behaviors are related to skills such as leadership, commitment, motivation, or ethics. Quality is in the technical competence. Quality is within the technical competence and should be considered as the foundation of the project. "Project quality management encompasses all stages (phases) and parts of the project, from initial design through all the project processes, project team management, subprojects to a closure" [7]. The organization that implements the projects sets the policy for quality. , defines methods of quality implementation (planning, operational procedures, indicators). The recommended method is to test the product at its manufacturing stage so that the subsequent versions and eventually the end product are defect-free and do not absorb repair costs [7].

Conclusion

Quality management in a project is a project management field that includes processes that meet the needs of the customer that caused the project to start. Quality management in the project involves most approaches such as planning, carrying out quality assurance and controlling it [1]. The processes are designed to identify the key quality standards for the project and the methods for their verification, the use of scheduled quality activities to ensure compliance, the monitoring of a specific project outcomes to determine whether they meet the quality standards and if unsatisfactory results are removed or reduced. In traditional methods, the scope is fixed and the time and cost are changed [6, 21]. Effective quality management allows you to monitor the progress of a project, determine whether the project is a good investment at every moment of its development. The project developer can make more efficient use of resources in the project by minimizing errors and losses and providing the recipient with a project that satisfies them. Modern approach to quality management is to minimize the deviation and to achieve specific results required by the stakeholders. Trends in project quality management include, but are not limited to: customer satisfaction, the pursuit of excellence through continuous improvement, management responsibility, mutual cooperation with suppliers [1].

Nevertheless, the presentation of different approaches to quality in project management gives us an insight into the possibilities of using and benefiting from the valuable solutions that are contained in the methodology.

CZ PL PŘEKRAČUJEME HRANICE
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JAKOŚĆ W TRADYCYJNYM PODEJŚCIU DO ZARZĄDZANIA PROJEKTAMI

Streszczenie: Zarządzanie jakością jest jednym z podstawowych składowych projektu, odnosi się zarówno do produktów, jak i do podejmowanych działań, które prowadzą do zrealizowania produktu końcowego. W artykule przedstawiono istotę zarządzania jakością w projektach w tradycyjnym podejściu do zarządzania projektami.

Słowa kluczowe: zarządzanie jakością, zarządzanie projektami, tradycyjne metodyki zarządzania projektami, jakość.

USE OF A DEFECTOSCOPE AS A TOOL FOR ELIMINATING THE HUNDRED-PERCENT QUALITY CONTROL IN A STEELWORKS

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Abstract: Technological progress in the vast majority of cases allows for eliminating destructive methods in the quality control of products in the metallurgical industry. At present we can use devices using e.g. ultrasounds or Eddy currents to test the quality of the produced product without destroying the sample. Properly calibrated control devices can check in real time whether a particular item complies with applicable standards and customer requirements, significantly improving the company's image and decreasing the costs associated with product returns. The article presents a solution aimed at improving the efficiency of quality control of the sealing of a product manufactured in a metallurgical production facility. The facility in question is located in Silesia, Poland.

Keywords: defectoscope, quality control, quality management methods

1 Introduction

The technological progress that we have observed over the last dozen or so years has had an impact on everything that surrounds us. New inventions can be found in many areas of everyday life. We try to use them as much as possible for the purpose of improving our quality of life, daily functioning and most importantly, to aid us at work. In the past, testing of products and manufactured goods was conducted using destructive methods. If it was not possible to check something using visual methods, destruction of the part, material or product was the only other choice.

Nowadays, when time plays an increasingly important role, we can use devices which utilise ultrasounds or Eddy currents to test the quality of a product without the need to destroy the sample [4,2]. Properly configured, they can check in real time whether a particular item meets the applicable standards or fails to do so and should be discarded or repaired. The article presents the results of the implementation of a defectoscope (automatic weld defect testing device) into the production cycle at the cambering line at the metallurgical plant's production site, to improve the quality control of the weld.

2 Cold-bent section line

Launched at the end of 2009, the FCF (Flexible Cold Forming) production line for the production of closed sections with a square or rectangular cross-section was the most modern in Poland and the third in Europe at that time. The advantage of these lines is their saving of the input material (5-20%) in the form of black or aluminum sheet. In lines of this type sections are produced directly from square and rectangular sheet in the ranges of 70x70 - 140x140 and 80x50 - 160x120 with wall thickness from 3.0 mm to 6.0 mm, made of S235, S275 and S355 grade steels from the JR, JO and J2 quality group according to PN-EN 10219-1, -2: 2007 ("Cold formed welded structural hollow sections of non-alloy and fine grain steels") for steel constructions. Shapes are produced in the roll forming process, and welded longitudinally using the inductive method.

The section production line consists of several main sections, such as:

1. Spreader - band preparation section. Includes a sheet metal swivel, spreader, straightener, guillotine to cut the beginning of the sheet and a welder. The output of this section is a single long sheet of metal, which is then rolled onto the battery.
2. Horizontal spiral battery - allows for storing more rolls of tape. It eliminates the need to stop at every consecutive joining of sheet metal in the previous section.
3. Forming section - These are three FCF blocks which, by alternatingly forming the sheet, give it the intended shape. The position of the plates is changed automatically, controlled from the desk at the welding section.
4. Welding section - In this section, a high frequency generator is used to heat the edge of the sheet and then, after plasticizing it by pressing it with the upper and lateral rollers and removing the flash, the most uniform surface possible is formed.
5. Calibration section - This section gives the final shape to the profile - its height and width. An important rule is that each of the four rolls is spinning.
6. Saw blade section - here the section is cut to the desired dimensions. Controlled by a CNC machine.
7. Packing machine - creates a package of sections, the number of pieces in each package depends on the quantity ordered by customers or the weight of the package. At the end of the process, each pack is weighed and a label with the necessary data is printed. Upon completion of the production and validation of the performance and non-destructive testing results, the label of each batch is attached by the quality assurance officer.

2.1 Product quality control

Sections intended for the construction market must meet the dimensional, material and delivery requirements specified in the standards set out in Table 1.

Table 1. Quality requirements for cold-bent sections

Cross-section of the section	Dimensional standard	Material standard	Delivery conditions
Square and rectangular	BN-79/0656-01	PN-88/H-84020	PN-EN 10219-1,-2:2007

In addition, the manufacturer confirms that the delivered products comply with the requirements stated in the order in the Factory Production Control document bearing a CE certificate number. The facility described from the beginning of the FCF line has been constantly monitoring the quality of its closed sections since the launch of the FCF line, which include:

- dimensional control,

- surface control,
- welding quality control.

The method of quality control testing in the initial period of operation of the Section Department was visual inspection carried out by employees of the production line. The effectiveness of this method depends on the level of training, knowledge, skills and experience of the staff. In the initial period, with a low number of orders, this method proved to be successful, but increasing production volumes and seeking maximum production capacity utilization, the company began to notice more and more problems with the quality of the sections and in particular with the quality of the weld consisting in weld discontinuity, as shown in Figure 1. Table 2 lists complaints due to defects in welds in the years 2011-2014.

Table 2. complaints pertaining to weld defects in the years 2011-2014

No.	Year of complaint	No. of complaints [pcs]	Returned amount [kg]	Production volume [Mg]	Ratio of defective products to production volume	Price discount given [%]	Cost* [PLN]
1	2011	3	10 400	600	1.73	3%	7 250
2	2012	4	9 540	6000	0.16	4%	5 400
3	2013	2	8 280	12 300	0.07	5%	40 000
4	2014	3	8 400	19 000	0.04	5%	30 100
						Total cost	**82 750**

**this position takes into account the approximate cost of the discount granted to the customer and provision of a defect-free product.*

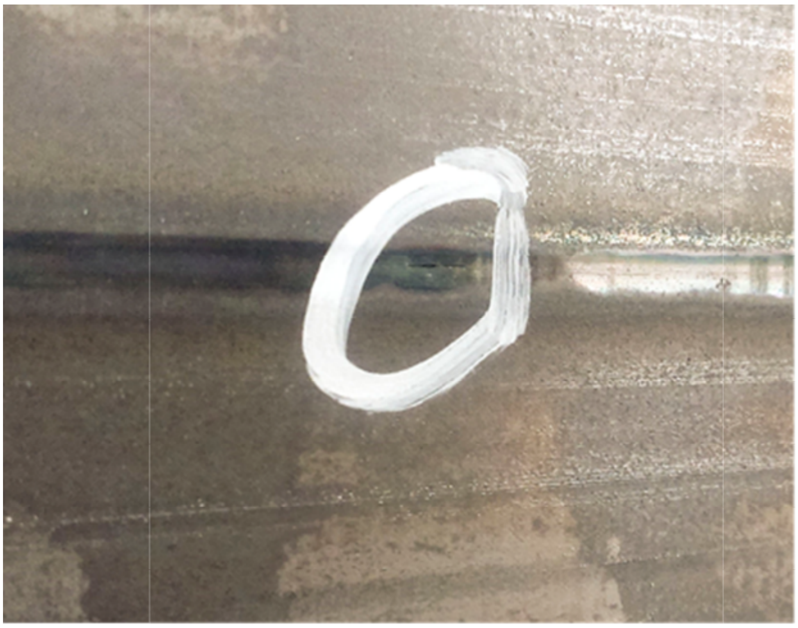

Fig. 1. Weld defects in form of holes in the flash

When analysing Table 2, it can be seen that the percentage of sections defective due to weld defects decreases year by year. However, with continuous increase in production volumes and increasing competition in the market, the occurrence of defects, even to a small extent, is undesirable and needs to be eliminated. Also presented in Table 2, the costs associated with complaints that the steelworks had to pay (over PLN 80,000) were too high for the problem of the defective to be ignored. In addition, the so-called hidden costs of credibility in the market should also be added to the identified costs. With this in mind, at the beginning of 2015, it was decided that a method for quality control of the seam should be developed, which, in addition to the visual inspection of the worker, would significantly increase the effectiveness of weld defect detection. Customer requirements, as well as the provisions in the standard for delivery conditions, indicate that it is best to use a device capable of measuring the quality of the weld during the operation of the line, which will allow for 100% control. There are two technologies available for conducting such testing in the market. These are devices that can perform ultrasonic or Eddy current testing [13]. The method of ultrasonic testing is applicable in cases where thick plates, i.e. ones over 8 mm, are used in the production This is a volume method. Eddy current testing using a coil is a surface-based method, which in this case is the best solution, therefore it was recommended to purchase, install and commission a device for measuring the weld quality using the eddy current method, and mount it in the production line. The cost of purchasing and installing such a solution is approx. 50 thousand PLN , which gives av return on the investment within two years of the department's work.

2.2 Closed section Eddy current testing device

After analyses, the EddyCheck 5 defectoscope with X-Y and Y-t imaging and LAB 3961R UN-type coil with 2-1000 kHz frequency and a differential and absolute circuit was purchased. Aspects such as price, implementation time, difficulty iof use, availability of technical support and possibility of mounting in the already existing technological line were taken into account when choosing the solution. EddyCheck 5 is a device used to measure weld quality obtained by applying a high frequency generator and pressing the edges of a section together. The resulting surface is smoothed using a lathe tool to remove the flash. The prepared surface is checked by the segment coil, part of the defectoscope. As a result of the measurements, the device receives the information from the coil, processes it and interprets whether weld discontinuity has occurred. In this method not only holes and bad welds, but also weld fractures on the inside of the test wall can be detected, which is an additional advantage of this solution [3,1,5].

Figure 2 shows the eddy current testing device's screen. On the left is a screen with the current information. At the top of the screen you can see the current speed of the line, the set length of the section, the number of good and bad pieces, and the total number of sections produced. Below, on the time chart we can see when a defect has occurred in the past. The moment of occurrence of defects in the weld is marked in red. Detecting a defect in the tested weld by the head caused rejection and preventing the given section from moving further in the production cycle.

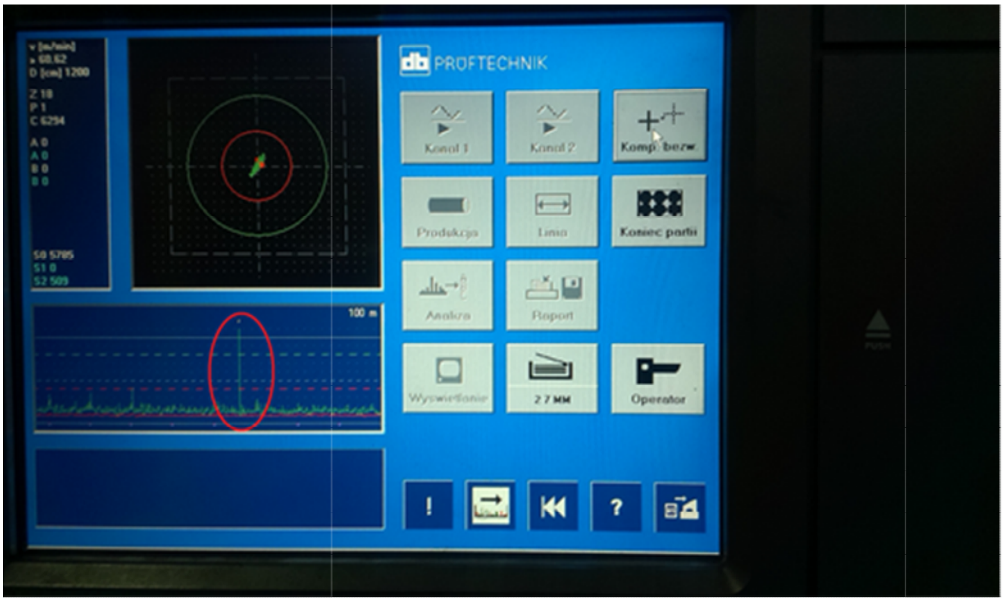

Fig. 2. Eddy current testing device screen showing the detected defect

3 The benefits of using the Pruftechnik EddyCheck 5 defectoscope

When launching a completely new production in the steel industry, where the plant had no experience in such production, it was assumed that in the initial period there might be some problems in terms of productivity, efficiency and quality of the manufactured products. The moment of the launch came at a time when the situation on the steel market was difficult. Each mistake brought costs and problems with establishing a presence in the market. In the fight for customers not only the price is a determinant but also the quality. The use of the defectoscope made it possible to completely eliminate the welds defects in the finished products, which, with successive increase of production volumes from year to year (Table 3) puts the steelworks in a good market position.

Table 3. Production volume of finished products in the period analysed

Year	2011	2012	2013	2014	2015
Production volume [Mg]	600	6000	12 300	19 000	22 300

Continuous improvement of the production process and application of solutions which help to achieve high quality of manufactured products allows the company to obtain positive opinions in the steel market. Today the steelworks enjoys a good brand in the market.

Conclusion

1. After the application of an automatic weld quality control device, the problem of complaints concerning bad welds or holes stopped. Employees responsible for proper production focus on the production process and do not need to personally check the quality of the weld

2. As a company entering the domestic market of cold-bend sections, the steelworks has shown that it cares about the quality of its products and is able to incur additional financial expenses to improve the testing process of the sections.
3. After several months since the launch of the eddy current testing device, there are no weld defects in the final product, which meets the applicable standards for sections.

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ZASTOSOWANIE DEFEKTOSKOPU JAKO NARZĘDZIA ELIMINUJĄCEGO STUPROCENTOWĄ KONTROLE JAKOŚCI W ZAKŁADZIE HUTNICZYM

Streszczenie: Postęp technologiczny pozwala w zdecydowanej ilości przypadków na wyeliminowanie metod niszczących w kontroli jakości produktów w branży hutniczej. Obecnie możemy za pomocą urządzeń wykorzystujących np. ultradźwięki bądź prądy wirowe dokonywać badań jakości wytwarzanego produktu bez potrzeby niszczenia próbki. Odpowiednio skalibrowane urządzenia kontrolne mogą sprawdzać w czasie rzeczywistym, czy dany detal jest zgodny z obowiązującymi normami i wymaganiami klienta, co w znaczny sposób poprawia wizerunek firmy i niweluje koszty związane z reklamacjami produktu.
W artykule zostanie zaprezentowane rozwiązanie mające na celu poprawienie efektywności kontroli jakości zgrzewu produkowanego wyrobu w zakładzie produkcyjnym branży hutniczej. Omawiana huta znajduje się w Polsce na terenie Śląska.

Słowa kluczowe: defektoskop, kontrola jakości, metody zarządzania jakością

ORGANIZATION IMPROVEMENT USING IT SYSTEMS TO SUPPORT THE HANDLING OF CASH REGISTER SERVICE COMPLAINTS

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Abstract: The article describes the problem of handling service complaints connected with cash registers manufactured for the needs of the retail branch. The improvement solution proposed by the author is to implement an IT system to support the recording of reported complaints and follow the process of IT system implementation. The collected data enabled determining the casual link between the variables of selected characteristics. The conclusions based on observation confirmed the effectiveness of the proposed solution. The increased effectiveness was achieved by focusing the company's activities on satisfying the clients' needs and fulfilling their expectations.

Keyword: fiscal device, complaint, application handling

1 Introduction

Clients are increasingly aware of their privileges in terms of consumer rights and interests protection [2], [5]. The producer's liability for the product contributes to the undertaking of actions which influence the safety of products placed on the market. However, the increasing degree of product complexity and the advancement of technological production process increases the risk related to product defects.

Complaints [1] always involve an uncomfortable situation for the buyer. Proper complaint handling may strengthen the client's loyalty and a positive image of the company. A satisfied client is a guarantee of profits, so the condition for success is developing plans of the company's activity which take clients' opinions into consideration. Improvement actions implementation in an organization is supported by a quality management system, compliant with ISO series 9000 standards [11], [12], [13]. As improving the customer service standards means adjusting the company's business processes to the needs of the environment, an important task faced by firms is monitoring buyers' satisfaction and an effective use of collected information. One of the methods for evaluating customer satisfaction is analysis of the complaint procedure [10]. The complaint management system is an integral part of the quality management system. It plays a key role in the improvement of provided services' quality. It is a source of information regarding the effects of the production activity. Consumers' comments can not only initiate the streamlining of the complaint procedure, but they can also provide inspiration for the directions of product development and manufacturing process improvement.

In the area of complaint procedure, the most important issue from the buyer's point of view is the effectiveness of undertaken service actions, promptness of response to a notification as well as the manner of communicating. For the seller an important thing is monitoring the progress of service works and eliminating the causes of customer dissatisfaction. From the producer's point of view, the complaint management system should ensure the possibility of analysing the collected data which describes quality nonconformities, accounting for among others the frequency of events, the type of machines or the conditions of use. Proper interpretation contributes to identifying the sources of failures, streamlining the manufacturing process as well as decreasing the number of non-conforming products. For each of the subjects participating in the complaint procedure an essential issue is efficient exchange of information.

The acquisition and transmission of data using IT tools support not only the subjects' communication and coordination of activities, but also the management of complaint handling scenarios. A scenario is a pattern of behaviours. Its developing requires defining the sequence of tasks and appointing the participants. Due to the final usefulness of the solution, scenario modelling should take into account factors which influence the effectiveness of service works' completion as well as the effectiveness of the collected data. Particular scenarios can be dedicated to various merchandise assortment groups, the types of nonconformities or even single stakeholders.

2 Previous research

Quality nonconformity also applies to cash registers, otherwise known as fiscal devices. They are used in retail to register the turnover and sums of tax due. The obligation of using cash registers results from the Goods and Services Tax Act [6] and the regulation on exemption from the obligation to keep a record if cash registers are applied [7]. The use of a cash register requires entering the device in fiscal controls and registering in a revenue office [9] as well as subjecting it to periodical technical inspections [7], [9]. The terms of organising and performing service activities for cash registers are specified in the regulation [8]. It provides a definition of a subject running the main service and a subject in charge of authorized service; it also specifies service technicians' obligations and the manner of documenting the conducted service works. The legal aspect of the issue has been described in detailed in the publication [3]. In the study also, the stages of technical intervention carried out by an authorized cash register service point have been presented. They include the following [3]:

- recording a reported complaint as well as assigning responsibility and the case classifier;
- verifying the legitimacy of the complaint and carrying out maintenance or repair works;
- preparing a cost estimate of service works and a settlement of the reported complaint;
- issuing a decision on the case, closing the reported complaint.

The collected knowledge allowed mapping the logical relationships between process participants and recording the effect of work in the form of a collaboration diagram in BPMN 2.0 [3]. The research continuation was developing a conceptual model enabling the recording and handling of reported service complaints, run by an authorized fiscal devices service point. The model diagram has been presented in the publication [4]. The need for designing a tool to support the complaint procedure additionally forced formulating the implementation requirements. Safety and data integrity were identified as important factors. The distinguished functionality was both authorization and concurrent access of users to the data. The mapping of limitations influenced the use of classifiers to describe an event and the models of serviced devices. A design of a database structure has been included in [4]. The solution to improve the process proposed by the author is creating variants in the handling of complaints by defining a set of scenarios. Harmonization of the type of activities undertaken in authorized service points was achieved by providing an initially configured implementation.

3 Evaluation of the effectiveness of the designed solution

The software implemented, supporting comprehensive complaint handling and processing of service requests, performs the following tasks:

- registering complaints and service requests based on defined classifiers describing, among others, models of serviced devices, types of events, types of non-compliances;
- gathering attachment documentation in the form of files;
- registering the customers' notes and remarks;
- handling the flow of information related to the coordination of activities carried out between authorized service points and the manufacturer;
- documenting the performance of service works;
- estimating the cost of an action, final account of the application;
- handling of expertise orders;
- informing the customer about the progress of the service works;
- generation of alerts initiating: information flow actions, reminders and warnings;
- transmission of messages informing the supervising stuff of transgressions and abuse;
- generation of periodic and statistical reports in a by-type format;
- generating summaries in graphical form;
- defining information flow paths;
- personalization of user permissions.

The effectiveness of the designed tool was assessed in two stages, through in-depth interview with representatives of system users and an analysis of system data collected when handling service and complaint requests. Three points of the authorized cash register service where dedicated software was implemented were selected for the study. The first stage of this study included defining the following:

- the dominant user group that uses the application;
- the system functionality with the highest utility for the unit;
- the way in which users respond to notifications and alerts;
- area of use of sheets, reports and summaries.

Table 1. Summary of the interview with representatives of the IT system users

No.	Entity name	Dominant user group	Prospective system functionality	Reaction to prompts	Area of report use
1	ASO 1	Service supervision staff	Documenting service works	Insignificant	Control of service technicians’ working time
2	ASO 2	Service technicians	Estimation of repair cost	Moderate	Admissibility of complaints
3	ASO 3	Service technicians	Documenting service works	Insignificant	Model failure reate

Source: author’s own study

In each organization, this stage was conducted in the form of an interview with a senior management representative responsible for maintaining and improving the quality management system in accordance with the requirements of ISO 9001: 2015-10 [12]. The application use report is presented in table 1.

As part of the second stage of the study, data were analysed that had been collected during the handling of complaints in the previously selected authorized service points. Estimates of the following characteristics were made:

- quotient of the quantity of accepted complaints to the sum of the number of accepted and dismissed complaints;
- the average cost of handling a complaint;
- quotas of quantities accepted for execution, to completed service requests;
- the average total service time of service personnel associated with the execution of a single service request;
- the failure rate of cash registers in relation to the number of registered devices, taking into account the conditions of their use.

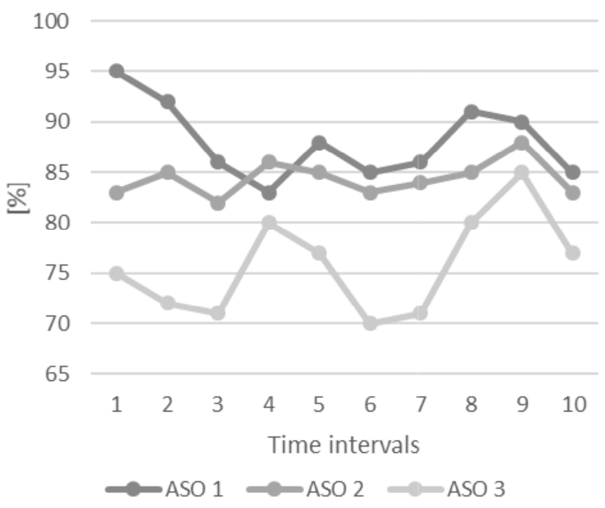

Fig. 1. Complaint acceptability [%]

Source: author's own study

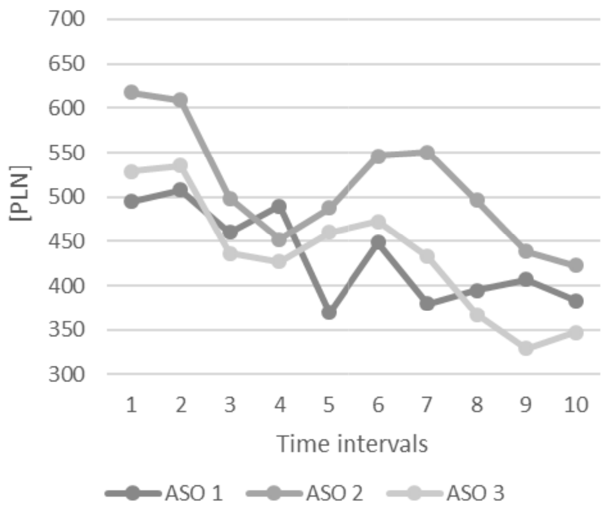

Fig. 2. Average cost of handling a complaint [PLN]

Source: author's own study

To illustrate the changes in each characteristic (Fig. 1 - 5), a linear graph was used. Placed on the horizontal axis are time intervals in which observations were made. An even distribution of time was assumed. Due to the confidentiality of data, variable scale unit lengths were used for each graph. Data aggregation was used for the spread of data along the vertical axis. The data series shown in Fig. 1 - 4 represent measurements taken at selected points of the authorized service. The data series shown in Fig. 5 describe the groups of devices that were created by the Polish cash register manufacturer.

Examining the effectiveness of the designed solution showed correlation of the complaint acceptance rate with the authorized service point as the receipt site. There was a clear downward trend for the complaint handling cost. The analysis of the effectiveness of service request execution has shown a cyclical nature of interferences that delay the completion of the work. Unfortunately, comparing the collected values describing the working time of the service technicians, neither a trend line nor the cause-effect relationships of the changes were identified. This is probably due to the fact that the handling of a complaint constitutes a small percentage of the actions taken compared to periodic technical inspections.

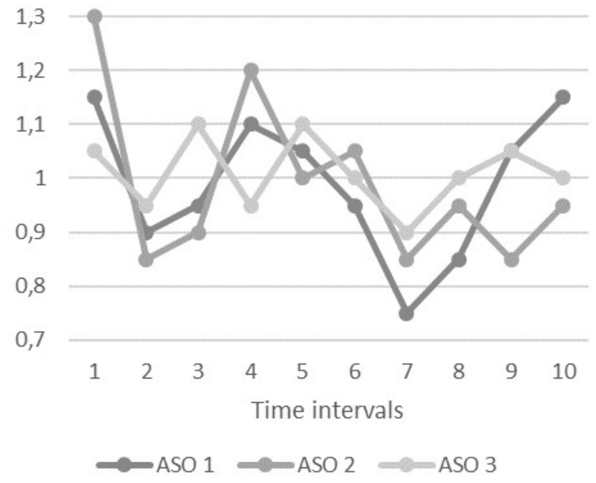

Fig. 3. Effectiveness of service request execution.

Source: author's own study

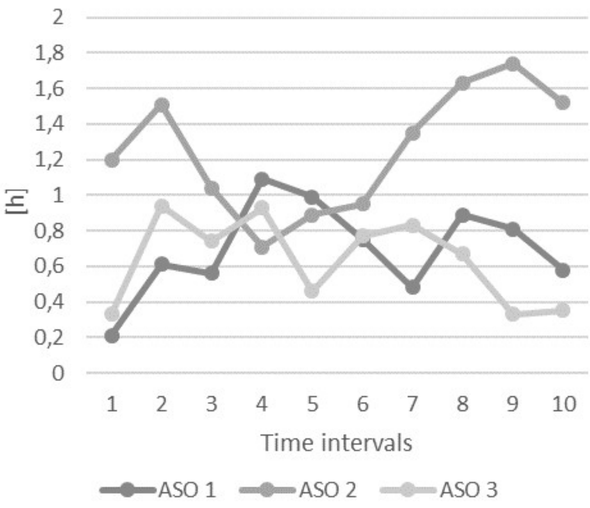

Fig. 4. Average total working time of service personnel per service request [h].

Source: author's own study

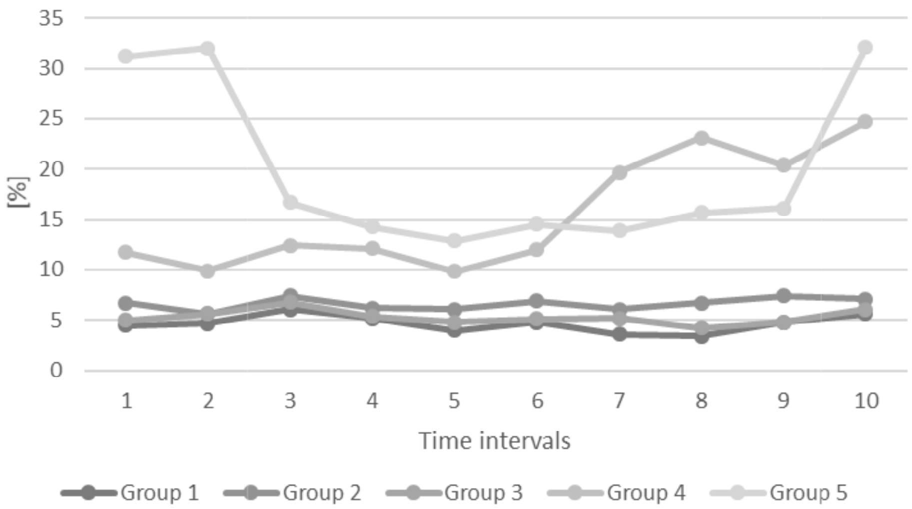

Fig. 5. Failure rate per number of registered devices [%].

Source: author's own study

Conclusion

The need to actively influence the quality of the services provided and the products supplied forced cash registers manufacturers to seek ways of eliminating the effects of quality noncompliance of cash registers and eliminating the causes behind the defects. Because complaint management plays a key role in the improvement of quality, an attempt was made to build a tool to support the registration of complaints and documentation of the service works. When creating the dedicated tool, the author used an analysis of the legal regulations in force in Poland as well as the identified needs of the Polish manufacturer of cash registers, produced for the retail industry. The implementation of the IT system was carried out in the company running the main service as well as in the entities running its authorized service. Analysis of the collected data revealed the existence of cause-and-effect relationships influencing the course of selected characteristics. The effectiveness of the solution was assessed by identifying the causes of non-compliances discovered during the operation of cash registers.

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DOSKONALENIE ORGANIZACJI Z WYKORZYSTANIEM SYSTEMÓW INFORMATYCZNYCH WSPOMAGAJĄCYCH OBSŁUGĘ ZGŁOSZEŃ SERWISOWO-REKLAMACYJNYCH KAS REJESTRUJĄCYCH

Streszczenie: W artykule opisano zagadnienie obsługi zgłoszeń serwisowo-reklamacyjnych urządzeń fiskalnych, wytwarzanych na potrzeby handlu detalicznego. Jako rozwiązanie doskonalące proces, autor zaproponował wdrożenie systemu informatycznego, wspomagającego ewidencjonowanie zgłoszeń i śledzenie przebiegu prac serwisowych. Zgromadzone dane umożliwiły wyznaczenie zależności przyczynowo skutkowych występujących pomiędzy zmiennymi wybranych charakterystyk. Wnioski z obserwacji potwierdziły skuteczność zaprojektowanego rozwiązania. Zwiększenie efektywności uzyskano poprzez ukierunkowanie działań przedsiębiorstwa na zaspokojenie potrzeb i spełnienie oczekiwań klientów.

Słowa kluczowe: urządzenie fiskalne, reklamacja, obsługa zgłoszeń

IMPROVEMENT THE PROCESS OF PAINTING CAR COCKPITS' ELEMENTS

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Abstract: The article presents an example of the improvement of the process of painting car dashboard components in one of the selected manufacturing companies. The article describes the painting process and provides figures indicating its initial low efficiency due to the fact that most of the painted details had paint defects in the form of material inclusions. The main part of the article is a description of causes of the high level of defect in the coating process. The study describes the actions taken in the investigated enterprise to improve the undesirable situation. The results achieved in the period of 2 months thanks to the implementation of a set of solutions improving the painting process is described in the last part of the study.

Keywords: quality, quality improvement, Ishikawa chart

1 Introduction

Improving the efficiency and effectiveness of the company's processes has now become a major task of managers nowadays. Continuous improvement became not only a function of normative management systems [1,2,3,4,5], it is also a condition of competitiveness and existence of enterprises on the market [6,7]. This article is an example of a process of improvement in the area of manufacturing, in particular the elimination of problems with defects in production. The company in which the research was carried out was a company in the automotive sector, which manufactures elements of cars' cockpits. The first part of the article describes problems with the quality of the process of painting plastic car dashboard elements. The following is a description of the main reasons for the unacceptable quality level with the implemented improvement measures. The effectiveness of these measures is assessed in the last section of this paper.

2 Problems in the painting process

The company specializes in manufacturing parts for the automotive industry. In the given case, they are interior fittings - cars' cockpit. The company has a hall producing moldings made of melted granules, plastic paintshop, assembly hall and auxiliary departments. This article describes the situation in the paintshop department, where problems with the quality of the coating applied to the plastic parts have been identified. The painting process is carried out in a three shifts system, and its schematic diagram is simplified in fig. 1. Preparation of the lacquered parts starts with degreasing them, setting them on a special rack. Then, by means of an automatic belt, the element is transported through blowing compressed air chamber. The next step is manual lacquering, automatic lacquering and baking in the oven at 90 ° C for about 1.5 hours. After this process, assessment of the quality of paint occurs.

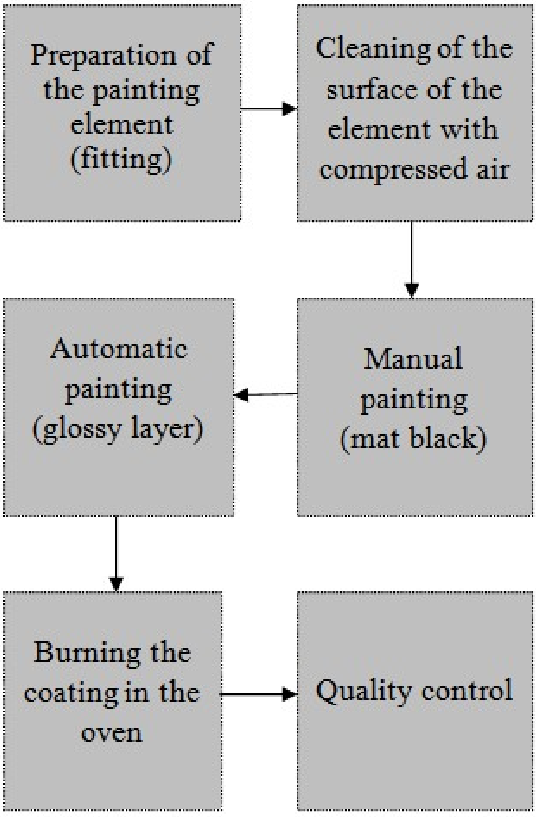

Fig. 1. Lacquering process
Source: Own study

During the inspection after the lacquering process a significant number of paint defects were identified - material inclusions. Due to the low effectiveness of lacquering, it was decided to analyze the causes of lacquering defects in order to improve the effectiveness of this process.

For this purpose, a special interdisciplinary task force was set up at the paintshop department, which consisted of: the manager, the controller of paint assembly line process, the foreman and the experienced painters. The team also included a specialist in quality assurance and accounting, the process controller in the assembly department and a specialist of the accounting department.

The team analyzed the quality control cards for each shift made in one month. Cards were the primary source of information on the number of incongruent items that required repair or re-paint. Figure 2 shows the effectiveness of lacquering - percentage of congruent elements in one month for each shift.

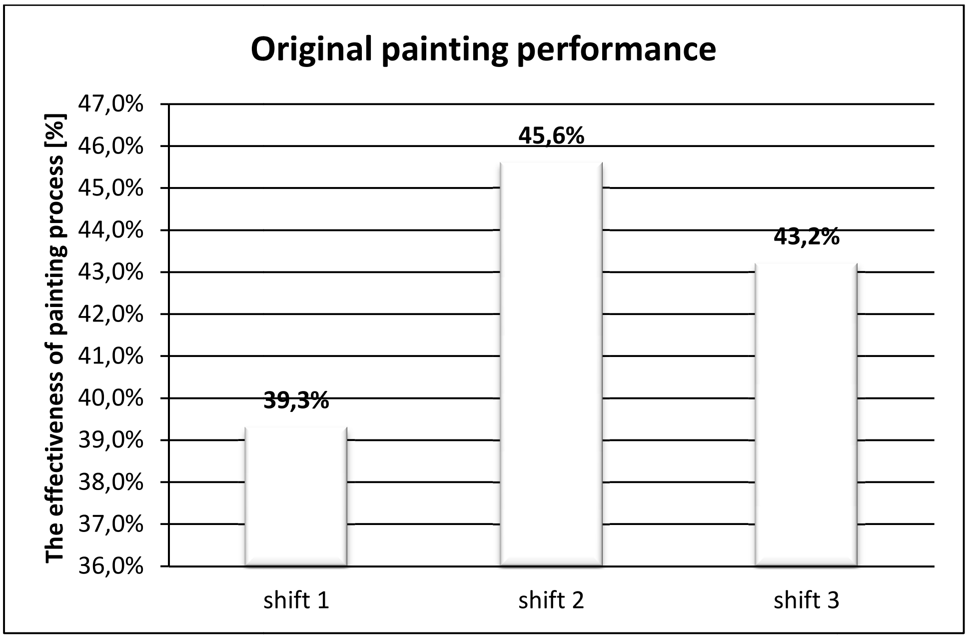

Fig. 2. Original painting performance
Source: Own study

The graph shows a highly adverse image of the painting process. Effectiveness of process in the range of 39.3% - 45.6% was rated by the management as highly unsatisfactory. The objective of the team in the first stage was to increase the efficiency of the varnishing process on each shift to a level of 60%.

A number of main causes and necessary corrective actions have been identified through the use of causal analysis. The causes of the defect of the painting process are shown in the Ishikawa diagram - Fig. 3

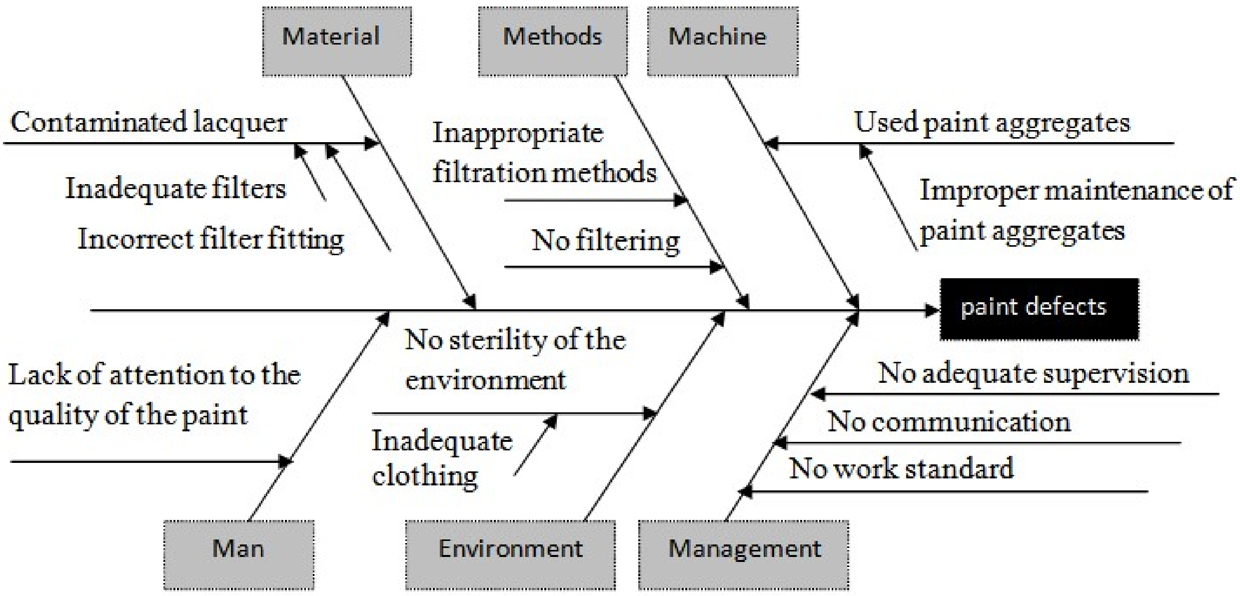

Fig. 3. Ishikawa chart for defect of the lacquering process
Source: Own study

The contaminants contained in the lacquer were the main cause of the inclusions. They were due to the improper and poorly installed filter, the material through which the lacquer was poured in the first phase of painting preparation. It was necessary to change the filter. The filter used initially had fibers with a density of 400µm and is currently 100µm. The new filter stops most contaminants. In addition, the filter was loosely mounted on the paint container, which caused the paint to float outside the filter. Thanks to the special stand the filter is stable and easy to clean. Many of the problems stemed from the fact that rigorous lacquer filtering standards have not been established. The filters were repeatedly used, what is worst, ignoring the direction of pouring. The problem was solved using a new filter design. Currently there is no possibility of confusing the direction of filtration.

On the pump for the paint to the aggregates have also been changed filters with the method of installation. The same density as with the paint filter is 100µm, the filter is double, and the assembly is more professional and durable.

Another cause of paint contamination was the lack of standards in the maintenance of equipment - guns (spray guns). It was necessary to replace the spray guns with the introduction of compulsory maintenance. Each cleaning of the equipment is currently listed in a special sheet under which the person responsible for cleaning is signed.

The next identified cause of the inclusions in the paint was the contaminated work clothes of the painters. Before applying corrective measures, 2-piece garments, i.e., trousers and lacquer blouse were used. The improvement in this case is the introduction of a workwear anti-static one-piece suit. As a result, it is no longer a source of contaminants in the form of textile fibers that get directly to the lacquer sprayed or floated in the air, settling on freshly lacquered elements.

As a consequence of the problems with the efficiency of the lacquering process, errors in communication and information flow were also recognized. Painters did not receive quality control feedback on the types of inclusions identified during quality control of painted items. Daily conferences were held to discuss current results of the lacquering process.

3 Effectiveness of corrective actions

During the next 2 months, the effectiveness of the lacquering process was analyzed. Based on the data collected in the control sheets, the indicators were compared for a total of 3 months. The results are shown in Figure 4.

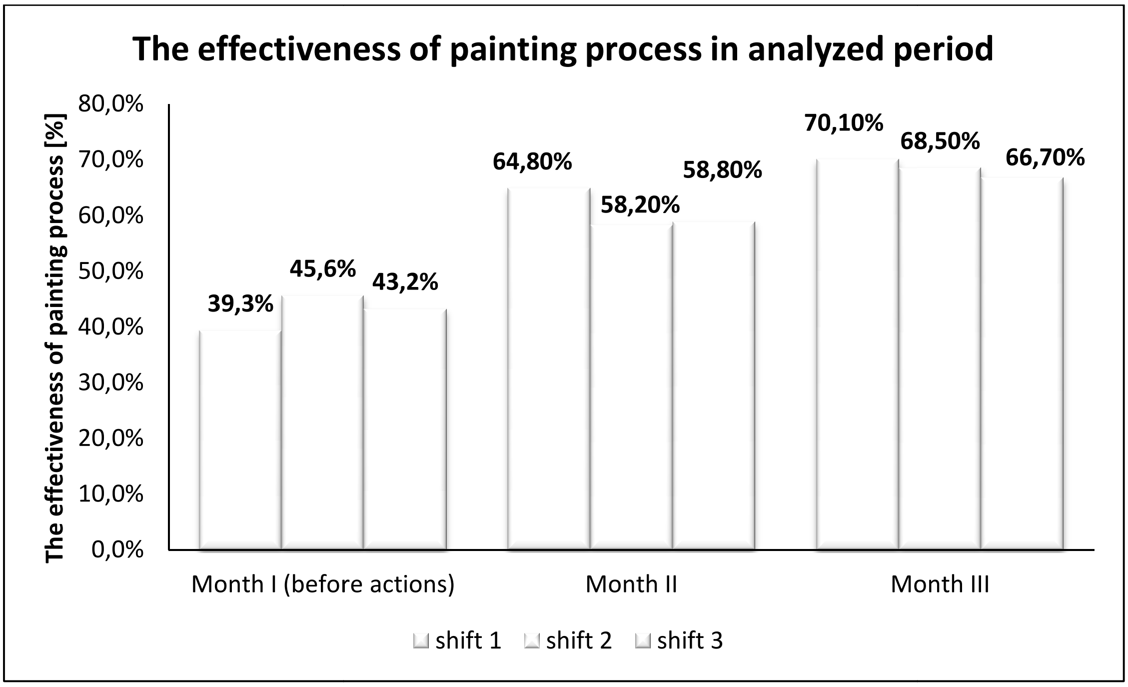

Fig. 4. The effectiveness of painting process in analyzed period
Source: Own study

Thanks to the introduction of a number of activities described in the article, the painting process has been improved. The improvement was noted already in the following month. The effectiveness of the lacquering process has reached the assumed level of 60% in all 3 shifts after 2 months. Not to mention the obvious benefits of less lacquer usage. The benefits of corrective actions are increasing the amount of space available in the product store where incongruent items are stored. Organizational benefits have also been gained, thanks to the mixing of the number of repainting has facilitated the planning of production of other elements.

Conclusion

The article is a good example of the process of improvement. Due to data obtained from quality control and well-chosen statistical methods, the scale of problems occurring in the process of painting in the examined production plant has been characterized. Teamwork and an analytical approach in the form of Ishikawa Diagram have identified the real causes of problems - inclusions, particles in the lacquer coating that disqualify the product. Analyzes of the causes by the specially appointed team identified the irregularities in all causal areas of the Ishikawa Diagram. Among other things, variations in lacquer filtering, changes in lacquer wear, and improvement of equipment maintenance standards, in addition to changes in communication, have resulted in significantly improved lacquering efficiency (60%) compared to the original. Although this value is far from what is expected, the improvement process should be considered effective as it has reached its original target.

The causes of defects in the lacquering process described in this paper and the described improvement actions may provide a useful source of knowledge for managers, who solve similar problems in the lacquering process.

PŘEKRAČUJEME HRANICE
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DOSKONALENIE PROCESU MALOWANIA ELEMENTÓW KOKPITU SAMOCHODOWEGO

Streszczenie: W artykule przedstawiono przykład doskonalenia procesu malowania elementów samochodowej deski rozdzielczej w jednym z wybranych przedsiębiorstw produkcyjnych. W artykule opisano proces lakierowania oraz podano dane liczbowe wskazujące na jego pierwotną niską skuteczność związaną z faktem, iż większość z malowanych detali posiadała wady lakiernicze w postaci wtrąceń materiałowych. Główna część artykułu stanowi opis zidentyfikowanych przyczyn wysokiego poziomu wadliwości procesu lakierowania. W opracowaniu opisano działania jakie podjęto w badanym przedsiębiorstwie w celu poprawy niepożądanej sytuacji. Efektów jakie osiągnięto w okresie 2. miesięcy dzięki wdrożeniu zestawu rozwiązań doskonalących proces lakierowania opisano w ostatniej części opracowania.

Słowa kluczowe: jakość, doskonalenie jakości, diagram Ishikawy

Modern Taylor Series Method in Numerical Integration: Part 2

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Abstract: The paper deals with extremely exact, stable, and fast numerical solutions of systems of differential equations with initial condition – initial value problems. Systems of ordinary differential equations are solved using variable order, variable step-size Modern Taylor Series Method. The Modern Taylor Series Method is based on a recurrent calculation of the Taylor series terms for each time interval. Thus, the complicated calculation of higher order derivatives (much criticized in the literature) need not be performed but rather the value of each Taylor series term is numerically calculated.
The paper present the solution of linear and nonlinear problems. As a linear problem, the telegraph equation was chosen. As a nonlinear problem, the behavior of Lorenz system was analyzed. All experiments were performed using MATLAB software, the newly developed nonlinear solver that uses Modern Taylor Series Method was used. Both linear and nonlinear solvers were compared with state of the art solvers in MATLAB.

Keywords: Taylor series method, ordinary differential equations, technical initial value problems.

1 Introduction

The paper deals with the solution of technical initial value problems (IVPs) representing the problems which arise from common technical practice (especially from electrical and mechanical engineering). To solve technical IVPs means to find the numerical solution of the system of ordinary differential equations (ODEs).

The best-known and the most accurate method of calculating a new value of the numerical solution of ODE [7]

$$y' = f(t, y), \qquad y(t_0) = y_0\,, \tag{1}$$

is to construct the Taylor series in the form

$$y_{i+1} = y_i + h \cdot f(t_i, y_i) + \frac{h^2}{2!} \cdot f'(t_i, y_i) + \ldots + \frac{h^n}{n!} \cdot f^{[n-1]}(t_i, y_i), \tag{2}$$

where h is the integration step.

The Taylor series can be very effectively implemented as the variable-order, variable-step-size numerical method [20] – Modern Taylor Series Method (MTSM). The method is based on a recurrent calculation of the Taylor series terms for each integration step. Therefore, the complicated calculation of higher order derivatives (much criticized in the literature) does not need to be performed, but rather the value of each Taylor series term is numerically calculated [13]. Equation (2) can then be rewritten in the form (3)

$$y_{i+1} = DY_0 + DY_1 + DY_2 + \cdots + DY_n\,. \tag{3}$$

Theoretically, it is possible to compute the solution of homogeneous linear differential equations with constant coefficients with arbitrary order and with arbitrary accuracy. Let us denote the ORD as the function which changes during the computation and defines the number of Taylor series terms in the current integration step ($ORD_{i+1} = n$). The resulting system of linear equations can be effectively solved either sequentially or in parallel.

An important part of the method is an automatic integration order setting, i.e. using as many Taylor series terms as the defined accuracy requires. Thus it is common that the computation uses different numbers of Taylor series terms for different integration steps of constant length.

The following paper is divided into several sections, which consider concrete technical IVPs and usage of MTSM. In Section 2, the effective numerical solution of a system of linear ODEs using higher order MTSM is shown and the problem of Telegraph equation is analyzed. The Section 3 considers the solution of quadratic nonlinear ODEs and the nonlinear Lorenz attractor problem is discussed. All algorithms of MTSM are efficiently implemented in MATLAB software [15] using vectorization. Finally, the MTSM algorithms are compared with MATLAB ode solvers.

Several papers focus on computer implementations of the Taylor series method in a variable order and variable step context (see, for instance TIDES software [3], TAYLOR [10] including detailed description of a variable step size version, ATOMF [6], COSY INFINITY [4], DAETS [17]. The variable step-size variable-order scheme is also described in [2] and [1], where simulations on a parallel computer are shown. This paper follows the article [5].

2 Solution of linear ODEs

Equation (2) for linear systems of ODEs in the form $\boldsymbol{y}' = \mathbf{A}\boldsymbol{y} + \mathbf{b}$ could be rewritten

$$\boldsymbol{y}_{i+1} \;=\; \boldsymbol{y}_i + h(\mathbf{A}\boldsymbol{y}_i + \mathbf{b}) + \tfrac{h^2}{2!}\mathbf{A}(\mathbf{A}\boldsymbol{y}_i + \mathbf{b}) + \tfrac{h^n}{n!}\mathbf{A}^{(n-1)}(\mathbf{A}\boldsymbol{y}_i + \mathbf{b})\,, \tag{4}$$

where $\mathbf{A}$ is the constant Jacobian matrix and $\mathbf{b}$ is the constant right-hand side.

Moreover, (4) can be rewritten in the form (3) where Taylor series terms could be computed recurrently

$$DY_0 = \boldsymbol{y}_i\,, \quad DY_1 = h(\mathbf{A}\boldsymbol{y}_i + \mathbf{b}), \quad DY_{j+1} = \frac{h}{j}\mathbf{A}DY_j\,, \quad j = 2, \ldots, n-1\,. \tag{5}$$

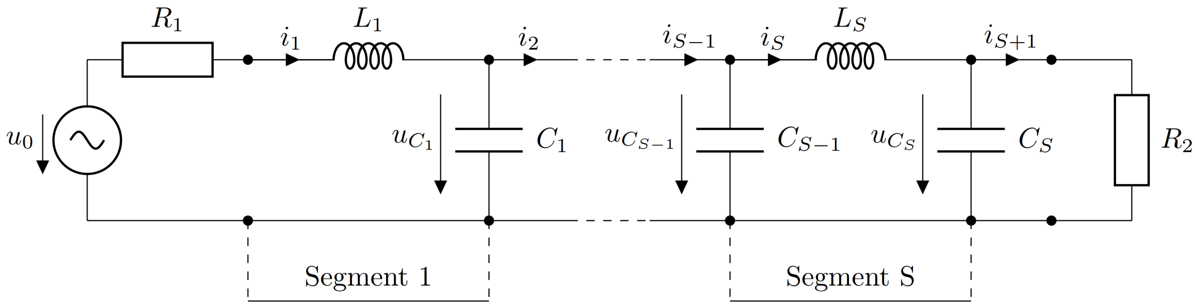


Figure 1: Model of the line−series of S segments

2.1 Telegraph equation

Let us solve the following electric circuit Figure 1, which represents a telegraph line [12,21]. The solution leads to the linear IVP

$$\boldsymbol{y}' = \mathbf{A}\boldsymbol{y} + \mathbf{b}, \quad \boldsymbol{y}(0) = \mathbf{y}_0\,, \tag{6}$$

where $\mathbf{A}$ is a matrix of constants (R, L, C parameters of circuit), $\boldsymbol{y}$ is a vector of variables (voltages and currents), $\mathbf{b}$ is a vector of constants and $\mathbf{y}_0$ is a vector of initial conditions. The block structure of matrix $\mathbf{A}$ and vectors $\boldsymbol{y}$ and $\mathbf{b}$ follows

$$\mathbf{A} = \left(\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{A}_{21} & \mathbf{A}_{22} \end{array}\right), \quad \boldsymbol{y} = \left(\begin{array}{c} u_{C_1} \\ \vdots \\ u_{C_S} \\ \hline i_1 \\ \vdots \\ i_S \end{array}\right), \quad \mathbf{b} = \left(\begin{array}{c} 0 \\ \vdots \\ 0 \\ \hline \frac{u_0}{L_1} \\ \vdots \\ 0 \end{array}\right), \tag{7}$$

where $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$ and $\mathbf{A}_{22}$ are individual block matrices with size $S \times S$

$$\mathbf{A}_{11} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{-1}{R_2 C_S} \end{pmatrix} \qquad \mathbf{A}_{12} = \begin{pmatrix} \frac{1}{C_1} & \frac{-1}{C_1} & 0 & \cdots & \cdots & 0 \\ 0 & \frac{1}{C_2} & \frac{-1}{C_2} & 0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \frac{1}{C_S} \end{pmatrix}$$

$$\mathbf{A}_{21} = \begin{pmatrix} \frac{-1}{L_1} & 0 & 0 & \cdots & \cdots & 0 \\ \frac{1}{L_2} & \frac{-1}{L_2} & 0 & 0 & \cdots & \vdots \\ 0 & \frac{1}{L_3} & \frac{-1}{L_3} & 0 & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \frac{1}{L_S} & \frac{-1}{L_S} \end{pmatrix} \quad \mathbf{A}_{22} = \begin{pmatrix} \frac{-R_1}{L_1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}.$$

For our experiments the capacitances and inductances are the same, $C = C_1 = \cdots = C_S = 1\,\text{pF}$ and $L = L_1 = \cdots = L_S = 10\,\text{nH}$. Moreover the transmission line is adjusted if $R_1 = R_2 = \sqrt{L/C} = 100\,\Omega$. The angular velocity is set $\omega = 3 \cdot 10^9 \text{rad/s}$. The input voltage u_0 should be generally constant (DC circuit) or harmonic (AC circuit) signal. In the case of DC circuit the input voltage u_0 is hidden in constant right hand side $\mathbf{b}$, see (7). In the case of AC circuit the input voltage $u_0 = U_0 \sin(\omega t)$ can be computed using auxiliary system of coupled linear ODEs

$$\begin{aligned} u_0' &= \omega x\,, & u_0(0) &= 0 \\ x' &= -\omega u_0\,, & x(0) &= U_0\,. \end{aligned} \tag{8}$$

In our example we use AC circuit with input voltage $u_0 = \sin(\omega t)$. The propagation constant per unit length of one segment for simple model of transmission line Figure 1 is known $t_{LC} = \sqrt{LC}$. Then the total delay of input signal could be computed as $t_{delay} = S\, t_{LC}$. The delay of output voltage $u_{C_{100}}$ for 100 segments is shown in Figure 2. The time of simulation was set $t_{max} = 2\, t_{delay}$ for all experiments.

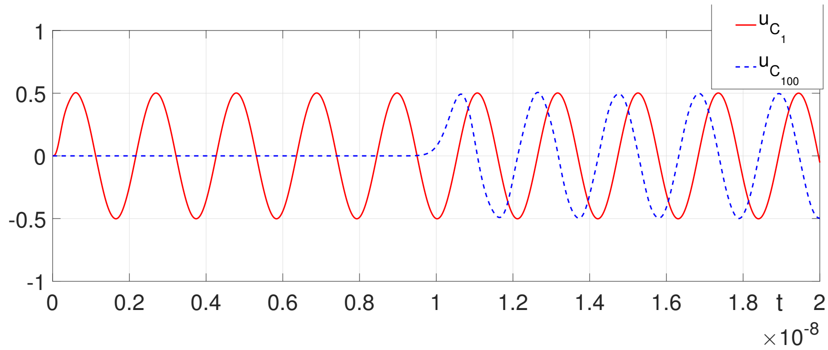


Figure 2: Delay of the signal on the transmission line with $S = 100$ segments

Vectorized MATLAB code of explicit Taylor series **expTay** with a variable order and variable step size scheme for linear systems of ODEs (6) has been implemented. This algorithm was tested on a set of examples of telegraph line with different number of segments S. The MTSM was compared with vectorized MATLAB explicit **ode** solvers. Both relative and absolute tolerances for all solvers were set to 10^{-7}. Benchmark results for MTSM with fixed number of integration steps $t_{max}/h = 200$ are shown in Table 1 and the results for MTSM with fixed h are shown in Table 2. Ratios of computation times *ratio* =**ode**/**expTay** > 1 indicate faster computation of the MTSM in all cases. Each reported runtime is taken as a median value of 100 computations. The MTSM average order ($mean(ORD)$) could be seen in the last columns of Table 1, 2. For the linearity and non-stiffness of the problem the ORD function was oscillating $mean(ORD) \pm 2$ during the computation.

Table 1: Time of solutions: explicit Taylor **expTay** and MATLAB explicit **ode** solver comparison; **expTay** with fixed number of steps $t_{max}/h = 200$

	ode23	**ode45**	**ode113**	**expTay**	**expTay**
S	*ratio*	*ratio*	*ratio*	[s]	$mean(ORD)$
200	18.9	7.1	6.2	0.056	20
600	18.6	9.2	6.9	0.199	41
1000	15.7	7.3	4.6	0.538	62
1400	15.9	7.5	4.1	1.005	83
1800	8.2	5.3	2.6	2.738	113

Table 2: Time of solutions: explicit Taylor **expTay** and MATLAB explicit **ode** solver comparison; **expTay** with fixed integration time step $h = 8 \cdot 10^{-10}$

S	**ode23** *ratio*	**ode45** *ratio*	**ode113** *ratio*	**expTay** [s]	**expTay** *mean*(*ORD*)
200	30.7	11.9	10.4	0.033	52
600	20.3	10.1	7.5	0.181	51
1000	13.6	6.4	4.1	0.606	51
1400	11.7	5.6	3.1	1.334	51
1800	8.8	5.5	2.6	2.498	51

More comparisons of MTSM numerical solutions of linear ODEs systems could be found in [16, 18].

3 Solution of nonlinear (quadratic) ODEs

In this section, the effective solution of special case of nonlinear quadratic systems of ODEs is described. The nonlinear quadratic system of ODEs is any first-order ODE that is quadratic in the unknown function. For such system Taylor series based numerical method can be implemented in very effective way.

Equation (1) for nonlinear-quadratic systems of ODEs can be rewritten in the form

$$\boldsymbol{y}' = \mathbf{A}\boldsymbol{y}^2 + \mathbf{B}\boldsymbol{y_{jk}} + \mathbf{C}\boldsymbol{y} + \mathbf{b}\,, \quad \boldsymbol{y}(0) = \mathbf{y}_0\,, \tag{9}$$

where $\mathbf{A} \in \mathbb{R}^{ne \times ne}$ is the matrix for pure quadratic term, $\mathbf{B} \in \mathbb{R}^{ne \times ne(ne-1)/2}$ is the matrix for mixed quadratic term, $\mathbf{C} \in \mathbb{R}^{ne \times ne}$ is the Jacobian matrix for linear part of the system, $\mathbf{b} \in \mathbb{R}^{ne}$ is the right-hand side for the forces incoming to the system and $\mathbf{y}_0$ is a vector of initial conditions and symbol *ne* stands for the number of equations in system of ODEs. The unknown function $\boldsymbol{y}^2$ represents the vector of multiplications $(y_1 y_1, y_2 y_2, \ldots, y_{ne} y_{ne})^T$; the unknown function $\boldsymbol{y_{jk}}$ represents the vector of mixed terms multiplications $(y_{j_1} y_{k_1}, y_{j_2} y_{k_2} \ldots, y_{j_{ne(ne-1)/2}} y_{k_{ne(ne-1)/2}})^T$. The indexes j, k comes from combinatorics $C(ne, 2)$. For simplification we suppose that the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and the vector $\mathbf{b}$ are constant.

Higher derivatives of such systems (9) can be effectively computed in MATLAB software [15] using matrix-vector multiplication, e.g. higher derivative $\boldsymbol{y}^{[p]}$ for pure quadratic term with matrix $\mathbf{A}$ can be expressed as

$$\boldsymbol{y}^{[p]} = \mathbf{A}\left(\sum_{i=0}^{p-2} \boldsymbol{y}^{[p-1-i]} .* \boldsymbol{y}^{[i]} \binom{p-1}{i} + \boldsymbol{y} .* \boldsymbol{y}^{[p-1]} \right), \tag{10}$$

where the operation '.∗' stands for *element-by-element* multiplication, i.e. $\boldsymbol{y}^{[p_1]} .* \boldsymbol{y}^{[p_2]}$ is a vector $(y_1^{[p_1]} y_1^{[p_2]}, \ldots, y_{ne}^{[p_1]} y_{ne}^{[p_2]})^T$. The binomial coefficients $\binom{p-1}{i}$ can be effectively precomputed using Pascal triangle, for more information see *pascal* function in MATLAB software [15].

3.1 Lorenz system

Lorenz system explains some of the unpredictable behavior of the weather. The Lorenz model supposes, that a planet atmosphere consists of a two-dimensional fluid cell which is heated from below and cooled from above [9]. The fluid motion can be described by three-dimensional system of ODEs (11)

$$\begin{aligned} x' &= \sigma(y - x) \\ y' &= \rho x - y - xz \\ z' &= xy - \beta z\,, \end{aligned} \tag{11}$$

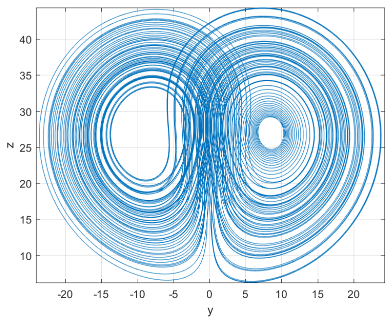
where σ is the Prandtl number, ρ is the Rayleigh number and β is the parameter related to the physical size of the system. The behavior of the system depends on the values of the parameters and initial conditions. Small changes in the initial conditions have a significant effect on the solution. The system (11) could be rewritten in the matrix form (9) where

$$\boldsymbol{y} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{A} = \mathbf{0}\,, \quad \mathbf{B} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} -\sigma & \sigma & 0 \\ \rho & -1 & 0 \\ 0 & 0 & -\beta \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

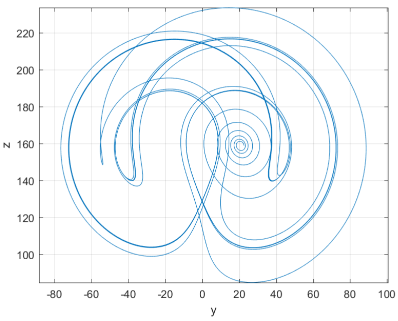
For the experiments, the parameters $\sigma = 10$, $\beta = 8/3$ were fixed. We change the parameter ρ to obtain different behavior of the system (11). For $\rho = 28$, the chaotic behavior could be observed (originally used by Lorenz [14]). For large ρ, e.g. $\rho = 160$, the solution is periodical (for more information see [11]). For $\rho = 23.7$, the solution is stable. Two equilibrium points can be calculated using (12). Initial conditions were then calculated by adding the constant vector $\vec{v} = (0, 2, 0)$ to the equilibrium point Q^+. For more information see [9], [8].

$$Q^{\pm} = (\pm\sqrt{\beta(\rho - 1)}, \pm\sqrt{\beta(\rho - 1)}, \rho - 1) \tag{12}$$

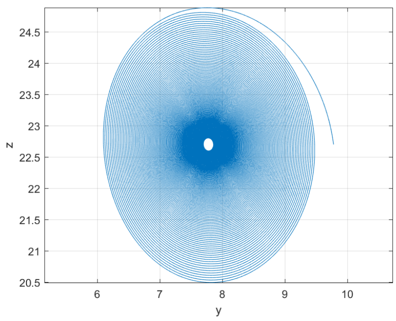
Figure 3 shows the solution of Lorenz system for different values of parameter ρ in yz-plane.



(a) $\rho = 28$



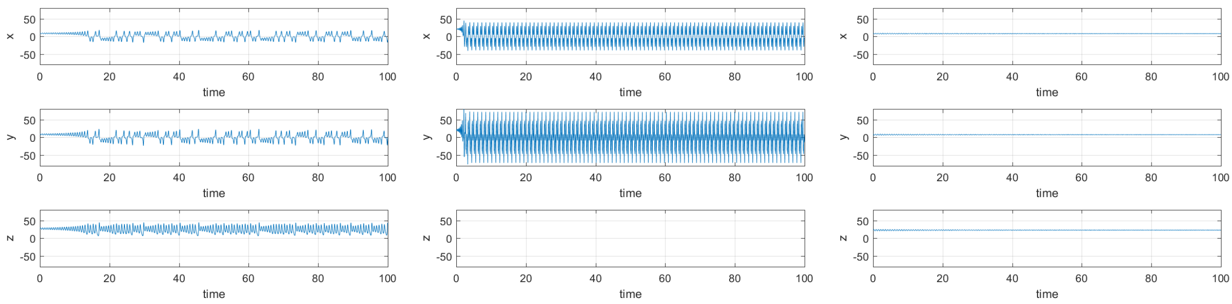
(b) $\rho = 160$



(c) $\rho = 23.7$

Figure 3: Behavior of Lorenz system, yz-plane

Solution in time domain could be seen in Figure 4. The maximum simulation time was set to $t_{MAX} = 100$ for all experiments.



(a) $\rho = 28$ (b) $\rho = 160$ (c) $\rho = 23.7$

Figure 4: Time domain

The MATLAB code of explicit Taylor series **expTay** with a variable order and variable step size scheme for nonlinear quadratic systems of ODEs (6) has been implemented. This algorithm was tested on a set of examples of Lorenz system with different ρ parameter. The MTSM was again compared with vectorized MATLAB explicit **ode** solvers. Both relative and absolute tolerances for all solvers were set to 10^{-10}. Results for comparisons MTSM with MATLAB ode solvers could be found in Table 3. Ratios of computation times $ratio$ =**ode**/**expTay** > 1 indicate faster computation of the MTSM in all cases. The number of integration steps could be found in Table 4. The MTSM order (ORD) is shown in Figure 5.

Table 3: Time of solutions: explicit Taylor **expTay** and MATLAB explicit **ode** solver comparison

ρ	**ode23** *ratio*	**ode45** *ratio*	**ode113** *ratio*	**expTay** [s]
28	200.8	7.3	1.9	0.933
160	196.3	6.9	1.9	2.363
23.7	15.7	7.3	4.6	0.538

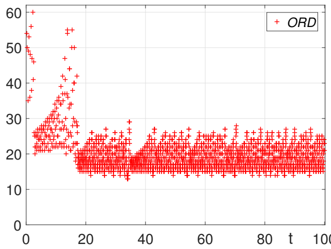
Table 4: Number of steps

ρ	**ode23**	**ode45**	**ode113**	**expTay**
28	3993135	322496	19794	2000
160	9907302	774340	48896	4000
23.7	1111529	147928	11029	500

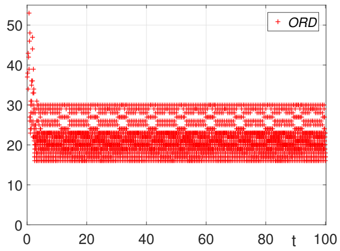
More comparisons of MTSM numerical solutions of non-linear ODEs systems could be found in [19].

Conclusion

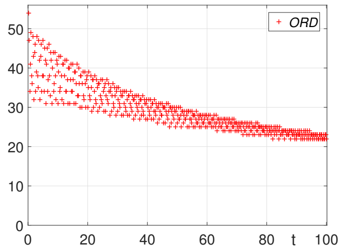
This article dealt with the numerical solution of linear and non-linear systems of ODEs. The model of the telegraph line was chosen as the example of linear problem, the Lorenz system



(a) $\rho = 28$



(b) $\rho = 160$



(c) $\rho = 23.7$

Figure 5: Lorenz system: ORD function

as the example of nonlinear one. All experiments were performed using MATLAB software. The MTSM solver for nonlinear systems of ODEs was successfully implemented. The experiments clearly showed, that MTSM is suitable for solving both linear and nonlinear systems. Moreover, the MTSM could be faster and more accurate than state-of-the art ode solvers in MATLAB.

Future work will be focused to the parallelization and hardware representation of the MTSM.

Acknowledgements

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MODERNÍ METODA TAYLOROVY ŘADY V NUMERICKÉ INTEGRACI: ČÁST II.

Abstrakt: Článek se zabývá přesným, rychlým a stabilním řešením obyčejných diferenciálních rovnic (Cauchyho úlohy). Soustavy těchto rovnic jsou řešeny pomocí Moderní metody Taylorovy řady. Tato metoda je proměnného řádu a využívá proměnný integrační krok. Členy Taylorovy řady se počítají iteratívně, díky tomu je možno vypočítat i vyšší derivace.
Článek prezentuje řešení lineárních a nelineárních problémů. Jako lineární problém bylo zvoleno řešení telegrafní rovnice, jako nelineární byl zvolen Lorenzův systém. Experimenty byly provedeny pomocí systému MATLAB s využitím nově implementovaných nástrojů. Moderní metoda Taylorovy řady byla porovnána s běžně používanými řešiči obyčejných diferenciálních rovnic v systému MATLAB.

Klíčová slova: Metoda Taylorovy řady, obyčejné diferenciální rovnice, technické problémy, Cauchyho úloha.

MODELLING OF NOISE THREAT ASSESSMENT IN SMALL INDUSTRIAL ROOMS

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Abstract: The research done in the area of analysis and assessment of noise threatened working place concentrate on identification of sources of sound. In order to perform the assessment of noise threat in the examined space, there is a need to gather additional information, among others about the construction features of the place, its equipment, etc. The process of selection of used ways and methods is highly determined by the acoustic parameters of the sources of sound, their characteristics and localizations. The information related to geometric models of the places as well as material features of the barriers create a set of complimentary parameters – necessary for proper selection of noise reducing methods. Due to variety of application of methods of modeling the acoustic fields it was stated, that each of them has limited applicability in the research because of the shape of the place and characteristics of the field. Its main simplification is related to the assumption of stability of acoustic parameters of the field in the domain of time simultaneously excluding the wave phenomena. This article is an attempt to identify the sources of sound as the emitters of acoustic energy in the working place. In this approach, the sources are treated as either points emitting acoustic energy.

Keywords: noise threat, model, source sound, assessment of noise

1 Introduction

The analysis of acoustic energy in small rooms is of the greatest importance because of existence of wave phenomena such as: radiation, absorption, refraction and dispersion of an acoustic wave. Taking into account the wave phenomena in assessment of noise threat in the working environment enables to identify the meaning of their emergence in the energy-related description of an acoustic field. Energy-related approach to acoustic field allows to perform a more detailed analysis of ways of transmission and dispersion of acoustic energy directly from the source in the examined environment. In particular, thanks to such a description of the field, it is possible to analyze the variability of the shape of the waves and streams of energy in the frequency domain. Application of advanced graphical methods and assessment of distribution of acoustic energy in relation with the measurement methods allows to shape the noise threatened working place according to the requirements of ergonomics [2, 3].

2 Evaluating noise threat using geometric methods

The proposed model using geometric methods of noise simulation applies in particular to stationary noise sources and ergodic stationary signals (the mean value and the autocorrelation function are independent of time). The sound intensity parameter for any number of sound beam reflections can be expressed by means of the following generalized formula (1).

$$I_{ki} = \frac{N_i}{\Omega_i \cdot R_{ki}^{\ 2}} \prod_{k=1}^{N}(1-\alpha_k) \qquad (1)$$

where:

N_i - *acoustic power of the j-th sound source,*

R_{ki} *-the distance of the sound beam from the i-th source to the k-th point,*

Ω_i i- *solid angle of the radiation of the i-th sound source,*

α_k *-sound absorption coefficients for the model surfaces.*

Isolating the attributes of acoustic parameters from (1), an a_{ik} coefficient is introduced, which describes the following relationship:

$$a_{ik} = \frac{(1-\alpha_k)}{\Omega_i \cdot R_{ki}^{\ 2}} \qquad (1.2)$$

After converting the (1.2) equation to a matrix form we receive the following:

$$|I| = |N| \cdot |A| \qquad (1.3)$$

where:

I – sound intensity vector

N- vector of acoustic sound sources,

A – coefficients matrix

With the matrix representation of equation (1.3) - it takes the following detailed form:

$$\begin{vmatrix} I_1 \\ I_2 \\ I_3 \\ \ldots \\ I_n \end{vmatrix} = |N_1 N_2 N_3 \ldots N_n| \cdot \begin{vmatrix} a_{11}a_{12}a_{13}\ldots a_{1m} \\ a_{21}a_{22}a_{23}\ldots a_{2m} \\ a_{31}a_{32}a_{33}\ldots a_{3m} \\ \ldots \\ a_{n1}a_{n2}a_{n3}\ldots a_{nm} \end{vmatrix} \qquad (2)$$

The (theoretical) component values of sound intensity coming from the individual sound sources can be obtained through a sound simulation post-process, using N = 1[W] as acoustic power values for the sources. In this case the sound intensity vector includes the theoretical values of acoustic parameters at reception points. In order to determine the real acoustic power values of the sound sources for the calculated sound intensity values at reception points, the operation of reversing the coefficients matrix [A] must be performed. This matrix contains a description of the geometric and acoustic parameters of the system discussed here. The essence of the proposed model consists in determining the inverse matrix [A], which includes the spatial relationships of geometric position in the source-receiver relationship as well as the geometry of the room, and takes into consideration the properties of the model's surface material [1].

$$|N| = |N_1 N_2 N_3 ... N_n| = |A|^{-1} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \\ ... \\ I_n \end{vmatrix} \tag{2.1}$$

The real values of acoustic power of sound sources on the basis of the values obtained for the intensity vector at reception points is determined using equation (2.1). For potential acoustic situations with 3 sound sources the noise exposure model using geometric methods of sound simulation takes the following form for the determined real values of acoustic power:

$$\begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = |N_1 N_2 N_3| \cdot \begin{vmatrix} a_{11} a_{12} a_{13} \\ a_{21} a_{22} a_{23} \\ a_{31} a_{32} a_{33} \end{vmatrix} \tag{3}$$

The obtained set of relationships of acoustic situations caused by the sources (3) for the simulated acoustic field in a room is described by an approximated noise exposure risk matrix model.

2.1 The analysis of the results of sound simulation in a model room

A theoretical model room was prepared for the purpose of a computer simulation with the following dimensions: 3.2[m]x2.2[m]x2.7[m]. The model room was designed as an ideal echo-free chamber with 3 spherical sources and 14 reception points situated at the height of 1 [m]; the α = 1 coefficient was used for all the walls. It was decided that sources S_1 and S_2 will be situated at the height of 0.5 [m] and source S_3 at the height of 1 [m]. The simulation parameters for the calculations were 5000 beams of sound from each source and 2000 reflections. According to equation (2) presented below equal acoustic power levels L_N of 120 dB (A) were used for all the sources.

$$L_N = |120 \quad 120 \quad 120| [dB(A)]$$

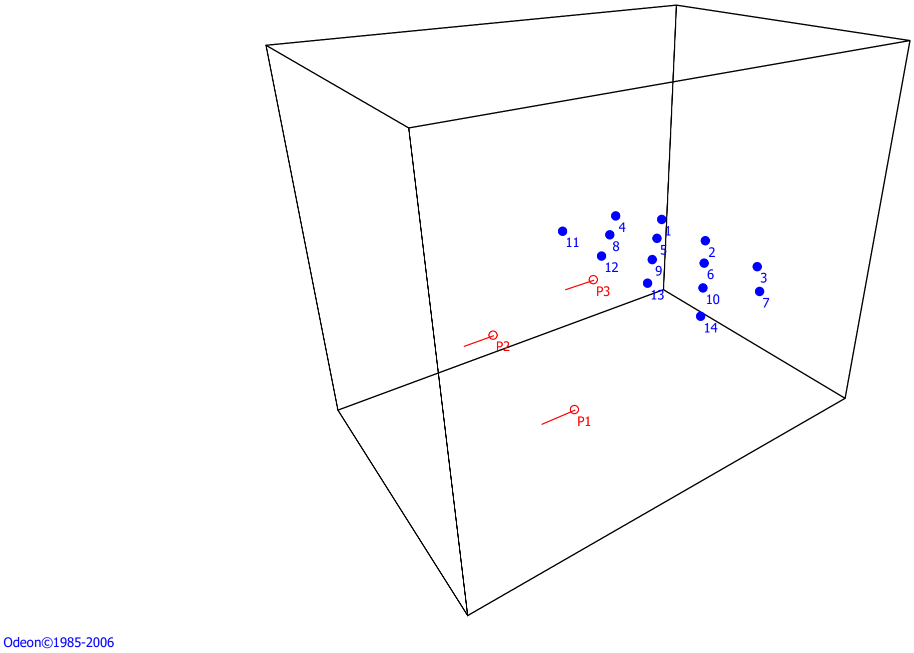

Fig. 1. Simulation model of the room

Following the sound simulation the sound levels coming from the respective sources S_1, S_2, S_3. were obtained for 3 selected reception points (1,2,3). The following values were obtained from source S_1 at 3 reception points:

$$S_1 = \begin{bmatrix} 101{,}5 \\ 102{,}6 \\ 103 \end{bmatrix} [dB(A)]$$

The following values were obtained from source *S2* at 3 reception points:

$$S_2 = \begin{bmatrix} 103 \\ 102{,}6 \\ 101{,}5 \end{bmatrix} [dB(A)]$$

The following values were obtained from source *S3* at 3 reception points:

$$S_3 = \begin{bmatrix} 106 \\ 107 \\ 106 \end{bmatrix} [dB(A)]$$

Using the above and converting the units in matrix [A], the matrix was then inverted:

$$|A|^{-1} = \begin{vmatrix} 10{,}922 & 182{,}530 & -70{,}238 \\ -153{,}665 & -153{,}655 & 131{,}537 \\ 182{,}530 & 10{,}922 & -70{,}238 \end{vmatrix} \left[1/m^2 \right]$$

After converting the units appropriately and substituting them in equation (3) and after multiplication total values of intensity vector in the individual points coming from sources S_1, S_2, S_3 are obtained.

$$|I| = \begin{vmatrix} 0{,}073 \\ 0{,}086 \\ 0{,}073 \end{vmatrix} \left[W\!\!\diagup_{m^2} \right]$$

The model can be verified by determining the value of acoustic power levels of the sources on the basis of the obtained results of the sound level at the same reception points in the model. The sound simulation yielded the following values of sound level at 3 reception points – successively from sources S_1, S_2, S_3:

$$S_1 = \begin{bmatrix} 63{,}5 \\ 64{,}6 \\ 65 \end{bmatrix} [dB(A)],\; S_2 = \begin{bmatrix} 65 \\ 64{,}6 \\ 63{,}5 \end{bmatrix} [dB(A)],\; S_3 = \begin{bmatrix} 63 \\ 64 \\ 63 \end{bmatrix} [dB(A)]$$

The total values of the intensity vector in the examined points are:

$$|I| = \begin{vmatrix} 0{,}000158 \\ 0{,}0000158 \\ 0{,}0000501 \end{vmatrix} \left[W\!\!\diagup_{m^2} \right]$$

After substituting them in equation (2.1), multiplying the matrix and converting the units, acoustic power values of sources S_1, S_2, S_3 are obtained.

$$L_N = |80 \quad 80 \quad 75| [dB(A)]$$

The verification of the proposed matrix model confirmed that the levels of acoustic power of the sources used for the simulation were correct.
After substituting them in equation (2.1) the values of acoustic power L_N for the 3 reception points were calculated taking into account various acoustic situations of the source activity.

Table 1. Combinations of acoustic situations, sound sources and sound levels

Acoustic situations	L_{N1} *[dB(A)]*	L_{N2} *[dB(A)]*	L_{N3} *[dB(A)]*
S_1	80	0	0
S_2	0	80	0
S_3	0	0	75
S_1,S_2	79,89	79,89	62
S_2,S_3	67,74	79,97	74,67
S_1,S_3	79,97	67,74	74,67
S_1,S_2,S_3	80,67	80,67	72,44

The values of acoustic power obtained for the reception points in question for the alternatives of two and three active sources (table 1) are similar to the values of the respective sources. However, in these acoustic situations there are sound level values from sources, which should yield zero values. An analysis of the results of the acoustic parameters simulation for the selected alternative of the operation of sources S_1, S_2, S_3 was carried out using the *Odeon 8.5* software.

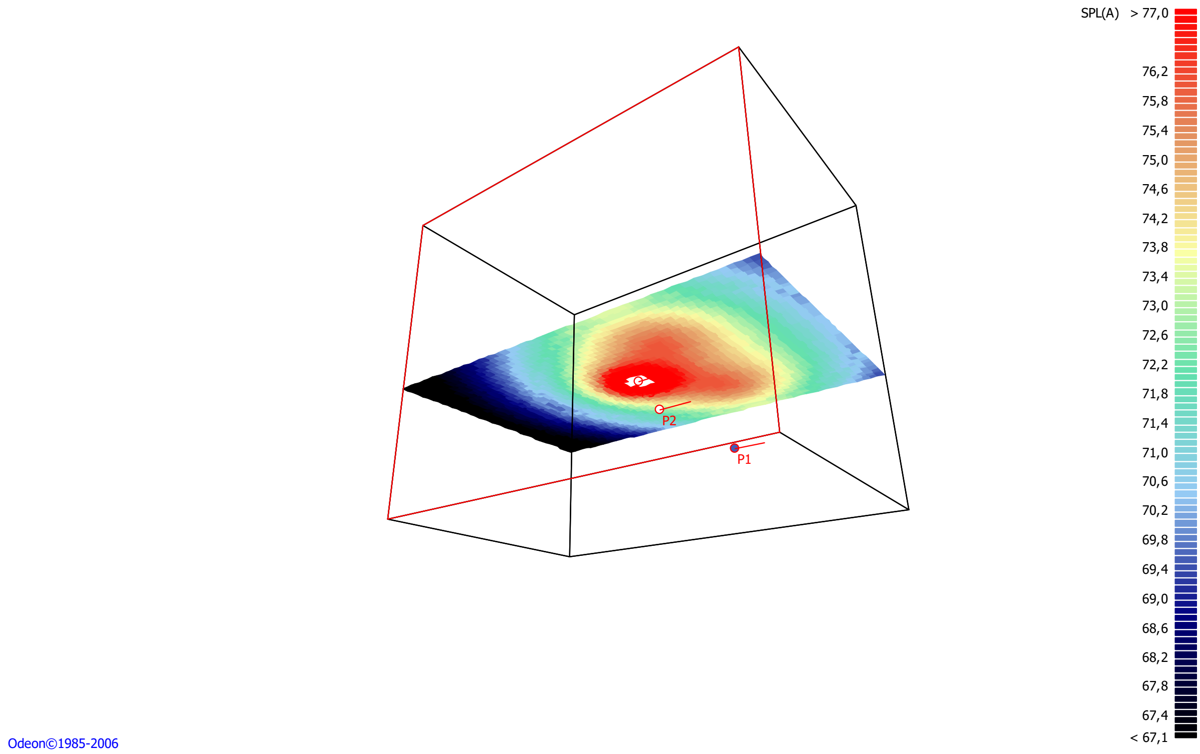

Fig. 2. Sound level distribution map at the height of source S_3 in the model room

In the simulation the sound level values were different at the reception points due to their position in space (Fig. 3).

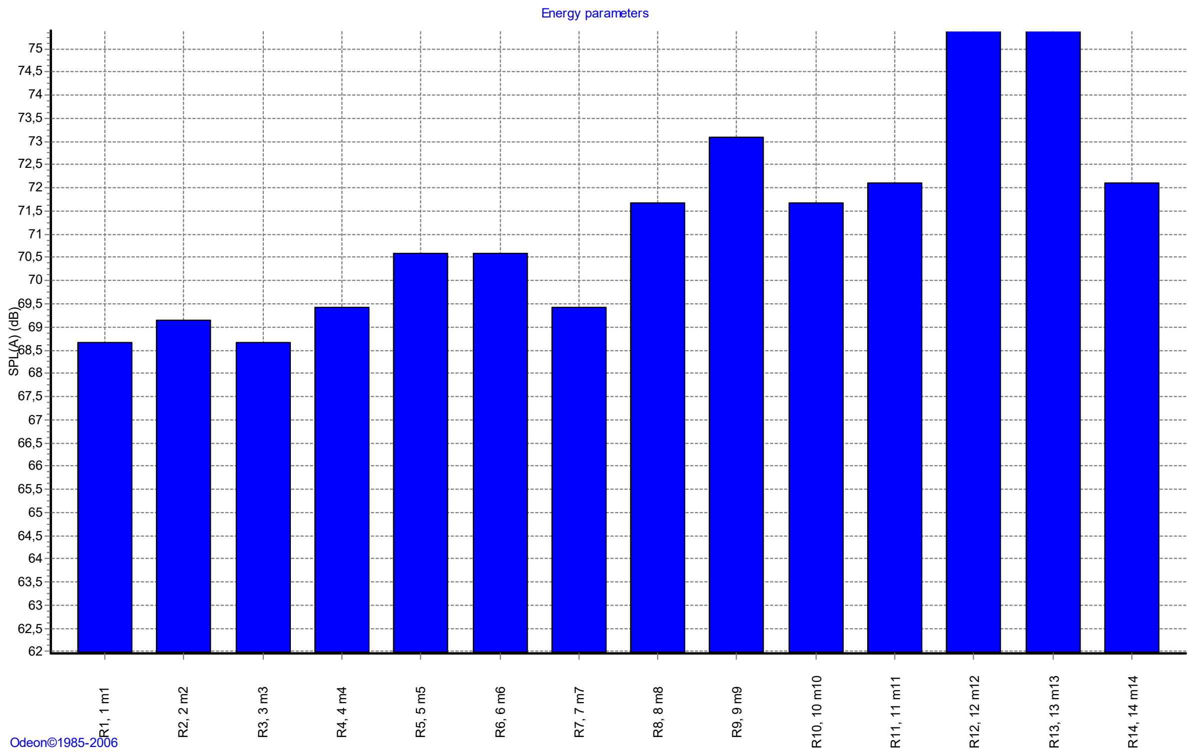

Fig. 3. Sound level distribution at reception points

The sound level value distribution at reception points in octave frequencies does not show variability (Fig. 4). The simulation results show that the largest energetic share of the sources in 1 reception point occurs at the 2000 [Hz] frequency. The results obtained from the simulation (table 1) at the reception points for the proposed noise threat evaluation model show some discrepancies. This is caused by the dimensions of the room and the limitations of the geometric simulation methods used. The distribution of acoustic energy from the sound sources in limited spaces mainly depends on the shape and the ratio between the room dimensions and the length of the emitted waves. In the case in question the room dimensions are relatively small in relation to the wavelengths. Additionally, in small rooms wave phenomena occur which are not accounted for in the geometric simulation methods.

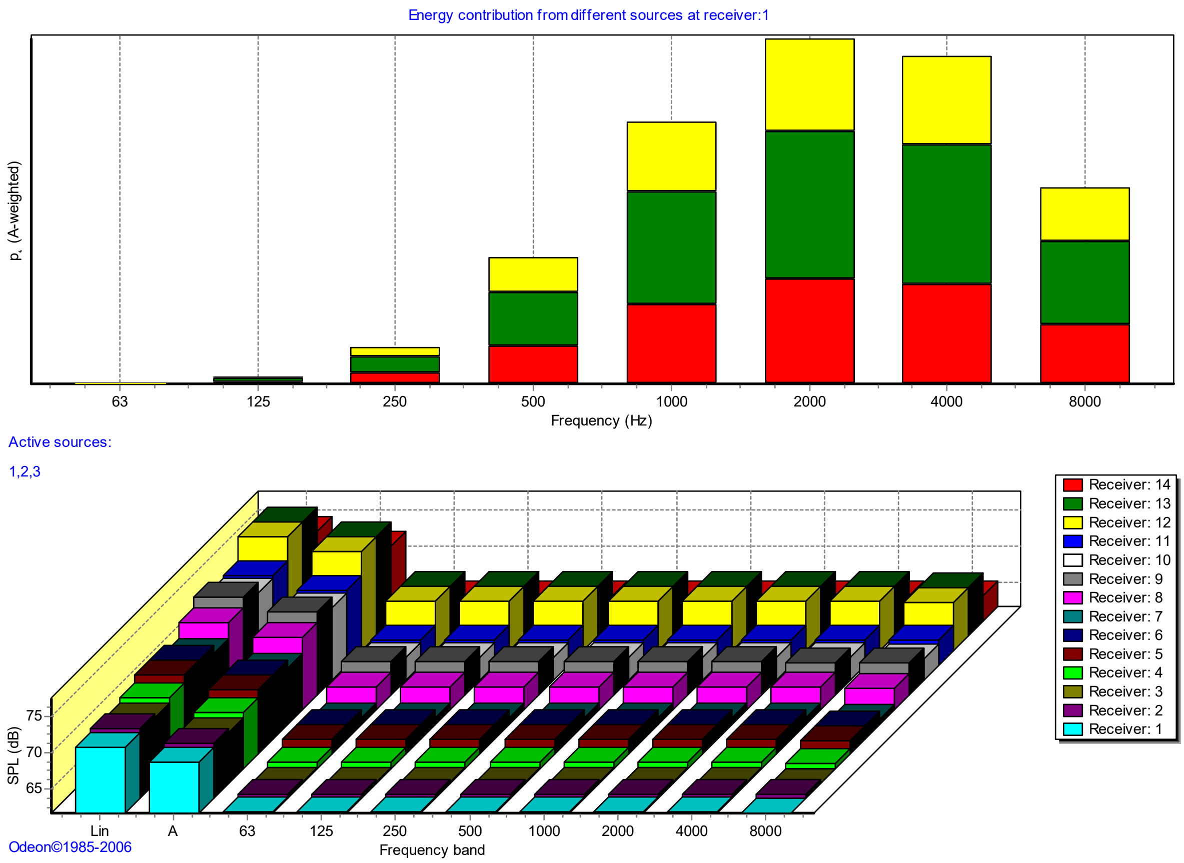

Fig. 4. The share of source energy in point 1 and sound level distribution at reception points for octave frequency bands

Conclusion

1. The proposed noise exposure matrix model can be used to evaluate the level of acoustic powers of the sources on the basis of the acoustic parameter values at the reception points. Another important element of the model is the coefficient matrix [A] of the analyzed room, which refers to the adopted relations: source-receiver and their invariable geometric and acoustic properties. This model was used in the formula to describe geometric methods of sound simulation, which are not sufficient in the case of the analyzed room due to the limited applicability of the method. The results obtained in the model analysis indicate certain discrepancies when acoustic alternatives of several active sources were analyzed.

2. It is proposed that MES/MEB numerical methods should be used in sound simulation when studying the interference of the sources in small rooms. The analysis of the interference of the sources in terms of frequency and effects will make it possible to evaluate a precise impact this phenomenon has on noise threat. For this purpose, it is planned that such analyses should first be performed on the analyzed model of the room (Fig.1) and comparing the results obtained by geometric methods with FEM/BEM.
3. Further research will be connected with creating simulation models of rooms representing real-life systems for which the sound simulation analyses will be verified by means of measurement methods. When the measurement values verify the simulation results a way can be elaborated to evaluate the impact of interference of the sources on noise threat.

PŘEKRAČUJEME HRANICE
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MODELOWANIE OCENY ZAGROŻENIA HAŁASEM W MAŁYCH POMIESZCZENIACH PRZEMYSŁOWYCH

Streszczenie: Badania przeprowadzone w zakresie analizy i oceny hałasu zagrażającego miejscom pracy koncentrują się na identyfikacji źródeł dźwięku. W celu przeprowadzenia oceny zagrożenia hałasem w badanej przestrzeni, istnieje potrzeba zebrania dodatkowych informacji, m.in. o cechach konstrukcyjnych miejsca, jego wyposażeniu, itp. Proces wyboru stosowanych sposobów i metod jest zdeterminowany przez parametry akustyczne źródeł dźwięku, ich charakterystyki i lokalizacje. Informacje niezbędne do modelowania, reprezentowane przez miejsca położenia pracownika i źródeł oraz cechy materiałowe barier akustycznych tworzą zestaw parametrów niezbędnych do doboru metod redukcji poziomu hałasu. Ze względu na różne zastosowania metod modelowania pól akustycznych stwierdzono, że każda z nich ma ograniczone zastosowanie w badaniach ze względu na ukształtowanie pomieszczenia i cechy pola akustycznego. Głównym uproszczeniem jest założenie stabilności parametrów akustycznych polu akustycznego w czasie oraz pominięcie zjawisk falowych. Niniejszy artykuł opisuje sposób identyfikacji źródeł dźwięku jak emiterów energii akustycznej w miejscach pracy. W proponowanym podejściu źródła są traktowane jako punkty emitujące energię akustyczną.

Słowa kluczowe: zagrożenie hałasem, model, źródło dźwięku, ocena hałasu.

ACE-R TESTS RESULTS AFTER CAS

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Abstract: The objective of this paper was to compare the results in ACE-R tests in the group of 32 patients of the Department of Neurology in Vítkovice Hospital, Ostrava, Czech Republic, who underwent carotid artery stenting (CAS) there from 2012 to 2015. Carotid artery stenting is a surgical treatment of the carotid artery (the artery that supplies the brain). A plaque can built up in the artery wall (this process is called atherosclerosis) and cause a narrowing (stenosis) of the artery. Pieces of plaque can break off, move to the brain and cause a stroke - one of the most serious diseases of present time (the third leading cause of death in industrialized countries and the major cause of functional impairment). Carotid artery stenting is an endovascular surgery where a stent (a tube-like metalic mesh) is deployed within the lumen of the affected carotid artery to dilate it and prevent a stroke. The effect of CAS on patients' cognitive functions was examined in various studies with different results. ACE-R test was used to asses the quality of cognitive functions of the patients in our study and significantly better results were found after CAS. Statistical analysis was carried out with the programs SPSS (Chicago, IL, USA) and Microsoft Excel (Redmont, WA, USA). A paired sample *t*-test was used and the value of $p = 0.05$ was taken as a level of significance. Bland and Altman plot was constructed to confirm our result and provide a graphical display of the data agreement.

Keywords: carotid artery stenting, ACE-R test, paired sample *t*-test, Bland and Altman plot

1 Introduction

1.1 Carotid artery stenting

Cerebrovascular accident (also ictus, brain attack or stroke) belongs to the most serious diseases of present time. It is the third leading cause of death in industrialized countries and the major cause of functional impairment [1]. It is a suddenly growing brain damage caused by a poor blood flow to the brain cells. This fault can occur either on the basis of cerebral artery occlusion (ie. ischemic stroke), or on the basis of cerebral vessels bleeding (ie. hemorrhagic stroke). It is an important diagnostic task to distinguish between ischemic and hemorrhagic stroke as the therapeutic approach is different in both cases and its inappropriate choice could worsen the patient's health state. Signs and symptoms of a stroke may include an inability to move or feel on one side of the body, problems with understanding and speaking, dizziness or impaired vision and hearing. Hemorrhagic strokes may also be associated with a severe headache. The main risk factor for stroke is high blood pressure, other risk factors include tobacco smoking, obesity, high blood cholesterol, diabetes mellitus and atrial fibrillation [2].

In the Czech Republic, stroke is the second leading cause of death and the leading cause of functional impairment and ischemic strokes (80%) predominate over hemorrhagic (20%) [3]. Annual incidence of ischemic stroke ranges from 250 to 300 per 100,000 persons [3] and annual mortality from 70 to 80 per 100,000 persons and the incidence increases with the age [4]. Ischemic stroke is very frequently caused by carotid artery stenosis. The carotid artery is an artery that supplies the brain. A plaque is often built up here (atherosclerosis) and causes a narrowing (stenosis) of the artery (see Fig. 1). Pieces of plaque can break off and block the blood flow in arteries, which leads to a stroke.

Fig.1 Arteries affected by atherosclerosis

There are three methods used in the treatment of carotid artery stenosis - conservative therapy, carotid endarterectomy (CEA) and carotid artery stenting (CAS). The classical conservative therapy is the primary step that is based on identifying and eliminating risk factors of atherosclerosis and prescribing special drugs - antiaggregants. Carotid endarterectomy is a classic surgical method consisting in surgical removal of the atherosclerotic plaque that narrows the artery. Carotid artery stenting is an endovascular surgery where a stent (a tube-like metalic mesh, see Fig. 2) is deployed within the lumen of the affected carotid artery to dilate (see Fig. 3) it and prevent a stroke. This minimally invasive method is often used to treat high-risk patients, when carotid endarterectomy is considered too risky.

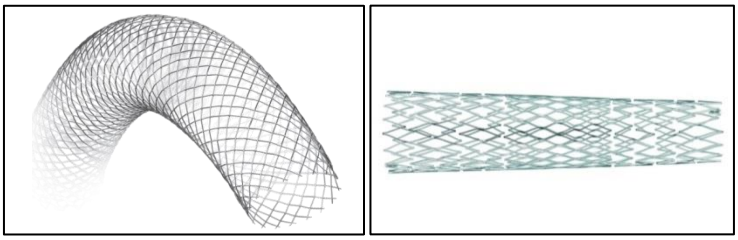

Fig.2 Two types of stents

Fig.3 Perioperative DSA angiography showing tight stenosis and Control DSA angiography after stent implantation

1.2 ACE-R test

ACE-R test belongs to the group of cognitive tests, which provide assessments of the cognitive capabilities of humans. The Addenbrooke's Cognitive Examination (ACE) [5] and its subsequent iteration, ACE-R [6], are easy to use, acceptable to patients, and have shown excellent diagnostic utility in identifying dementia and cognitive impairment in a variety of clinical situations (Alzheimer's disease, frontotemporal lobar degenerations, Parkinsonian syndromes, stroke and vascular dementia, and brain injury). The ACE-R test is used worldwide and is available in a number of languages. The ACE-R takes about 15 min to administer and score in a clinical setting. It contains 5 sub-scores, each one representing one cognitive domain: attention/orientation (18 points), memory (26 points), fluency (14 points), language (26 points) and visuospatial (16 points). ACE-R maximum score is 100, composed by the addition of the all domains, a higher score denotes better cognitive function [6].

2 Methodology

2.1 Aim of the study

The aim of our study was to determine the impact of carotid artery stenting on patients' cognitive functions.

2.2 Study design

The examined group of 32 patients were indicated to CAS by a committee consisting of an interventional radiologist, a vascular surgeon and a neurologist in Vítkovice Hospital, Ostrava from 2012 to 2015. The indication criteria for CAS followed the current recommendations of the American Stroke Association. Addenbrooke's Cognitive Examination Revised (ACE-R) was used to test the level of the patients' cognitive functions. The patients underwent the ACE-R testing immediately before CAS and one month after the surgery. The test was performed by a clinical speech therapist with long-term experience in cognitive functions testing. The original tables of the ACE-R test by Mioshi et al [6] were used as a template for the testing.

2.3 Statistical analysis

Statistical analysis was carried out with the programs SPSS (Chicago, IL, USA) and Microsoft Excel (Redmont, WA, USA). A paired sample *t*-test was used and the value of $p = 0.05$ was taken as a level of significance. Bland and Altman plot was constructed to confirm our result and provide a graphical display of the data agreement.

3 Results

The paired sample *t*-test is a statistical procedure used to determine whether the mean difference between two sets of observations is zero. In a paired sample *t*-test, each subject or entity is measured twice, resulting in pairs of observations. The main assumption of this test is, that the differences between the paired values should be (approximately) normally distributed.

In our case the ACE-R score was measured immediately before CAS (ACE-R_1 variable) and one month after CAS (ACE-R_2 variable) at each of 32 patients and the differences in these two scores were computed for each patient. As the differences meet the assumption of normality (Kolmogorov-Smirnov test, $p = 0.816$), a paired sample *t*-test was performed and the following results were obtained:

Tab.1 Paired Samples Test

	ACE-R_1 (Average ± SD)	ACE-R_2 (Average ± SD)	ACE-R_2 - ACE-R_1 (Average ± SD)	*p* - value
ACE-R score	87.8 ± 7.1	91.6 ± 5.4	3.8 ± 3.9	< 0.001

Table 1 summarizes the conclusion of one-sided paired samples test. As the calculated *p*-value (< 0.001) is much smaller than the stated level of significance (0.05), we can consider the patients' ACE-R score results after CAS significantly higher than the score results before the surgery.

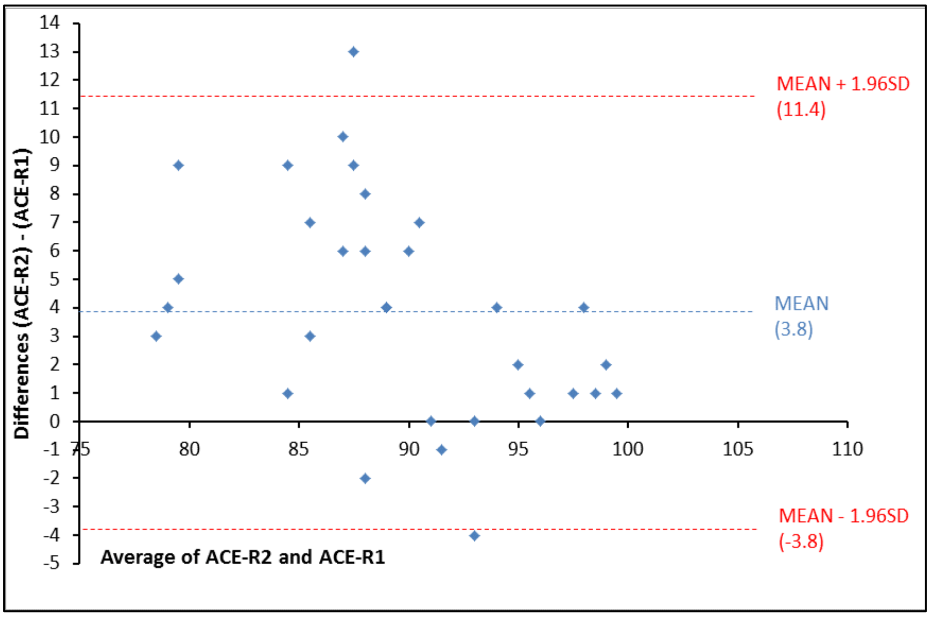

Fig.4 Bland and Altman plot

Our analysis was supplemented by Bland and Altman plot (Fig. 4), which was constructed to provide a graphical display of the difference between the two data sets and confirm our result.

Bland and Altman plot is the plot of the differences between the paired values against their mean. It is often accompanied by the 95% limits of agreement (average difference ± 1.96 standard deviation of the difference), represented by two red horizontal lines in Fig. 4, which tells us how far apart the observations were more likely to be for most individuals.

Fig. 4 displays considerable lack of agreement between the values of the ACE-R scores before and after CAS. Almost all the points tend to be above zero, so the difference ACE-R_2 - ACE-R_1 is positive in most cases. It corresponds with the conclusion of *t*-test that the patients' ACE-R score results are higher after CAS than before it.

Conclusion

Thirty-two patients of the Department of Neurology in Vítkovice Hospital, Ostrava, Czech Republic, who underwent carotid artery stenting from 2012 to 2015, underwent the ACE-R testing of their cognitive functions. Recent studies focused on the effect of CAS on patients' cognitive functions differ in their results. Our study found significantly better ACE-R score results at the patients after CAS. The improvements in cognitive performance in most patients can be attributed to the improvements in cerebral perfusion after CAS.

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VÝSLEDKY ACE-R TESTŮ PO KAS

Abstrakt: Cílem této studie bylo porovnat výsledky ACE-R testů u skupiny 32 pacientů Neurologického oddělení Vítkovické nemocnice v Ostravě, kteří zde v letech 2012 - 2015 podstoupili karotický stenting (KAS). Karotický stenting je chirurgická léčba karotické tepny (tepny zásobující mozek). Na vnitřní stěně této tepny se může usazovat plak (tento proces se nazývá ateroskleróza), což způsobuje zúžení (stenózu) tepny. Kousky plaku se mohou odlomit a krevním řečištěm doputovat k mozku, kde mohou zapříčinit mozkovou mrtvici - jedno z nejzávažnějších onemocnění dnešní doby (ve vyspělých zemích je to třetí nejčastější příčina úmrtí a nejčastější příčina invalidity). Karotický stenting je endovaskulární výkon, při kterém se do postižené cévy zavede stent (kovová síťka tvaru trubičky), jehož úkolem je cévu rozšířit a udržet její průchodnost. Vliv KAS na kognitivní funkce pacientů byl zkoumán v četných studiích s různými výsledky. V naší studii byly kognitivní funkce pacientů posuzovány pomocí ACE-R testu a statisticky významně lepší výsledky v těchto testech byly u pacientů shledány po provedeném KAS. K statistické analýze byl použit párový *t*-test (s hladinou významnosti p = 0,05), Bland Altmanův graf byl sestrojen k následné vizualizaci rozdílu mezi porovnávanými hodnotami a k potvrzení našeho výsledku. Výpočty byly provedeny pomocí programů SPSS (Chicago, IL, USA) a Microsoft Excel (Redmont, WA, USA).

Klíčová slova: karotický stenting, ACE-R test, párový *t*-test, Bland Altmanův graf

SELECTED QUALITY PROBLEMS OF MECHANIZED ENCLOSURES

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Abstract: The article presents selected operating problems of mechanical enclosure components. Attention was drawn to the causes of the most common inconsistencies and the suggested range of potential for their elimination.

Keywords: housing, wear, layout, corrosion

1 Introduction

The Investigation of mechanized enclosures for both failure and failure and functionality is a multi-step process that begins with the single-element testing phase and ends with the entire section [1]. These tests are analytical-measuring. They are carried out before the housing is allowed to work, after any modernization and after a failure to find the cause.

The scope of basic research that is carried out in the phase before the new housing is commissioned for operation or after its modernization or general renovation is included in the construction requirements [3].

The research cycle begins with the simplest of actions: product identification, measurement of geometric features in the next order is carried out tests: material, fatigue, overload, functionality, stability and galvanic coatings. Testing and testing of all hydraulic components or finished sections is intended to reflect the natural working conditions of these components and the emergency situations in which they may be exposed. With the use of the latest technology, computerized measurement systems equipped with specialized software are measured and analyze the data received by comparing them with the requirements that must be fulfilled before allowing for further use.

Due to continuous improvement in the process of modernization, during the repair of the section, a test of the worn parts is carried out and tests of the hydraulic and control hydraulics have been carried out for some time. The tests are designed to check the component's integrity and visual assessment.

In order to carry out the examination of individual components, it is necessary to analyze the discrepancies in the individual cycles, from the beginning to the end of the product life cycle [2]. Due to the complexity of the whole system, the focus was on selected components..

2 Discrepancies that occurred at the manufacturing stage of the casing components

These are defects, defects of individual components, parts or fragments of the structure that arise in the subsequent stages of the production process or after its completion, before they become part of the entire enclosure and begin to function.

Most often they occur during machining, when turning, milling, drilling are also the result of malfunction due to poor weld joints or lack of care during transport.

These discrepancies are a problem that occurs directly during the production cycle, so they are not usually the subject of a complaint because they are identified and are eliminated on a regular basis by people involved in the production process.

2.1 Damage caused by machining - uneven surface

The most common damage caused by machining is an uneven, undulating surface (Figure 1), which, consequently, does not correspond to the exact dimensions imposed by the constructor and can lead to damage to other components working against the surface such as seals.

The causes of such a phenomenon can be:

- improper cutting parameters for machined material, inadequate feedrate for rotation and rotation relative to material diameter and hardness,
- inadequate type of tool or knife plate,
- the spindle of the machine or bearings supporting the workpiece.

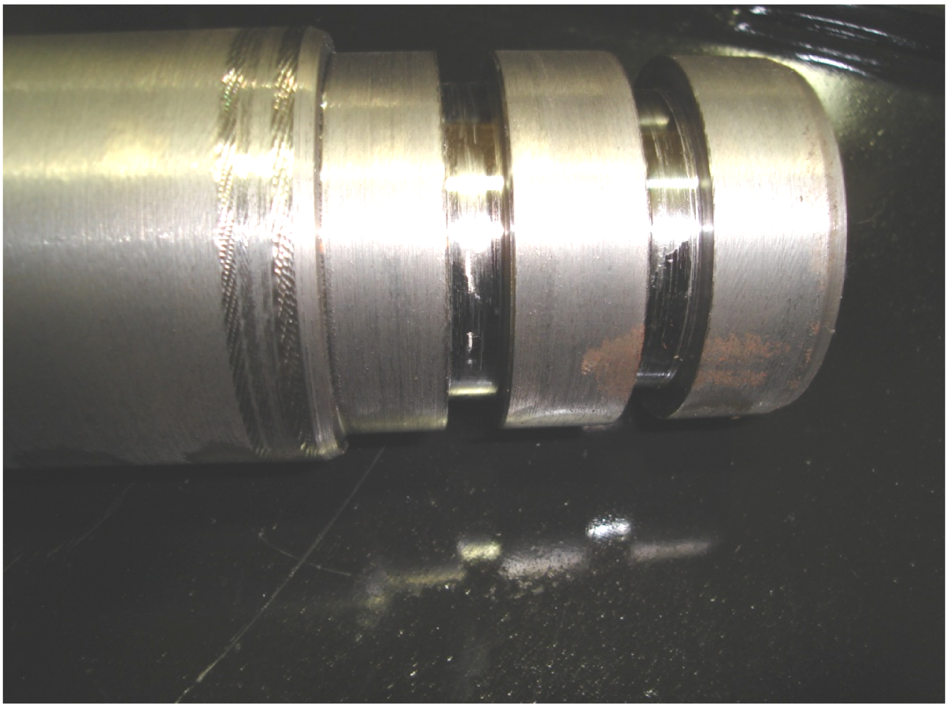

Fig.1 Mechanical damage (piston rods) due to vibrations

The cause of the faults is different, so a thorough analysis is needed to give a true cause and indicate whether the fault lies with the manufacturer, the repairer or the user.

In addition to the illustrated example, fragments of the structure are damaged and steering hydraulics or hydraulic hydraulics are damaged too.

The research and analysis have shown that the most common cause of failure is damage to the steering hydraulics followed by damage to the hydraulic hydraulics seals.

2.2 Damage to the steering hydraulics

The following Table 1 shows the cause of the damage.

Table 1. List of causes of damage to hydraulic control components

	Four-way manifolds	Valve block	Shut off valves	Throttle valves
Corrosion	1890	1410	270	175
No leakage	1871	857	202	80
Loss of funkcionality	1428	705	74	30
Mechanical damage	626	170	119	130

Source: [4]

The data in the table shows that the most common damages are:

- four-way manifolds,
- valve blocks,
- shut off valves,
- throttle valves.

The reason for these defects are:

- corrosion,
- no leakage,
- loss of functionality,
- mechanical damage.

Figure 2 shows the cause of damage to the hydraulic control.

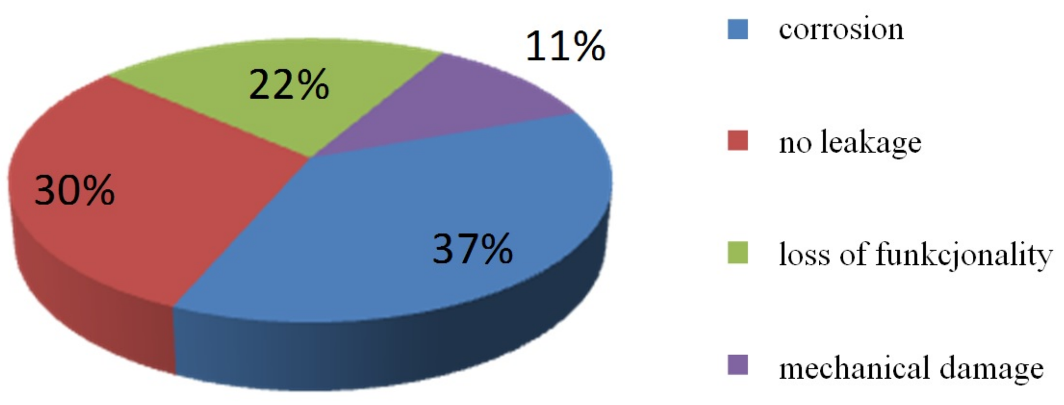

Fig. 2 Main causes of damage to the hydraulic control components

The Corrosion

- inadequate grade of steel;

most of the hydraulic control elements are made of normal low-alloy construction steel (S355, 45). Only the working elements determining the reliability of the system are made of stainless steel: martensitic 2H13 and 4H13 and austenitic 1H18N9 [5].

- improper material handling;

as a result of improper selection of stainless steel material in the grinding or polishing process, free carbon is introduced, which, when combined with chromium, forms chromium-reduced grains, if the chromium content falls below 12% then the corrosion process begins [6,7].

- nadequate proportions of working fluid components;

the working fluid is a mixture of:
95% water of adequate hardness,
4.25% of the base oil of the refined petroleum product,
0.75% emulsifier, substance reducing the surface tension between oil and water [8].
The oil has an additional preservative function and if it is too low relative to water or the water is too hard then the corrosion of the components begins.

No leakage

- accumulation of impurities in the working fluid
- inadequate proportions of working fluid components;

when there is more oil in the working fluid compared to the other components of the concentrate, the hardness and volume of the seals changes.

- mechanical damage.

Loss of functionality

- mechanical damage to internal components,
- suspension of springs,
- clogging of internal flow channels.

Mechanical damage

- the impact of external forces;

as a result of falling dirt, crushes, breakages, crushes, particularly exposed to elements not permanently attached to the structure, such as throttle valves return 31% damaged in this wayseals.

2.3 Damage to the hydraulic hydraulics

Hydraulic components belong to the basic and essential equipment of mechanized housings, they correspond to the support and safety [9]. Their proper functioning and failure is a priority.
Hydraulic hydraulics include:

- hydraulic stands,
- support,
- auxiliary cylinders.

All these elements are known under the common name hydraulic cylinders, they combine common similarity in terms of construction and principle of operation.

It is the seals that are responsible for the proper operation of the actuator and their damage causes dangerous external leaks or internal leaks of the working fluid resulting in rapid changes in the pressure in the cylinder leading to loss of stability of the hydraulic stand or support.

The main causes of damage to the seals of the actuators are damage caused by external forces or mechanical damage and caused by harmful environmental factors such as dust and sand.

Mechanical damage

- damage to the chrome protective layer;

it comes to it due to improper protection of elements during transport, they are mainly scratches and dents, during the operation comes to the splinters in piston rods appear spallions, corrosion process begins. All resulting in inequalities act on the surface of the seal.

- damage to the actuator components;

piston rod deflection, the gland causes incorrect deformation of the piston seals and gland when the actuator is operating.

Damage caused by environmental factors

- damage caused by sand and dust;

due to the long-term impact of sand particles and dust, scratches on the parts of the actuator operating on the outside, such as the piston rod on the chrome protective layer, appear scratches,

- dirty hydraulic fluid;

there is damage to the inner surface of the cylinder and abrasion of the seals piston.

As in the case of hydraulic steering components, preventive actions are taken in the form of sealing damage analysis among randomly selected actuators. The purpose of the analysis is to identify the defects and to find the cause of them while showing the scale of the phenomenon. The cognitive function is to take preventive actions reducing the number of defects, which will significantly affect the number of complaints.

The results of the analysis are presented in Table.2. The test consisted in a leakproof test provided for the repair of the actuators and then a visual inspection of the already dismantled parts.

Table 2 Results of identifying sealing defects and probable causes their formation

Type of damage	Quantity	Mechanical damage		Soiled hydraulic fluid (internal refrigerant)	Impact of sand, dust (external factor)
		Damage to the finishing (failure of chrome layer continuity)	Damage to the actuator components (piston curl, etc.)		
Damage to the scraper ring	82	20	4	0	58
Damage to the guide ring	45	0	35	5	5
Damage to packing gland	30	14	0	8	8
Damage to the piston seal	25	16	0	9	0
Damage of static seal of two fixed surfaces	12	0	12	0	0
SUM	**194**	**50**	**51**	**22**	**71**

Conclusion

1. The main cause of faulty housing is corrosion. The highest occurrence of this phenomenon is in the case of hydraulic steering components, about 37% of the cases. In order to reduce the scale of the phenomenon, stainless steel materials should be used, increased control of the quality and proportion of the individual components of the working fluid in the hydraulic and power hydraulics, and the instructions for proper assembly. Damage to the guide ring is caused by mechanical damage to the actuator components, that is the piston rod.

2. Gland and piston seal defects are caused mainly by defects in the protective layer caused by scratches or corrosion. In the case of stuffing the piston is damaged and in the case of piston damaged cylinder.

3. Damage to the static seal in the case of gland is 100% due to mechanical damage to the gland.

Important - practical application

The results of the study can be used in the heavy machinery and mining industries. This will limit the development of quality defects in mechanized housing subassemblies. As indicated in the article of cause their formation can be very varied. This is due, among other things. from the heavy working conditions of the mechanized teams.

PŘEKRAČUJEME HRANICE
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2014—2020

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WYBRANE PROBLEMY JAKOŚCI OBUDÓW ZMECHANIZOWANYCH

Streszczenie: Artykuł prezentuje wybrane problemy eksploatacyjne elementów obudów mechanicznych. Zwrócono uwagę na przyczyny najczęstszych niezgodności oraz sugerowany zakres potencjalnych możliwości ich eliminacji.

Słowa kluczowe: obudowa, zużycie, układ, korozja

Hamiltonian system in dimension 4

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Abstract: The aim of this paper is announce some recent results concerning the Hamiltonian theory. The case of first order Hamiltonian systems related to an affine second order Euler–Lagrange form is studied. In dimension 4 the structure of Hamiltonian systems (i.e., Lepagean equivalent of an Euler–Lagrange form) is found.

Keywords: Euler–Lagrange form, Lepagean equivalent of Euler–Lagrange form, Hamiltonian system.

1 Introduction

Throughout the paper all manifolds and mappings are smooth and summation convention is used. We consider a fibered manifold (i.e., surjective submersion) $\pi : Y \to X$, dim $X = n$, dim $Y = n + m$, its r-jet prolongation $\pi_r : J^rY \to X$, $r \geq 1$ and canonical jet projections $\pi_{r,k} : J^rY \to J^kY$, $0 \leq k \leq r$ (with an obvious notations $J^0Y = Y$). A fibered chart on Y (resp. associated fibered chart on J^rY) is denoted by (V, ψ), $\psi = (x^i, y^\sigma)$ (resp. (V_r, ψ_r), $\psi_r = (x^i, y^\sigma, y^\sigma_i, \ldots, y^\sigma_{i_1 \ldots i_r})$).

A vector field ξ on J^rY is called π_r-*vertical* if it projects onto the zero vector field on X. A q-form η on J^rY is called π_r-*horizontal* if $i_\xi \eta = 0$ for every π_r-vertical vector field ξ on J^rY.

The fibered structure of Y induces a morphism h, of exterior algebras, defined by the condition $J^r\gamma^*\eta = J^{r+1}\gamma^* h\eta$ for every section γ of π, and called *horizontalization*. Apparently, horizontalization is an $\mathbb{R}$-linear wedge product preserving mapping such that applied to a function f and to the elements of the canonical basis of 1-forms $(dx^i, dy^\sigma, dy^\sigma_i, \ldots, dy^\sigma_{i_1 \ldots i_r})$ on J^rY gives

$$hf = f \circ \pi_{r+1,r},\ hdx^i = dx^i,\ hdy^\sigma = y^\sigma_l dx^l, \ldots, hdy^\sigma_{i_1 \ldots i_r} = y^\sigma_{i_1 \ldots i_r l} dx^l.$$

A q-form η on J^rY is called *contact* if $h\eta = 0$. A contact q-form η on J^rY is called 1-*contact* if for every π_r-vertical vector field ξ on J^rY the $(q-1)$-form $i_\xi \eta$ is horizontal. A contact q-form η on J^rY is called i-*contact* if for every π_r-vertical vector field ξ on J^rY the $(q-1)$-form $i_\xi \eta$ is (i-1)-contact.

Recall that every q-form η on J^rY admits a unique (canonical) decomposition into a sum of q-forms on $J^{r+1}Y$ as follows:

$$\pi^*_{r+1,r}\eta = h\eta + \sum_{k=1}^{q} p_k\eta,$$

where $h\eta$ is a horizontal form, called the *horizontal part of* η, and $p_k\eta$, $1 \leq k \leq q$, is a *k-contact part of* η (see [3]).

We use the following notations:

$$\omega_0 = dx^1 \wedge dx^2 \wedge \cdots \wedge dx^n,\ \omega_i = i_{\partial/\partial x^i}\omega_0,\ \omega_{ij} = i_{\partial/\partial x^j}\omega_i,$$

and

$$\omega^\sigma = dy^\sigma - y^\sigma_j dx^j,\ \ldots\ ,\ \omega^\sigma_{i_1i_2\ldots i_k} = dy^\sigma_{i_1i_2\ldots i_k} - y^\sigma_{i_1i_2\ldots i_kj}dx^j$$

For more details on fibered manifolds and the corresponding geometric stuctures we refer e.g. to [6].

In this section we briefly recall basic concepts on Lepagean equivalents of Euler–Lagrange forms and generalized Hamiltonian field theory, due to Krupková [4].

By an *r-th order Lagrangian* we shal mean a horizontal n-form λ on J^rY.

A *closed* $(n+1)$-form α is called a *Lepagean equivalent of an Euler–Lagrange form* $E = E_\sigma\omega^\sigma \wedge \omega_0$ if $p_1\alpha = E$.

Recall that the Euler–Lagrange form corresponding to an r-th order $\lambda = L\omega_0$ is the following $(n+1)$-form of order $\leq 2r$

$$E = \left(\frac{\partial L}{\partial y^\sigma} + \sum_{l=1}^{r}(-1)^l d_{p_1}d_{p_2}\ldots d_{p_l}\frac{\partial L}{\partial y^\sigma_{p_1\ldots p_l}}\right)\omega^\sigma \wedge \omega_0.$$

The family of Lepagean equivalents of E is also called a *Lagrangian system*, and denoted by $[\alpha]$. A (single) Lepagean equivalent α of E on J^sY is also called a *Hamiltonian system of order s.*

2 Hamiltonian Systems.

We shall consider dim $X = 4$ and a second order Euler–Lagrange form $E = E_\nu\ \omega^\nu \wedge \omega_0$ which coefficients E_ν are affine in the second derivatives, i.e.,

$$E_\nu = A_\nu + B^{kl}_{\nu\sigma}y^\sigma_{kl}, \tag{1}$$

where A_ν and $B^{kl}_{\nu\sigma}$ do not depend on second derivatives.

In what follows, we shall study first order Hamiltonian systems (i.e., $s = 1$) corresponding to a Lepagean equivalents of such Euler–Lagrange form. In dimension 4 the 1st order Hamiltonian systems admit a decomposition

$$\pi^*_{2,1}\alpha = p_1\alpha + p_2\alpha + p_3\alpha + p_4\alpha + p_5\alpha,$$

Keeping notations introduced so far, we write

$$
\begin{aligned}
\alpha &= E_\sigma \, \omega^\sigma \wedge \omega_0 + F^i_{\sigma\nu} \, \omega^\sigma \wedge \omega^\nu \wedge \omega_i + F^{ij}_{\sigma\nu} \, \omega^\sigma \wedge \omega^\nu_i \wedge \omega_j \\
&+ F^{ijk}_{\sigma\nu} \, \omega^\sigma_i \wedge \omega^\nu_j \wedge \omega_k + G^{ij}_{\sigma\nu\kappa} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega_{ij} \\
&+ G^{ijk}_{\sigma\nu\kappa} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa_i \wedge \omega_{jk} + G^{ijkl}_{\sigma\nu\kappa} \, \omega^\sigma \wedge \omega^\nu_i \wedge \omega^\kappa_j \wedge \omega_{kl} \\
&+ G^{ijklm}_{\sigma\nu\kappa} \, \omega^\sigma_i \wedge \omega^\nu_j \wedge \omega^\kappa_k \wedge \omega_{lm} + K^{ijk}_{\sigma\nu\kappa\beta} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega_{ijk} \\
&+ K^{ijkl}_{\sigma\nu\kappa\beta} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta_i \wedge \omega_{jkl} \\
&+ K^{ijklm}_{\sigma\nu\kappa\beta} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa_i \wedge \omega^\beta_j \wedge \omega_{jkl} \\
&+ K^{ijklmn}_{\sigma\nu\kappa\beta} \, \omega^\sigma \wedge \omega^\nu_i \wedge \omega^\kappa_j \wedge \omega^\beta_k \wedge \omega_{lmn} \\
&+ K^{ijklmno}_{\sigma\nu\kappa\beta} \, \omega^\sigma_i \wedge \omega^\nu_j \wedge \omega^\kappa_k \wedge \omega^\beta_l \wedge \omega_{mno} \\
&+ M^{ijkl}_{\sigma\nu\kappa\beta\gamma} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma \wedge \omega_{ijkl} \\
&+ M^{ijklm}_{\sigma\nu\kappa\beta\gamma} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma_i \wedge \omega_{jklm} \\
&+ M^{ijklmn}_{\sigma\nu\kappa\beta\gamma} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta_i \wedge \omega^\gamma_j \wedge \omega_{klmn} \\
&+ M^{ijklmno}_{\sigma\nu\kappa\beta\gamma} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa_i \wedge \omega^\beta_j \wedge \omega^\gamma_k \wedge \omega_{lmno} \\
&+ M^{ijklmnop}_{\sigma\nu\kappa\beta\gamma} \, \omega^\sigma \wedge \omega^\nu_i \wedge \omega^\kappa_j \wedge \omega^\beta_k \wedge \omega^\gamma_l \wedge \omega_{mnop} \\
&+ M^{ijklmnopq}_{\sigma\nu\kappa\beta\gamma} \, \omega^\sigma_i \wedge \omega^\nu_j \wedge \omega^\kappa_k \wedge \omega^\beta_l \wedge \omega^\gamma_m \wedge \omega_{nopq},
\end{aligned} \tag{2}
$$

Lemma 2.1. *Let* dim $X = 4$. *Let $E = E_\nu \omega^\nu \wedge \omega_0$ be a second order Euler–Lagrange form with coefficients E_ν satisfying (1), and let α be a Hamiltonian system of the form (2). Then the functions $F^{ijk}_{\sigma\nu}$, $G^{ijkl}_{\sigma\nu}$, $G^{ijklm}_{\sigma\nu}$, $K^{ijklm}_{\sigma\nu}$, $K^{ijklmn}_{\sigma\nu}$, $K^{ijklmno}_{\sigma\nu}$, $M^{ijklmn}_{\sigma\nu}$, $M^{ijklmno}_{\sigma\nu}$, $M^{ijklmnop}_{\sigma\nu}$, $M^{ijklmnopq}_{\sigma\nu}$ do not depend on E_σ.*

Proof. Proof of the lemma follows from the explicit computation of $d\alpha = 0$. □

One can see from the above lemma that the functions $F^{ijk}_{\sigma\nu}$, $G^{ijkl}_{\sigma\nu}$, $G^{ijklm}_{\sigma\nu}$, $K^{ijklm}_{\sigma\nu}$, $K^{ijklmn}_{\sigma\nu}$, $K^{ijklmno}_{\sigma\nu}$, $M^{ijklmn}_{\sigma\nu}$, $M^{ijklmno}_{\sigma\nu}$, $M^{ijklmnop}_{\sigma\nu}$, $M^{ijklmnopq}_{\sigma\nu}$ do not depend on E_σ (cf. [4]). The invariant choice is

$$
\begin{aligned}
&F^{ijk}_{\sigma\nu} = G^{ijkl}_{\sigma\nu} = G^{ijklm}_{\sigma\nu} = K^{ijklm}_{\sigma\nu} = K^{ijklmn}_{\sigma\nu} = K^{ijklmno}_{\sigma\nu} = 0, \\
&M^{ijklmn}_{\sigma\nu} = M^{ijklmno}_{\sigma\nu} = M^{ijklmnop}_{\sigma\nu} = M^{ijklmnopq}_{\sigma\nu} = 0,
\end{aligned}
$$

and we obtain new Hamiltonian system of the form

$$
\begin{aligned}
\bar{\alpha} &= E_\sigma \, \omega^\sigma \wedge \omega_0 + F^i_{\sigma\nu} \, \omega^\sigma \wedge \omega^\nu \wedge \omega_i + F^{ij}_{\sigma\nu} \, \omega^\sigma \wedge \omega^\nu_i \wedge \omega_j \\
&+ G^{ij}_{\sigma\nu\kappa} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega_{ij} + G^{ijk}_{\sigma\nu\kappa} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa_i \wedge \omega_{jk} \\
&+ K^{ijk}_{\sigma\nu\kappa\beta} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega_{ijk} + K^{ijkl}_{\sigma\nu\kappa\beta} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta_i \wedge \omega_{jkl} \\
&+ M^{ijkl}_{\sigma\nu\kappa\beta\gamma} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma \wedge \omega_{ijkl} \\
&+ M^{ijklm}_{\sigma\nu\kappa\beta\gamma} \, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma_i \wedge \omega_{jklm},
\end{aligned} \tag{3}
$$

where $F^i_{\sigma\nu}$ are skew-symmetric in the indices $\sigma\nu$, $G^{ij}_{\sigma\nu\kappa}$ are skew-symmetric in the indices $\sigma\nu\kappa$ and skew-symmetric in the ij, $G^{ijk}_{\sigma\nu\kappa}$ are skew-symmetric in the indices $\sigma\nu$ and skew-symmetric in the jk, $K^{ijk}_{\sigma\nu\kappa\beta}$ are skew-symmetric in the indices $\sigma\nu\kappa\beta$ and skew-symmetric in the ijk,

$K^{ijkl}_{\sigma\nu\kappa\beta}$ are skew-symmetric in the indices $\sigma\nu\kappa$ and skew-symmetric in the jkl, $M^{ijkl}_{\sigma\nu\kappa\beta\gamma}$ are skew-symmetric in the indices $\sigma\nu\kappa\beta\gamma$ and skew-symmetric in the $ijkl$, $M^{ijklm}_{\sigma\nu\kappa\beta\gamma}$ are skew-symmetric in the indices $\sigma\nu\kappa\beta$ and skew-symmetric in the $jklm$.

Now the functions on the Hamiltonian system depend on coefficients of Euler–Lagrange form. In the following theorem the stucture of the functions on the Hamiltonian system $\bar{\alpha}$ (3) is studied.

Theorem 2.2. *Let* $\dim X = 4$*. Let* $E = E_\nu \omega^\nu \wedge \omega_0$ *be a second order Euler–Lagrange form with coefficients* E_ν *satisfying (1), and let* $\bar{\alpha}$ *be a Hamiltonian system of the form (3). Then the following coeffient conditions are satisfied*

1) $F^{ij}_{\sigma\nu} = \frac{\partial E_\sigma}{\partial y^\nu_{ij}} + f^{ij}_{\sigma\nu}$*, where* $f^{ij}_{\sigma\nu}$ *are arbitrary functions satisfying* $(f^{ij}_{\sigma\nu})_{\mathrm{sym}(ij)} = 0$ *and* $f^{ij}_{\sigma\nu}$ *do not depend on second derivatives.*

2) $F^{i}_{\sigma\nu} = -\frac{1}{2}\left(\frac{\partial E_\nu}{\partial y^\sigma_i} - d_j F^{ij}_{\nu\sigma}\right)_{\mathrm{alt}(\sigma\nu)}$.

3) $G^{ij}_{\sigma\nu\kappa} = \frac{1}{6}\left(\frac{\partial F^{j}_{\nu\kappa}}{\partial y^\sigma_i} - \frac{\partial F^{ij}_{\nu\sigma}}{\partial y^\kappa}\right)_{\mathrm{alt}(\nu\kappa)} + g^{ij}_{\sigma\nu\kappa}$*, where* $(g^{ij}_{\sigma\nu\kappa})_{\mathrm{alt}(\nu\kappa)} = 0$ *and* $g^{ij}_{\sigma\nu\kappa}$ *do not depend on second derivatives.*

4) $G^{ijk}_{\sigma\nu\kappa} = \left(\frac{\partial F^{ij}_{\nu\kappa}}{\partial y^\sigma_k}\right)_{\mathrm{alt}((\sigma k)(\kappa i))} + g^{ijk}_{\sigma\nu\kappa}$*, where* $(g^{ijk}_{\sigma\nu\kappa})_{\mathrm{alt}((\sigma k)(\kappa i))} = 0$ *and* $g^{ijk}_{\sigma\nu\kappa}$ *do not depend on second derivatives.*

5) $K^{ijq}_{\sigma\nu\kappa\beta} = -\frac{1}{12}\left(\frac{\partial G^{ij}_{\nu\kappa\beta}}{\partial y^\sigma_q} - \frac{\partial G^{qij}_{\nu\kappa\sigma}}{\partial y^\beta}\right)_{\mathrm{alt}(\nu\kappa\beta)} + k^{ijq}_{\sigma\nu\kappa\beta}$*, where* $(k^{ijq}_{\sigma\nu\kappa\beta})_{\mathrm{alt}(\nu\kappa\beta)} = 0$ *and* $k^{ijq}_{\sigma\nu\kappa\beta}$ *do not depend on second derivatives.*

6) $K^{ijql}_{\sigma\nu\kappa\beta} = -\frac{1}{9}\left(\frac{\partial G^{ijq}_{\nu\kappa\beta}}{\partial y^\sigma_l}\right)_{\mathrm{alt}(\nu\kappa),\ \mathrm{alt}((\sigma l)(\beta i))} + k^{ijql}_{\sigma\nu\kappa\beta}$*, where the functions* $(k^{ijq}_{\sigma\nu\kappa\beta})_{\mathrm{alt}(\nu\kappa),\ \mathrm{alt}((\sigma l)(\beta i))} = 0$ *and* $k^{ijq}_{\sigma\nu\kappa\beta}$ *do not depend on second derivatives.*

7) $M^{ijkl}_{\sigma\nu\kappa\beta\gamma} = \frac{1}{20}\left(\frac{\partial K^{ijk}_{\kappa\beta\gamma\nu}}{\partial y^\sigma_l} - \frac{\partial K^{lijk}_{\sigma\kappa\beta\gamma}}{\partial y^\nu}\right)_{\mathrm{alt}(\nu\kappa\beta\gamma))} + m^{ijql}_{\sigma\nu\kappa\beta\gamma}$*, where the functions* $(m^{ijq}_{\sigma\nu\kappa\beta\gamma})_{\mathrm{alt}(\nu\kappa\beta\gamma))} = 0$ *and* $m^{ijq}_{\sigma\nu\kappa\beta}$ *do not depend on second derivatives.*

8) $M^{ijklq}_{\sigma\nu\kappa\beta\gamma} = \frac{1}{16}\left(\frac{\partial K^{ijkl}_{\nu\kappa\beta\gamma}}{\partial y^\sigma_q}\right)_{\mathrm{alt}(\nu\kappa\beta),\ \mathrm{alt}((\sigma q)(\gamma i))} + m^{ijklq}_{\sigma\nu\kappa\beta\gamma}$*, where functions* $(k^{ijq}_{\sigma\nu\kappa\beta\gamma})_{\mathrm{alt}(\nu\kappa\beta),\ \mathrm{alt}((\sigma q)(\gamma i))} = 0$ *and* $m^{ijklq}_{\sigma\nu\kappa\beta\gamma}$ *do not depend on second derivatives.*

Where sym() *means the symmetrization in the indicated multiindices and* alt() *means skew-symmetrization in the indicated multiindices.*

Proof. Proof of the theorem follows from facts that the Hamiltonian systems is of first order and from the explicit computation of $d\bar{\alpha} = 0$. □

The Hamiltonian system (3) $\bar{\alpha}$ admits noninvariant decomposition

$$\bar{\alpha} = \alpha_E + \phi \tag{4}$$

where ϕ does not depend on the Euler–Lagrange form and

$$\begin{aligned}
\alpha_E &= E_\sigma\, \omega^\sigma \wedge \omega_0 - \frac{1}{2}\left(\frac{\partial E_\nu}{\partial y^\sigma_i} - d_j \frac{\partial E_\nu}{\partial y^\sigma_{ij}}\right) \omega^\sigma \wedge \omega^\nu \wedge \omega_i \qquad (5)\\
&+ \frac{\partial E_\sigma}{\partial y^\nu_{ij}}\, \omega^\sigma \wedge \omega^\nu_i \wedge \omega_j \\
&- \frac{1}{12}\left(\frac{\partial^2 E_\kappa}{\partial y^\sigma_i \partial y^\nu_j} - d_k \frac{\partial^2 E_\kappa}{\partial y^\sigma_i \partial y^\nu_{jk}}\right) \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega_{ij} \\
&+ \left(\frac{\partial^2 E_\nu}{\partial y^\sigma_k \partial y^\kappa_{ij}}\right)_{\mathrm{alt}((\sigma k)(\kappa i))} \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa_i \wedge \omega_{jk} \\
&+ \frac{1}{144}\left(\frac{\partial^3 E_\beta}{\partial y^\sigma_k \partial y^\nu_i \partial y^\kappa_j} - d_l \frac{\partial^3 E_\beta}{\partial y^\sigma_k \partial y^\nu_i \partial y^\kappa_{jl}}\right) \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega_{ijk} \\
&- \frac{1}{9}\left(\frac{\partial^3 E_\kappa}{\partial y^\sigma_l \partial y^\nu_k \partial y^\beta_{ij}}\right)_{\mathrm{alt}((\sigma l)(\beta i))} \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta_i \wedge \omega_{jkl} \\
&+ \frac{1}{2880}\left(\frac{\partial^4 E_\nu}{\partial y^\sigma_l \partial y^\kappa_k \partial y^\beta_i \partial y^\gamma_j} - d_p \frac{\partial^4 E_\nu}{\partial y^\sigma_l \partial y^\kappa_k \partial y^\beta_i \partial y^\gamma_{jp}}\right) \\
&\quad \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma \wedge \omega_{ijkl} \\
&- \frac{1}{144}\left(\frac{\partial^4 E_\kappa}{\partial y^\sigma_m \partial y^\nu_l \partial y^\beta_k \partial y^\gamma_{ij}}\right)_{\mathrm{alt}((\sigma m)(\gamma i))} \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma_i \wedge \omega_{jklm},
\end{aligned}$$

depends on derivatives of coefficients of the Euler–Lagrange form.

Proposition 2.3. *Let* dim $X = 4$. *Let* $E = E_\nu\, \omega^\nu \wedge \omega_0$ *be a second order Euler–Lagrange form with coefficients* E_ν *satisfying (1), and let* $\bar{\alpha}$ *be a Hamiltonian system (3) admitting the decomposition* $\bar{\alpha} = \alpha_E + \phi$, *then* α_E *is closed.*

Proof. We have $\bar{\alpha} = \alpha_E + \phi$, $d\bar{\alpha} = 0$ and ϕ does not depend upon E. For $E = 0$: $\alpha_E = 0$, yielding $d\phi = 0$. Hence $d\alpha_E = d\bar{\alpha} - d\phi = 0$. This is completes the proof. □

Conclusion

The differential geometry tools are very useful for application to Hamilton (field) theory. The "geometrization" of Euler–Lagrange and Hamilton theory is used e.g. in [1] - [5], [7], [8].

The paper is generalization of classical Hamiltonian field theory on fibred manifold. The regularization procedure and Lepagean equivalent of the first order Lagrangians was proposed by Krupková and Smetanová [5]. The concept of the Lepagean equivalent of the Euler–Lagrange forms was given in [4]. In the paper the case of first order Hamiltonian systems related to an affine second order Euler–Lagrange form is studied. In dimension 4 the structure of Hamiltonian systems (i.e., Lepagean equivalent of an Euler–Lagrange form) is found.

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Hamiltonovy systémy v dimenzi 4

Abstrakt: Článek je věnován Hamiltonovým systémům v dimenzi 4. Je zde studován systém afinní Eulerovy–Lagrangeovy formy druhého řádu. V dimenzi 4 je nalezena struktura Hamiltonových systémů (tj. Lepageových ekvivalentů Eulerovy–Lagrangeovy formy).

Klíčová slova: Eulerova–Lagrangeova forma, Lepageův ekvivalent Eulerovy–Lagrangeovy formy, Hamiltonův systém.

USE OF THE 8D METHOD FOR THE IDENTIFICATION OF QUALITY PROBLEMS IN THE PRODUCTION PROCESS

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Abstract: The article presents the methodology of the 8D method. This method is used to solve quality problems that may occur in the production process or over the course of product manufacturing. The advantage of the 8D method is that a problem can be identified at various stages of the analysis and the end result is a report.

Keywords: method, 8D, problem, quality, process, production, report.

1 Introduction

The 8D method is used to analyse quality problems to improve and streamline a process or product. In this method, problems are identified over the course of its subsequent stages and a possible way of eliminating them is proposed [5, 7, 11]. The methodology of 8D method is organized into eight disciplines and the advantage of using this method in enterprises is the possibility of teamwork and the final report it provides. Within each discipline, a list of current questions is created [1, 3, 4].

The following tools are used in the 8D method to solve the problem: Pareto chart, punched card, Ishikawa diagram, histograms, scatter plots, graphs, control charts [8 ,9, 10, 13]. Depending on the type of problem, appropriate tools are used at each stage of the 8D method [12, 14, 15, 17].

The 8D methodology was originally developed by the US Department of Defence in 1974. This method is generally used in the automotive and aerospace industries [2, 6, 8, 16]. [2, 6, 8, 16].

2 Stages of the 8D method

Analysis using the 8D method is carried out in eight stages enumerated in Tab. 1 [8, 18]. Fig.1 below shows the subsequent steps:

Step 1 – D1

In the first stage, a team is created to solve the problem. The team should consist of several employees who know the problem as well as the production process and product parameters. The team must know the individual tools used in the 8D method. A leader should be selected from the team members to coordinate work. Specific tasks of the leader include: creating a list of the participants, assigning each team member with a task and creating a report after the analysis has been concluded [8].

Step 2 – D2

Step 2 consists in specifying the problem in detail, identifying what is wrong and determining the cause of irregularities. It is important that the report describes the problem that has been presented [8].

Step 3 – D3

In this step, it is necessary to revise measures that have been taken to minimize inappropriate quality operations and introduce preventive measures in order to remove the errors that have arisen. After the measures have been taken, they need to be verified. A schedule of actions needs to be drawn out by defining and describing the most important and effective preventive measures and assessing them, and subsequently implemented. It is important to create a register of actions and to continuously analyse the improvements. The report must describe actions with beginning and conclusion dates [8].

Step4 – D4

In this step, the underlying causes that have resulted in nonconformities and errors should bne determined and verified, then corrective actions must be taken to eliminate the most important causes. This is done in the following way: Corrective action is determined on the basis of pre – established preventive actions, if necessary, the cause of the problem is identified again. At this stage a real list of causes should be established, and the main causes and their percentage of errors and incompatibilities determined on its basis. Finally, describe the selected causes in the report [8].

Step 5 – D5

In stage five, corrective actions should be selected and verified. The most important part of this step is to find corrective measures to eliminate all the causes of irregularities. This step should also assess the probability that the corrective measures will yield results. The most important actions include: developing other corrective actions in case the previous ones do not work, assessing corrective measures, identifying those responsible for carrying out the actions, describing all the actions developed in the report [8].

Step 6 – D6

The most important action in this step is to establish a plan for continuous implementation of the corrective actions laid out in the previous stage. All corrective actions need to be monitored in the long term. Actions to be taken in this phase include: establishing a timetable for the introduction of corrective actions, continuous monitoring of the implemented actions, monitoring the execution of the actions according to the agreed timetable and recording of all actions in the report [8].

Step 7 – D7

Measures implemented in stage 7 are to prevent the problems that have occurred in the past from reoccurring. The most important activities include: analysing monitored corrective actions, developing a list of actions to eliminate all potential causes of problems, making a list of preventive actions, implementing preventive actions, and describing all actions taken in the registry [8].

Step 8 – D8

In step 8, a report of actions taken as part of the previous stages is drawn up and the work of the team must be evaluated. The work of each team member has to be evaluated and the report submitted to the designated person and kept [8].

Table 1. Stages of execution in the 8D method

The problem has been detected by the employees/quality control/the client	**Action within the 8D method**
1D – Work group	A team of 2 –10 people is chosen from the employees of various departments and the leader is selected
2D – Describing the problem	The problem is precisely described
3D – Immediate action	Implementing various measures adequate to the problem
4D – Cause	Determining the real cause of the problem
5D – Corrective action	Determining and implementing corrective measures
6D – Verification of corrective action	Verification of the corrective measures implemented
7D – Preventive action	Implementing preventive measures in order to prevent the problem from reoccurring in the future
8D – Evaluation and report of the 8D	Evaluation of all the measures taken and drawing up of the report

Source: [8]

Figure 1 shows the course of action in the 8D method. The plan does not take into account the tools used depending on the type of problem encountered.

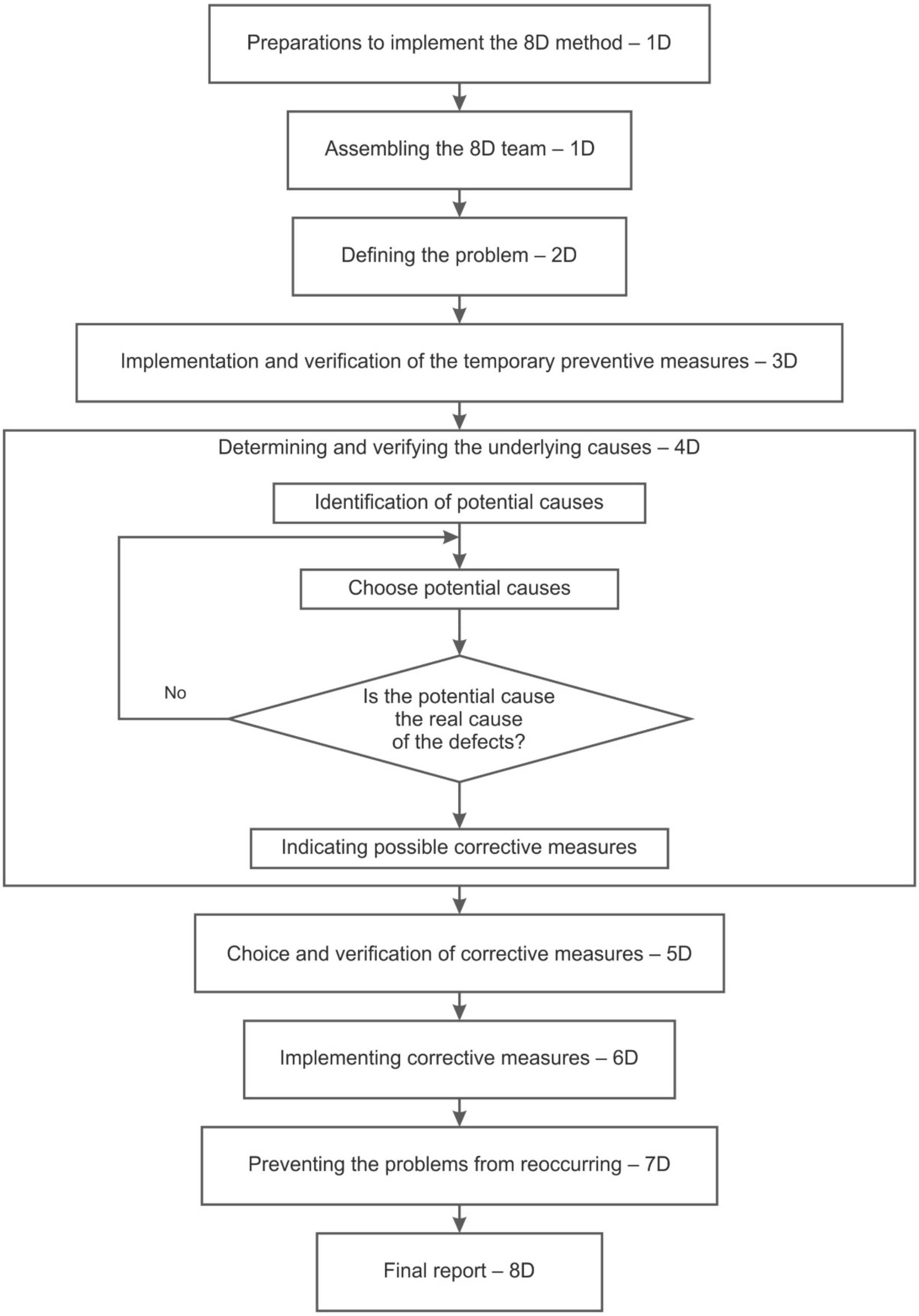

Fig. 1. Workflow chart for the 8D method
Source: [8]

The last stage of the 8D method is drawing up the report. The report contains all of the information discussed during the team meetings, shown in Tab. 2.

Table 2. An example 8D report in basic form

Solving a problem using the 8D method		
Company name		Client/Shipment
Order No.		Supplier/client order No.
Report No. and date		Supplier/client report No.
Part designation		Supplier/client part designation
Date		Date
1D Team assembled: Persons responsible: Contact person:		
2D Description of the problem/noncompliance		
3D Immediate corrective actions Person responsible	Performed from:	Performed until:
4D Description of the cause		
5D Corrective measures Person responsible	Performed from:	Performed until:
6D Verification/confirmation of the corrective measures introduced	Quality manager	Date:
7D Preventive measures Person responsible	Performed from:	Performed until:
8D Verification/confirmation of the preventive measures introduced	Quality manager	Date:
Notes:		
Report closure date:		Quality manager
Report confirmation date:		Company president/director

Source: own elaboration based on [8, 18]

All solutions of problems using the 8D method must be documented in the form of an 8D report (Table 2). Depending on the need the form can be extended with a list of questions. Each stage of the 8D method is documented and most stages contain appendices in the form of charts and pictures. A drawn up and closed 8D report is sent to all the parties concerned 20 days after its closure [8, 18].

Conclusion

Based on the analysed stages of the 8D method it can be stated that it should be used for:

- solving difficult problems connected with complaints,
- detecting noncompliances in a product,
- solving quality problems both inside and outside of an organisation,
- creating the possibility to verify implemented measures.

The advantage of this method is that many people from the company in which the problem has arisen are involved in its execution as well as the great method of reporting noncompliances and corrective measures to suppliers. The method's disadvantage is its labour intensiveness.

Precise implementation of this method results in a large increase in the quality of products or services in most cases. This has a significant impact on the continuity of production, the company being perceived as a reliable partner by its clients and positive financial results [8, 18].

PŘEKRAČUJEME HRANICE
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WYKORZYSTANIE METODY 8D DO IDENTYFIKACJI PROBLEMÓW JAKOŚCIOWYCH W PROCESIE PRODUKCJI

Streszczenie: W artykule przedstawiono metodologię postepowania w metodzie 8D. Metoda ta służy do rozwiązywania problemów jakościowych jakie mogą wystąpić w procesie produkcyjnym, bądź też w trakcie wytwarzania produktu. Zaletą metody 8D jest to, że w poszczególnych etapach analizy można zidentyfikować problem a efektem końcowym analizy jest raport wyników.

Słowa kluczowe: metoda, 8D, problem, jakość, proces, produkcja, raport

MODELING OF PRODUCTION PROCESSES – REVIEW ARTICLE

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Abstract: The purpose of this paper is to present an available methodologies that may be the basis for modeling production processes. In literature there are many affordable methods and notations for modeling processes, but they are still not widely used in companies. Choosing the right methodology and tool for modeling processes is not as easy as it appears. It requires, first and foremost, a thorough analysis of the production process, an understanding of the purpose for which the process is modeled and the appropriate selection of that method, which will allow for a simple and transparent presentation of even a complex production process. Although the methodologies available on the market are not the most difficult and actually allow for a clear presentation of the production process, the companies do not always decide to use them. The high cost of buying the right modeling software and the time it takes to devote to the specific activities to create the model of the production process are quite significant. Few companies see the real benefits that a given methodology can bring, and most of them present the production process in the simplest form – using only a flowchart.

Keywords: process modeling, production processes, methods and notations of process modeling, modeling tools.

1 Introduction

According to M. Hammer, the process is "... a related task group which common result is value for the customer" [6]. In the *Reengineering the Corporation – A Manifesto for Business Revolution*, M. Hammer and J. Champy give a slightly more detailed definition of the process by formulating it as inter-related activities that have their own inputs and outputs, and create value for customers [5] or organization [3]. The graphic representation of the production processes (mapping of their position by means of graphic symbols) allows for the rapid identification of critical areas, and make the necessary changes that are closely linked to improving their position. Companies in which production processes are carried out should first of all focus on their continuous improvement, in that it enables them to achieve additional benefits for customers and the company itself. Therefore, the selection of appropriate methods and tools for modeling production processes seems to be crucial for the undertakings engaged in improvement activities.

To choose the right modeling methods, you must first understand what self-modeling is. Incorrectly in the literature, the model is often brought into the normal flowchart or process map. While mapping only serves as a mapping of the process itself (subsequent actions/steps) and identifying owners of each stage, modeling will also allow to identify some process-specific parameters (indicators). These parameters allow to control the production process (its automation) as well as in the future for the implementation of supporting tools, including appropriate tools and IT systems, as well as for optimization of production processes with respect to the selected criteria, in this case predefined parameters (indicators) [8].

Modeling allows not only to map the actual course of the production process (successive operations) and the relationships that take place between the processes identified in the process but also to give the process concerned the proper characteristics (parameters/indicators) relevant to the production process and its users. However, it is still only a mapping – i.e. the transfer of the actual production process to the most accurate graphic model. In this context, the model will only reflect the most important features of the analyzed production process from the point of view of the task (goal) that it pursues [4]. With this dependency, one production process can be described by different models, and each one can be equally useful in a particular enterprise or an individual. It is worth pointing out here that it is also common in the literature that the model should be a compromise between the best and the simplest mapping of a given phenomenon or a process [7].

The first step in modeling a production process is to gather as much data and information about the modeled process. Correct identification and detailed analysis of elements - material and information flows (including its inputs and outputs), its objectives and participants will allow the most accurate representation of the process using the chosen model. The knowledge of these elements and the purpose of modeling itself determines the choice of the appropriate method or approach to be applied. A well-structured production process will allow for better understanding of the process and will significantly improve later on. The benefits of modeling production processes will be reflected in the added value that the firm will achieve – the speed at which decisions are made and related actions, minimizing manufacturing costs and increasing the quality of products and services [1, 2, 9]. By applying appropriate methods of process modeling in an enterprise, it should not be limited to the purpose of changes in production processes but also, and perhaps above all, to the potential for the creation and subsequent implementation of IT systems that allow for the monitoring and Controlling production processes and controlling them [8, 9, 11].

Turning to issues related to modeling of production processes, it seems important to define also the basic terminology here. As in the literature, the notion of the method is often interwoven with notation, and the term is also misleading, it is worth pointing out at the outset how these concepts will be distinguished. The notation is therefore a set of graphical elements available for the diagrams to be created, together with a description of the relationships between them. The method determines the approach to modeling; It outlines the principles that should be followed when modeling - describe what elements (e.g. diagrams) and in what order they should be used so that the model created is as precise and consistent as possible. Often, along with the notation, there is also a suitable method or methodology, but remember that these are two different concepts. Also, the term tool is associated with some of the methods and notations – software vendors often dedicate their programs to selected methods and associated notations. The tool will thus be understood as a computer program, by which it is possible to present a model of a selected manufacturing process [9].

For ease and due to the existing literature divisions, and actually lack a certain uniqueness, the method and notation will be in this article described in one category, the tool, however, will be in a synthetic way presented as a tool to create a model in the context of specific methods and notation (corresponding software).

2 Types and levels of modeling

There are many different models in the literature, due to the variety of problems that these models try to solve. An interesting classification for modeling systems and also identified within these systems processes is presented in a publication of A. Burduk, distinguishing the three most commonly used classifications – with respect to their form (appearance), the time and type of variables. The criterion of form (appearance) divides models into three groups [4]:

- physical models (physical) – they are a physical representation of the characteristics of the studied phenomenon, of appropriate scale and degree of accuracy (e.g. product model)
- schematic (graphical) models – take the form of diagrams, block diagrams, maps (e.g. model of flow of information in the system)
- symbolic models – in the form of mathematical and algorithmic writing, define relations between selected variables taking the form of mathematical formulas (e.g. simulation model of production process).

As it is emphasized by A. Burduk, the most universal models used at many levels of the company are schematic models that allow analysis of the problems that arise and provide the basis for further action. Equally often used in companies there are simulation models that belong to a group of symbolic models. With them, it is possible to perform simulations and to verify planned changes and decisions. Simulation models allow control of the process, assuming that the appropriate outputs of the process can be optimally adjusted for its inputs [4].

Among the schematic models we can distinguish three levels [4]:

- block diagrams – which represent the graphical course of activities identified in the selected process, enable identification of elements in the process and the relationship between them;
- process maps – which present the graphical activities identified in the process and their owners and contractors, improve the process and compare it with its previous versions;
- process models – which represent the graphical process of activities identified in the process and their contractors in a highly advanced form, consisting of not many different types of schemes, allowing for taking into account relations with external entities – depending on the chosen method or notation and allow to give the processes certain characteristics (parameters/indicators), enable building of information systems, reorganization of processes, optimization and control of processes.

3 Commercially available methods and tools for process modeling

As it was mentioned earlier, the creators of the given method/notation dedicate it to the appropriate IT tools, so this is also worth mentioning in this statement. The following table (Tab. 1) shows the methods/notations of process modeling and the corresponding IT tools that are most commonly used and widely described in the literature.

The variety of literature-based methods and notations of process modeling introduces some chaos and makes it difficult to choose the best solution. In scientific publications, however, the benefits of BPMN notation are very often emphasized – this approach is very comprehensive compared to other available methods. Its great detail, standardization and extensive usage make it a very popular method for business process modeling. In its favor, it also speaks of the fact that it is a constantly evolving notation that has a large base of elements and objects, which allows to reproduce even very complex production processes. The simple record of the process makes it easy to understand the model, also by people who have never before been modeled. BPMN is a very precise standard, and there are many tools on the market that support it. However, this method is

quite time consuming and the software for commercial use is relatively expensive. By the way, most companies do not decide to use it [11].
Also, some of the presented above methods/notations require some degree of know-how from their users, understanding the rules and rules used. All this involves additional costs and is time consuming, which many entrepreneurs cannot afford. These methods are very helpful when you are going to make far-reaching changes in the production process or deploy software that will support this process. Therefore, in most enterprises, no process modeling is used, and only a simple flowchart is generated that shows the process flow or is created in a slightly more advanced process map.

Table 1. Selected methods/notations of process modeling and corresponding tools

Method/notation	Characteristics	Selected tools
Business Process Modeling Notation	▪ allows to create many different patterns / models ▪ a rich set of objects and elements ▪ allows you to automatically generate program code used for application development ▪ allows to assign selected parameters / indicators to a modeled process ▪ allows to describe even very complicated processes ▪ allows the contractor to identify the individual activities identified in the process ▪ very detailed and quite popular method	▪ Microsoft Visio ▪ Bizagi Modeler ▪ iGrafx Process ▪ Oracle Designer ▪ ARIS Platform
Data Flow Diagrams	▪ allows for graphical presentation of data flow and information in the process ▪ a small number of available objects and elements ▪ allows you to create diagrams on three levels of detail ▪ does not represent the relationship between the individual activities identified in the process	▪ Power Designer ▪ Microsoft Visio ▪ SmartDraw ▪ Visual Paradigm
The Integrated Definition for Function Modeling	▪ a collection of several methods (most commonly used IDEF0 and IDEF3) ▪ is used to graphically display functions, activities and activities in the process/system ▪ allows the process control and necessary resources to be included in the diagram ▪ allows for multi-level modeling of processes	▪ AIØ WIN ▪ Edraw Max ▪ Microsoft Visio
The Architecture of Integrated Information Systems	▪ a collection of several solutions for process modeling ▪ is a graphical representation of processes in the form of events ▪ a small number of available objects and elements ▪ enables modeling on different levels (data, functions, organization, products / services, processes) ▪ allows for a simulation modeling	▪ Platforma ARIS ▪ ARIS Express ▪ Microsoft Visio

Source: Own elaboration based on [1, 2, 3, 4, 8, 10, 11].

Conclusion

The appropriate selection of methods and tools for modeling production processes guarantees their correct reproduction and gives rise to a further improvement activities. Before selecting the appropriate method, it is necessary to analyze in a detail the particular production process, the elements of which are to be modeled. Equally important is the knowledge of the modeling goal itself, since each of the modeling methods allows for different results. The next step will be selecting the right modeling method, if there are no major problems with IT tools, which are in fact many (both free and paid version) on the market, the choice of the right notation or method is already a bit more complicated.

This choice should be therefore determined not only by the company's financial capabilities or the time, which can be devoted to modeling itself, but also the complexity of the modeled production process, the reason for deciding on modeling – the goal to be met by the model and its future customers. However, it is important to address the key question – Is modeling of the production process necessary? If the process is smooth and the benefits are satisfactory, it is not enough to prepare a map of the production process.

Process maps and common diagrams are a commonly used method for presenting production processes, mainly because they are not complicated enough to use time-consuming and cost-effective modeling methods in most of small and medium enterprises. Everything really depends on the ability of the company and often emphasized in the design of the purpose of modeling. Larger companies whose production processes are complex can, and should, allow themselves to be modeled accordingly – in line with the idea of continuous improvement and added value that process modeling undoubtedly brings.

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MODELOWANIE PROCESÓW PRODUKCYJNYCH – ARTYKUŁ PRZEGLĄDOWY

Streszczenie: Celem niniejszego artykułu jest przedstawienie dostępnych na rynku metodologii mogących być podstawą przy modelowaniu procesów produkcyjnych. W branżowej literaturze znajduje się wiele przystępnych metod i notacji pozwalających na modelowanie procesów, jednak w dalszym ciągu w przedsiębiorstwach nie znajdują one szerszego zastosowania. Wybór odpowiedniej metodologii i narzędzia do modelowania procesów nie jest jak się okazuje wcale łatwy. Wymaga on przede wszystkim dokładnej analiza procesu produkcyjnego, zrozumienie celu dla którego proces ten jest modelowany i odpowiedniego dobranie do tego metody, która pozwoli na proste i przejrzyste przedstawienie nawet skomplikowanego procesu produkcyjnego. Choć dostępne na rynku metodologie nie należą do najtrudniejszych i faktycznie pozwalają na przejrzyste przedstawienie procesu produkcyjnego, to przedsiębiorstwa nie zawsze decydują się na ich wykorzystanie. Dość znaczną przeszkodą są tutaj wysokie koszty związane z zakupem odpowiedniego oprogramowania do modelowania i czas, jaki należy poświecić na poszczególne działania zmierzające do stworzenia modelu danego procesu produkcyjnego. Niewiele przedsiębiorstw widzi realne korzyści, jakie może przynieść użycie danej metodologii, a większość przedstawia procesy produkcyjne w najprostszej postaci – jedynie za pomocą schematu blokowego.

Słowa kluczowe: modelowanie procesów, procesy produkcyjne, metody i notacje modelowania procesów, narzędzia modelowania.

THE ROLE OF STANDARD COSTING IN AN EXEMPLARY IT COMPANY

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Abstract: In today's economic reality, enterprises based on modern technologies are increasingly gaining on the importance. Access to modern technology is often offered through outsourcing to IT companies. Despite offering modern, often virtual solutions, cost accounting issues are as real as possible. The aim of this paper is to present the general principles of standard costing in contemporary enterprises and to define the possibility of using standard cost accounting in companies offering specific IT solutions. The paper uses the method of literature analysis in relation to the problem of standard costing. The paper contributes to the management accounting literature in two ways. Firstly, it points out that in the face of widespread criticism of the use of traditional tools such as standard costing and the use of contemporary accounting tools (such as ABC, JIT, BSC and others) in the era of globalization, the established practice may not reflect the popular opinion. Secondly, the paper presents a case study of standard costing in an IT service company that can be implemented in practice or be further developed theoretically.

Keywords: standard costing, cost accounting, IT, valuation, management accounting.

1 Introduction

Standard cost accounting is a classic instrument of management accounting. It was predominantly used in manufacturing companies since the end of the nineteenth century. With the advancement of civilization, apart from traditional companies, there have developed companies based on modern technologies as well as offering IT solutions that combine the features of a product and service. The author of the paper raises the question whether in today's modern enterprise in the IT industry the standard cost accounting has lost its importance or whether it is still applicable. Two objectives of the study were formulated:

- presentation of the role of standard costing in contemporary enterprises,
- determining whether it is possible to adopt standard cost accounting in companies offering specific IT solutions.

The paper uses the method of literature analysis in relation to the addressed problem of standard costing. In the empirical part, a case study on an exemplary IT enterprise is presented, in which methods of standard costing can be applied. The analysis deals with 6 different IT solutions offered to customers within a comprehensive IT service. The paper contributes to the management accounting literature in two ways. Firstly, it points out that in the face of widespread criticism of the use of traditional tools such as standard costing and the use of contemporary accounting tools (such as ABC, JIT, BSC and others) in the era of globalization, the practice may not reflect the popular opinion. Secondly, it provides examples of standard costing in an IT service enterprise that can be implemented in practice or be further developed theoretically.

2 Theoretical rules of standard costing

The standard cost accounting is the base for the key performance indicators, which are helpful in making decisions pertaining to the planning, designing and controlling production processes in the case of varied production and/or service [8].

The standard cost accounting consists of planning the cost of products and services based on reasonably justified consumption norms and postponed purchase prices as well as of defining the deviations between the standard costs and the actual costs incurred. Apart from the accounting definition of costs, it is assumed that costs are the financial expression of the resource consumption within the enterprise activities. Therefore, the cost model may take the form of the so-called basic cost equation [14]:

Cost = Resource consumption x Resource unit price

The above equation is the basis for modelling the standard cost of manufacturing a product unit. This model assumes certain operational conditions of the company's activities, resulting, inter alia, from qualitative requirements, applied technology, qualifications of employees, parameters of remaining resources. As noted by Piosik [10], from the point of view of management objectives, the adoption of standards at expected values levels is arguable. This usually results in standards being set on an average level of difficulty. In order to motivate managers to achieve management objectives, higher-level standards can be set, at least for some cost groups. Ideal standards are not, in turn, achievable under real conditions.

From the standard cost equation it follows, firstly, the need to distinguish the quantitative standard for the physical quantity of a resource with the specified quality parameters necessary for the production of the product. Secondly, the price standard for the proposed unit price of a resource with specific characteristics should be distinguished.

When using the variable cost accounting instead of the full cost accounting, the standards for variable costs as well as for fixed costs should be set separately [1].

For example, in order to set standard direct material costs, consumption standards and price standards are set separately. In order to establish consumption standards, the method of average consumption per production unit can be used on the basis of historical data. This method requires detailed quantitative and qualitative records and material flow records, but it is relatively simple to use. The second method is the technical analysis of consumption (industrial-engineering method) [9]. In this case, the projected net consumption is calculated taking into account technological losses, useful and useless waste, quality deficiencies. Losses can be estimated based on the formula for the percentage of planned consumption. Estimation of losses can also be based on statistical data, especially for bulk production. Standard purchase prices, standard purchase costs and expected price discounts should be considered in order to determine the standard purchase price. To estimate the standard purchase price one can apply an extrapolation of the linear trend to determine the trend. If the purchase price does not show the trend, the standard will be determined based on the expected value [10].

Similarly, for payroll cost, cost standards can be set, consisting of a labour intensity standard, which is the equivalent of the consumption standard and a standard remuneration rate. In determining the labour intensity standard, usually defined as man-hours, the non-productive time for working breaks should be taken into account. In turn, the method of linear trend extrapolation can be used to determine the remuneration rate standard, as in the case of material prices.

In practice, the standard costs of fuel and technological energy, the cost of special tools, the cost of third-party tooling, the cost of production preparation and production related overheads can be determined.

3 Criticism of standard costing in the contemporary companies

The literature from the late 1980s contains a lot of criticism of the inconsistency of the standard costing in today's manufacturing environment. The standard cost account was criticised for being based on historical data and cost data deformation leading to erroneous decisions. The standard cost account was also blamed for fulfilling the support and service function towards financial accounting. According to Kaplan and Johnson [7] as well as Ferrara [5], the standard costing methods and the variance analysis appeared to be insufficient for cost control and performance evaluation. The mentioned authors stated such reasons for the reduced usability of standard cost accounting as shorter product life cycles, advanced manufacturing technologies, decreasing emphasis on labour in the production process. The wave of criticism aimed at standard costing has been accompanied by popularisation of new management concepts (e.g. Just-in-time, Kaizen) and alternative cost accounting such as Activity Based Costing (ABC) [12, 13], Time Driven Activity Based Costing (TABC), or Target Costing connected with the lean management concept.

There have also emerged alternative views. For example, Bromwich and Bhimani [3] believed that the low level of management accounting development and the negative effects of standard costing were due to poor quality of management and lack of skills among senior executive staff, and consequently they disagreed with the view that management accounting was at the service of financial accounting.

Similarly, although Bowhill and Lee [2] perceived the strategic importance of management accounting for modern companies, they did not confirm the need to implement new cost accounting systems. They stated that: 'Although not necessarily fully "compatible" with new manufacturing methods, standard costing systems can still continue to be useful.'

Despite the emerging trends, companies have continued to use the standard cost accounting, which, among others, was proved by Drury [4]. Based on the conducted research, he found that 76% of organisations responding to his survey operated a standard cost system. Drury emphasized that in addition to budgeting and variance analysis, standard costing also has many other purposes than cost control. The most important for him were decision-making purposes.

In Poland, Sobańska [11] carried out a study on the popularity of various cost accounting models in the period of 1999-2005, which showed that the standard cost account was still the dominant cost account in the 93 surveyed companies, and only 3.3 % of them decided to implement alternative cost accounts.

Nowak [9] emphasized that an important area of cost accounting is the cost accounting for decision-making, which does not have systematic nature but concerns specific decisions taken under specific management conditions. In decision-making tasks that are limited to cost minimization, costs can occur as a goal function. In other more complex situations, costs are values occurring under the conditions restricting decision-making models.

Based on the literature analysis, the question arises as to whether companies are able to manage without the traditional standard cost accounting, or whether it is better to treat the alternative cost accounting as parallel cost accounting for decision-making. Is it possible to use traditional methods of standard costing in modern service enterprises, for example in IT sector? To answer this question the case study of an exemplary IT company is discussed in the paper,.

4 Case study of Exemplary IT Company

The analysed company is engaged in outsourcing IT services. The goal of the company is a complex provision of IT services for small and medium enterprises. The activity is carried out within the framework of long-term contracts concluded with customers for the provision of standardized IT services. Thanks to signing a comprehensive contract, the customer has a professional IT service provided without the need of hiring IT professionals and can focus on the core business. Sample services included in the standard outsourcing agreement are included in Table 1.

Table 1. Types of business services offered to customers

Business service ID	Type of Business Service	Description of Business Service
BS_1	Internet Sharing	The Internet sharing services are provided to clients on an ongoing basis, i.e. 24 hours a day, 7 days a week. The basic parameter of the service is the bandwidth determined by the minimum and maximum transfer of downloaded and sent data.
BS_2	Electronic mail	The service consists of setting up and configuring mail accounts, reporting service levels, configuring email clients at workstations, providing ongoing support, maintaining services within established quality and capacities.
BS_3	Provision and maintenance of workstations	The service includes rental and maintenance of desktops and laptops. The inventory of equipment and licenses of installed software is included in the service.
BS_4	Supply of consumables	As part of the service, supplies of consumables such as printer toners and spare parts for computers are delivered.
BS_5	Applications development and servising	The service consists of the data collection of customer needs to clarify functional and environmental needs. Then the architecture and the environment of created application are proposed. The application is created in selected languages and using the indicated technologies.
BS_6	IT consulting	The service deals with broadly understood IT consulting.

Source: own elaboration

Taking into account the theoretical assumptions for standard costs set out in point 2, one should consider whether they will be applicable to determining the costs of business services presented in Table 1. Undoubtedly, payroll costs are very significant cost item in the IT service. The remuneration rate of employees of a given department is calculated according to the formula:

$$\boldsymbol{C_{MH}} = \frac{\frac{\sum C_D}{\boldsymbol{FTE}}}{\boldsymbol{168}}$$

where: C_{MH} – cost of one man-hour; C_D – costs of department; *FTE* – number of full-time employees.

The payroll costs and overhead expenses from a given department are collected from the accounting system. Table 2 presents the remuneration rates for one of the divisions of the analysed company in the period of April-December of 2016.

Table 2. Observations of monthly man-hour rate

No.	Month	Man-hour rate PLN
4	April	45.30
5	May	46.10
6	June	47.00
7	July	47.70
8	August	50.00
9	September	51.20
10	November	51.80
11	October	52.40
12	December	54.00

Source: own elaboration

In order to set a standard rate based on historical data, the linear trend extrapolation method (using the REGLINP program, statistical functions in Microsoft Excel tools) was used [10].

The following parameter values were calculated:

– a = 40.59 (α)
– c = 1.11 (γ)
– average estimate errors D(c) = 0.49; D(a) = 0.058
– multiple determination coefficient: 0.98

The standard remuneration rate can be described using the function [10]:

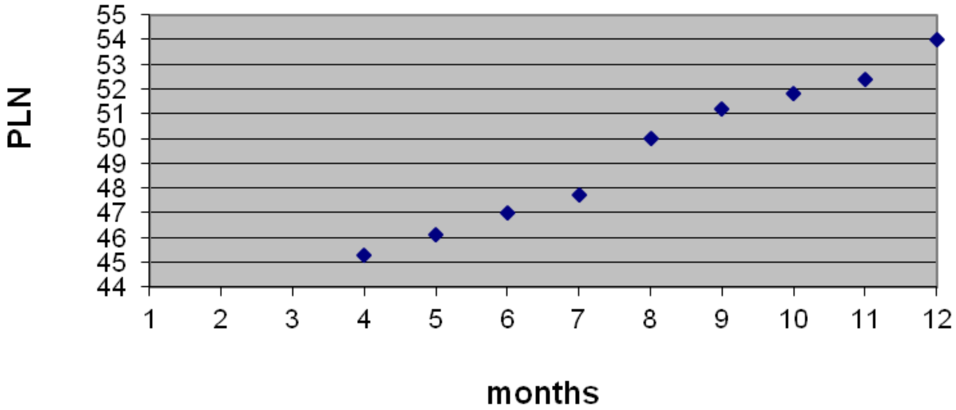
$$\varepsilon(y_t) = \propto + \gamma t$$

After substituting the parameters, the function looks like this:

$$\varepsilon(y_t) = 40.59 + 1.11t$$

A graph of the function is presented below.

Figure 1. Development of standard remuneration rates for an exemplary department



Source: own elaboration

In order to calculate how much payroll costs amount to a business service BS_5, one should estimate the labour cost in man-hours to create the application and multiply it by the predicted remuneration rate. For example, if a service requires 90 man-hours and is scheduled to be executed in March 2017 (12 consecutive months from April), then the payroll costs will be:

$$\boldsymbol{R} = \mathbf{90}\ \boldsymbol{x}\ (40.59 + 1.11x12) = 90\ x\ 53.91 = 4\,851.90$$

The second important factor affecting the cost of the analysed company is the cost of licenses paid to subcontractors in the form of subscriptions. Subscriptions can be paid for a period of one to five years. Customers do not pay one-off amount for the licenses they have purchased, as they are included in the monthly payments for the comprehensive service contract. In order to determine the standard monthly payment for a license, account should be taken of the change in the time value of money and discounted at the adopted rate of IRR (Internal Rate of Return)[1].

Computers and laptops made available by the analysed company are installed with office software, whose price per license is PLN 1650.00. The license is available for 33 months. Under the agreement with the subcontractor the IT company managed to negotiate a favourable rate of 1597.00 PLN on the purchase of 100 licenses. The payment for the license supplier is settled monthly. The manner in which monthly license payments are made is shown in Tables 3 and 4.

Table 3. Example of calculation of licenses' cost to rent

No.	Name	Quantity	Price PLN	BS	Purchase cost PLN
1.	purchase of license X	100	1 579.00	BS_...	157 900.00
				SUM	157 900.00

Source: own elaboration

Table 4. Example of calculation of licenses' rental instalments

NPV cost		Margin	NPV revenue	
payments in months	33		payments in months	33
IRR	6.05%		IRR:	6.05%

[1] Internal rate of return is a discount rate that makes the net present value (NPV) of all cash flows from a particular project equal to zero.

instalment PLN	5194.42	5,80%	instalment PLN	5514.24
sum of instalments PLN	171415.70	5,80%	sum of instalments PLN	181969.96
			payment for 1 license PLN	55.14

Source: own elaboration

The value of the customer's monthly fee as part of a comprehensive IT contract shall be equal to the product of PLN 55.14 and the number of licenses he/she uses.
In order to value the BS_3 service, the company must determine the standard cost of the computers being made available. The standard monthly cost of sharing a computer set is based on purchase prices from supplier, but also includes the "Risk of Exchange Rate" and the "Risk of Computer Set Change" rates. They should also include the costs of final disposal, unless the entity returns used computer sets free of charge. Table 5 shows an example of the cost of a standard computer set.

Table 5. Example of calculation of cost of computer set to rent

Computer Set – Standard	Purchase price EUR	Quantity	Unit	Cost PLN
HP 600G2PD MT i36100 500G 4.0G 54 PC	464.6	1	pcs	2072.12
HP 4GB DDR4-2133 DIMM	27.27	1	pcs	121.62
HP 5y NextBusDay Onsite DT Only HW	15.15	1	pcs	67.57
plug strip	5.00	1	pcs	22.30
HP EliteDisplay E240c Monitor	156.55	1	pcs	698.21
HP warranty 5 years	15.00	1	pcs	66.90
Risk of Exchange Rate and Risk of Computer Set Change	2.76%	1	%	84.14
Insurance	0.25%	5	years	39.63
Disposal	0.00	15	kg	0.00
One-time man-hours	57.00	2	mh	114.00
Sum				**3286.49**

Source: own elaboration

This way established sharing cost over time should include the margin, as well as account for the changes in the time value of money through the assumed IRR rate, as in the case of providing the license. The standard monthly rental rate for the computer set should be added to the standard cost of the support offered, for example 0.5 man-hour multiplied by the standard remuneration rate from the support department of PLN 57.00.

IT companies providing the so-called Server Hosting or offering rental of servers are also required to evaluate the standard cost of hardware and software platforms. Standard items valued for the so-called Virtual Machine include a particular collection of virtualized hardware resources available to a virtual machine (VM) instance, including the memory size, virtual CPU count, and maximum persistent disk [6].

The valuation is based on purchase costs of hardware resources and software licenses, as well as labour costs associated with the maintenance. On this basis, price lists are created for customers. Table 6 lists examples of virtual machines.

Table 6. Exemplary types of virtual machines

Machine type	Virtual CPUs	Memory	Price (USD)	Preemptible price (USD)
n1-highmem-2	2	13GB	$0.1184	$0.0250
n1-highmem-4	4	26GB	$0.2368	$0.0500
n1-highmem-8	8	52GB	$0.4736	$0.1000
n1-highmem-16	16	104GB	$0.9472	$0.2000
n1-highmem-32	32	208GB	$1.8944	$0.4000
n1-highmem-64	64	416GB	$3.7888	$0.8000

Source: [6]

Due to limitations of the paper, it is not possible to include detailed calculations for all the analysed business services. Table 7 summarizes what cost items can be standardised for each of the services.

Table 7. Standardized cost items per service

Business service ID	Cost items that are subject to standardisation included in the valuation of the service
BS_1	The valuation of the services is based on costs depending on the capacity parameters of the service provided, i.e. the so-called bandwidth of the connection. A significant component of the standard cost is the cost of purchasing connections (fibre) from the subcontractor, which should be accounted for by the distribution keys in reference to the bandwidth and the number of users. The service support is calculated on the basis of standard payroll costs.
BS_2	The price lists of email boxes of a given capacity are based on platform costs and support costs (standard payroll costs)
BS_3	Sharing and maintenance of workstations. The price lists for customers are created based on the costs of sharing computers and licenses (depending on purchase costs) and their maintenance costs (payroll costs).
BS_4	The supply of consumables. The price lists are based on the standard cost of purchasing toners and other consumables.
BS_5	Application development and technical support. In order to value the service, information on employees' payroll costs, license costs and hardware platform costs are required.
BS_6	IT consulting is primarily based on the employees' payroll cost.

Source: own elaboration

The presented case study is an example of how diverse the applications of standard costing can be found in modern enterprises providing IT services.

Conclusion

The literature on management accounting has been questioning the applicability of traditional standard costing since the 1980s. According to many authors, standard costing became irrelevant and cumbersome. They often promote lean technology like lean management, just-in time (JIT) or activity based costing. Despite the criticism, standard costing is still very popular in many manufacturing companies. The aim of the paper was to indicate the possible applications of standard costing also in service enterprises in the IT sector. The presented case study regarded a company, which carries out its activity under long-term contracts concluded with clients for the provision of standardized IT services. The discussed company offers services such Internet sharing and electronic mail, rental and maintenance of computers, supplies of consumables, as well as application development and provision of consulting services. For most of these services, it is necessary to set up standard costs, as they often result from costs incurred in the past such as hardware purchase costs, server maintenance costs, employee costs. The author has shown, according to Drury [4] as well as Bowhill and Lee [2], that standard cost accounting has additional uses primarily for the standardization of offered services and their valuation. The valuation issues in modern IT companies are gaining momentum as they often involve high-value intangible assets. Standardization of costs under such conditions may be subject to further in-depth scientific research.

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ZNACZENIE KOSZTÓW STANDARDOWYCH W PRZYKŁADOWYM PRZEDSIĘBIORSTWIE INFORMATYCZNYM

Abstrakt (Streszczenie): We współczesnej rzeczywistości gospodarczej coraz bardziej zyskują na znaczeniu przedsiębiorstwa oparte na nowoczesnych technologiach. Dostęp do nowoczesnych technologii oferowany jest często w ramach outsourcingu przed przedsiębiorstwa informatyczne. Pomimo oferowania nowoczesnych, często wirtualnych rozwiązań, problemy w zakresie rachunku kosztów są jak najbardziej rzeczywiste. Celem artykułu jest przedstawienie ogólnych zasad standaryzacji kosztów we współczesnych przedsiębiorstwach oraz określenie możliwości zastosowania rachunku kosztów standardowych w przedsiębiorstwach oferujących konkretne rozwiązania informatyczne. W artykule wykorzystano metodę analizy literatury w odniesieniu do poruszanej problematyki standaryzacji kosztów. Artykuł wnosi wkład do literatury z zakresu rachunkowości zarządczej na dwa sposoby. Po pierwsze w artykule zwrócono uwagę, że w obliczu powszechnej krytyki stosowania klasycznych narzędzi takich jak standaryzacja kosztów, i popularyzacji wykorzystywania alternatywnych narzędzi rachunkowości zarządczej (takich jak ABC, JIT, BSC i inne) w erze globalizacji, praktyka nie odzwierciedla opinii popularnych w literaturze. Po drugie w artykule zaprezentowano przykłady wykorzystania standaryzacji kosztów w przedsiębiorstwie usługowym branży informatycznej, które można wdrożyć w praktyce lub w dalszym stopniu rozwijać teoretycznie.

Klíčová slova (Słowa kluczowe): koszt standardowy, rachunek kosztów, IT, wycena, rachunkowość zarządcza

SPREADSHEET AS A MEANS TO SUPPORT NONCONFORMITY ANALYSIS – DATA INTEGRITY IN THE TOOL

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Abstract: The solution discussed in the article is an example of a tool created in accordance with the author's signature concept of Microtools Based on the Relational Data Model to support the nonconformity analysis process regarding rolled products. The study opens with an explanation of the data structure implemented in the tool. The entities thus identified as well as their attributes and interlinks between them have been depicted using an adequate Entity Relationship Diagram. What has also been discussed in the manner in which the proposed data structure has been implemented in a spreadsheet. Further paragraphs of the publication describe the solutions applied to ensure integrity of the data stored in the tool. The conditions to be met by the data as well as individual mechanisms created to verify them have been described. All the solutions proposed in the paper have been developed using standard spreadsheet features and components, without involving any additional code created in any programming language.

Keywords: information processes, process improvement, relational data model, MiRel.

1 Introduction

Functioning of contemporary businesses is inextricably linked with the notion of information. Processes where information is generated, collected, stored, processed, transferred, rendered, interpreted or used are referred to as information processes [4, 5]. Different kinds of IT tools are commonly used to support their implementation. There is a group of computer programs used in nearly every organisation and familiar to nearly every employee, namely spreadsheets. They enable relatively quick creation of tools matching the given company's current needs, which prove particularly useful in situations when application of other IT solutions is impossible. Using spreadsheets to support the functioning of organisations, as proposed in the literature of the subject, is a concept which encompasses a very wide range of applications. The diverse solutions proposed are, among other spheres, related to finance management, controlling [2, 3, 14], sales, marketing [2, 14, 15] or quality management [2, 10, 11].

The tools thus created may be developed in an intuitive manner, without any openly pre-defined data model. They may also be preceded by a thorough problem analysis leading to formulation of an adequate relational data model which – once it has been implemented in the spreadsheet – provides grounds for the tool to be built [6, 12, 13]. And this is exactly the kind of procedure assumed as a basis for the author's signature concept of Microtools Based on the Relational Data Model, abbreviated to MiRel [7]. An example of a tool conforming with this concept is one which supports the process of product nonconformity analysis. Individual solutions which make it possible to develop the reports assumed as the process deliverables have been discussed in the author's previous publication [8]. Further sections of this article describe the data structure implemented in the tool in question as well as the solutions applied to ensure integrity of the data entered into and stored in the tool.

2 Data structure applied in the tool

In the development of the tool addressed in the paper, it was originally proposed that it should comprise such entities as Order, Nonconformity Card, Cause, Decision, Production Line, Control, Cause on a Card, Nonconformity for Cause on a Card and Decision for Cause on a Card. The structure of the above entities along with the attributes they comprise has been depicted in a diagram conforming with the CASE Method [1], as provided in Figure 1.

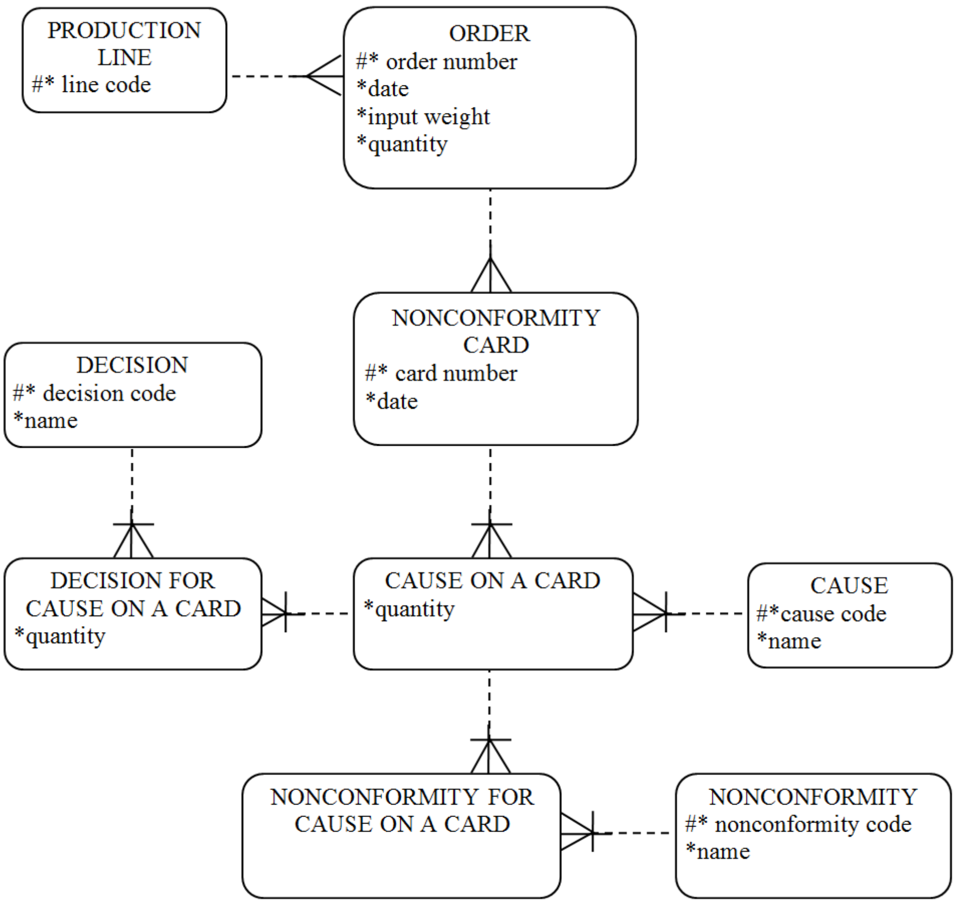

Fig. 1. Structure of entities in the tool discussed

The MiRel concept is based on an assumption that entities can be represented in a spreadsheet in two ways [9]. In this case, all the identified entities have been implemented as traditional tables, as it is handled in classical relational databases. Each table has been entered into a separate worksheet. The worksheet names are, at the same time, table names. Their arrangement has been shown in Figure 2.

Worksheet "O"

	A	B	C	D	E
1	Order_no	Data	Line	Input_wgt	Quant
2	16/0001	2016-01-02	B	6,461	23
3	16/0002	2016-01-02	C	20,811	53
4	16/0003	2016-01-02	B	1,908	4

Worksheet "CC"

	A	B	C
1	Card_no	Cause_code	Quant
2	KN/16/0001	PRO	5
3	KN/16/0002	ACC	1
4	KN/16/0003	PRO	3

Worksheet "DCC"

	B	C	D	E
1	Card_no	Cause_code	Decision_code	Quant
2	KN/16/0001	PRO	VER	4
3	KN/16/0001	PRO	REC	1
4	KN/16/0002	ACC	REC	1

Worksheet "D"

	A	B
1	Decision_code	Name
2	OFF	Offering
3	VER	Verification
4	REC	Recycling

Worksheet "NCC"

	B	C	D
1	Card_no	Cause_code	NC_code
2	KN/16/0001	PRO	T
3	KN/16/0002	ACC	S
4	KN/16/0003	PRO	T

Worksheet "CA"

	A	B	C
1	Card_no	Order_no	Data
2	KN/16/0001	16/0002	2016-01-11
3	KN/16/0002	16/0004	2016-01-11
4	KN/16/0003	16/0006	2016-01-12

Worksheet "C"

	A	B
1	Cause_code	Name
2	ACC	Accident
3	PRO	Programm
4	DFI	Defective input

Worksheet "L"

	A
1	Line_Code
2	B
3	C

Worksheet "N"

	A	B
1	NC_code	Name
2	T	Out of tolerance
3	R	Rhombus
4	S	Deflections

Fig. 2. Representation of entities in a spreadsheet

3 Solutions ensuring data integrity

An important problem tackled when developing a tool in a spreadsheet is how to create solutions that enable validation of the data being entered. In this respect, the correctness may pertain to different aspects, one of which is the conformity of foreign key values with the values of simple primary keys contained in source tables. In order to ensure this kind of conformity in columns, where foreign key values are entered, a *data validation* mechanism has been implemented, so that one can enter values selected from a drop-down list. The data sources for these lists are specific named worksheet ranges, as defined in the name manager. These ranges have been dynamically defined by means of suitable functions. The named ranges have been listed along with their definitions and target columns in Table 1.

Table 1. Named ranges used as data sources for data validation

Name	Reference	Target table	Target column
Card_Number	=OFFSET(CA!$A$2;0;0; COUNTA(CA!$A$2:$A$1001);1)	CC	Card_no
Cause_Code	=OFFSET('C'!$A$2;0;0; COUNTA('C'!$A$2:$A$11);1)	CC	Cause_code
Decision_Code	=OFFSET(D!$A$2;0;0; COUNTA(D!$A$2:$A$11);1)	DCC	Decision_code
Line_Code	=L!$A$2:$A$3	O	Line
Nonconformity_Code	=OFFSET(N!$A$2;0;0; COUNTA(N!$A$2:$A$11);1)	NCC	NC_code
Order_Number	=OFFSET(O!$A$2;0;0; COUNTA(O!$A$2:$A$1001);1)	CA	Order_no

The other data integrity problem one must solve is connected with conformity between foreign key values and values of corresponding composite primary keys. In the tool in question, this is the case of the composite primary key in table "CC". This key comprises attributes "Card_no" and "Cause_code", and it migrates to tables "DCC" and "NCC". The combinations of attributes "Card_no" and "Cause_code" entered into these tables must already exist in table "CC". A solution proposed in order to ensure this conformity is to add auxiliary columns in worksheets which contain the three aforementioned tables.

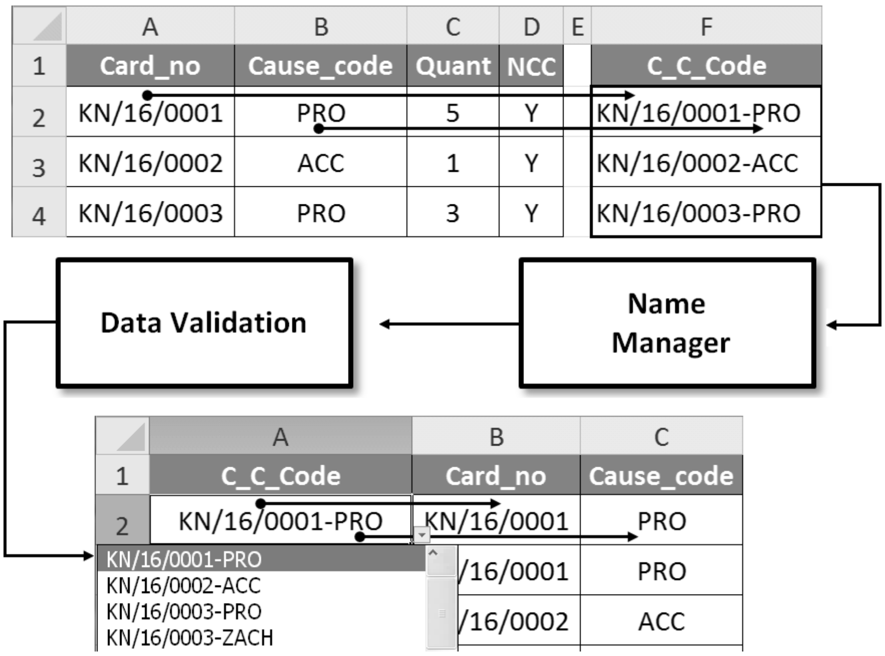

Fig. 3. Schematic representation of the solution ensuring conformity of key values in tables "CC", "NCC" and "DCC"

In worksheet “CC”, in the added column, a code is created as a combination of attributes “Card_no” and “Cause_code” using an appropriate formula. The named worksheet range that contains this code becomes the data source for the drop-down list used by the data validation mechanism. In worksheet “DCC” or “NCC”, in the added column, the user enters a value by selecting it from a list of permissible existing combinations of attributes “Card_no” and “Cause_code”. The value thus entered is broken down into values of individual attributes by means of suitable formulas. This solution has been schematically illustrated in Figure 3. The formulas applied in it have been depicted in Figure 4 using the example of tables “CC” and “DCC”.

Worksheet "CC"

	A	B	C	D	E	F
1	Card_no	Cause_code	Quant	NCC	FA	C_C_Code
2	KN/16/0001	PRO	5	Y		KN/16/0001-PRO
3	KN/16/0002	ACC	1	Y		KN/16/0002-ACC

Worksheet "DCC"

	A	B	C	D	E
1	C_C_Code FB	Card_no FC	Cause_code	Decision_code	Quant
2	KN/16/0001-PRO	KN/16/0001	PRO	VER	4
3	KN/16/0001-PRO	KN/16/0001	PRO	REC	1

FA	IF(A2<>"";CONCATENATE(A2;"-";B2);"")
FB	IF(A2<>"";LEFT(A2;SEARCH("-";A2)-1);"")
FC	IF(A2<>"";MID(A2;SEARCH("-";A2)+1;LEN(A2)-SEARCH("-";A2));"")

Fig. 4. Formulas applied in the solution to ensure conformity of keys in tables “CC” and “DCC”

Another issue that one must probably address in terms of data integrity is the unrepeatability of primary key values. In order to solve it, auxiliary tables have been added to individual tables, where – in each row – the algorithm counts the number of instances of the primary key from the given row in the table. In the event that a value repeats itself, the cells which contain it become highlighted by conditional formatting. An example of this solution with reference to table “CC” has been illustrated in Figure 5.

Conditional Formatting Rule		
Rule	Format	Applies to
Formula: =$H2>1	AaBbCcYyZz	=$A$2:$B$3001

	A	B	C	D	G	H
1	Card_no	Cause_code	Quant	NCC	FD	I1
2	KN/16/0001	PRO	5	Y		1
3	KN/16/0002	ACC	1	Y		1
4	KN/16/0003	PRO	3	Y		2
5	KN/16/0003	PRO	3	Y		2

FD	COUNTIFS($A$2:$A$3001;A2;$B$2:$B$3001;B2)

Fig. 5. Solution applied to verify primary key uniqueness in table “CC“

The solutions discussed above have all concerned uniqueness of primary keys and conformity between their values and those of foreign keys. Another condition pertains to the relationship between the data entered into tables “CC” and “DCC”. The number of pieces assigned to a specific cause on the given card in table “CC” must correspond to the sum of pieces assigned to different decisions for the given cause and card in table “DCC”. In order to verify this condition, auxiliary columns have been added both in worksheet “CC” and in worksheet “DCC”. Based on the values to be found in the auxiliary columns, in both tables, cells in columns “Quant” whose values do not match one another are highlighted using conditional formatting. The added auxiliary columns along with the formulas applied and the conditional formatting rules have all been depicted in Figure 6.

Conditional Formatting Rule		
Rule	Format	Applies to
Formula: =C2<>J2	AaBbCcYyZz	=$C$2:$C$3001

Worksheet "CC"

	A	B	C	D	E	F	G	J
1	Card_no	Cause_code	Quant	NCC		C_C_Code	FE	I3
2	KN/16/0001	PRO	5	Y		KN/16/0001-PRO		4
3	KN/16/0002	ACC	1	Y		KN/16/0002-ACC		1
4	KN/16/0003	PRO	3	Y		KN/16/0003-PRO		3

Conditional Formatting Rule		
Rule	Format	Applies to
Formula: =$K2>$J2	AaBbCcYyZz	=$E$2:$E$3001
Formula: =$K2<$J2	AaBbCcYyZz	=$E$2:$E$3001

Worksheet "DCC"

	A	B	C	D	E	F	I	J	K
1	C_C_Code	Card_no	Cause_code	Decision_code	Quant		FF I3	FG I4	FH I5
2	KN/16/0001-PRO	KN/16/0001	PRO	VER	3		1	5	4
3	KN/16/0001-PRO	KN/16/0001	PRO	REC	1		1	5	4
4	KN/16/0002-ACC	KN/16/0002	ACC	REC	1		2	1	1

FE	IF(F2<>"";SUMIF(DCC!$A$2:$A$3001;CC!F2;DCC!$E$2:$E$3001);"")
FF	MATCH(A2;CC!$F$2:$F$3001;0)
FG	INDEX(CC!$C$2:$C$3001;I2;1)
FH	SUMIF($A$2:$A$3001;A2;$E$2:$E$3001)

Fig. 6. Arrangement of formulas and formatting rules of the data validating mechanism for tables “CC” and “DCC”

Another verification mechanism applied in the tool is intended to check whether each combination of values of attributes “Card_no” and “Cause_code” that appears in table “CC” also appears at least once in table “NCC”. In order to test this condition, the “NCC” auxiliary columns has been added to table “CC”. Both the column and the formula applied have been shown in Figure 7.

	A	B	C	D
1	Card_no	Cause_code	Qu FI	NCC
2	KN/16/0001	PRO	5	Y
3	KN/16/0002	ACC	1	N

FI	=IF(A2<>"";IF(COUNTIF(NCC!$A$2:$A$3001;CC!F2)=0;"N";"Y");"")

Fig. 7. Additional column and formula of the mechanism verifying conformity of data in tables “CC” and “NCC”

The cells of auxiliary column "NCC" which contain the "N" value have been highlighted by the conditional formatting mechanism. This implies that an appropriate combination of "Card_no" and "Cause_code" has not been entered into table "NCC" yet.

The last of the solutions proposed verifies whether values of all attributes have been entered in all the tables previously introduced. What has been used in this solution is the conditional formatting mechanism based on the added auxiliary column. An example of such a solution assumed for table "CC" has been illustrated in Figure 8.

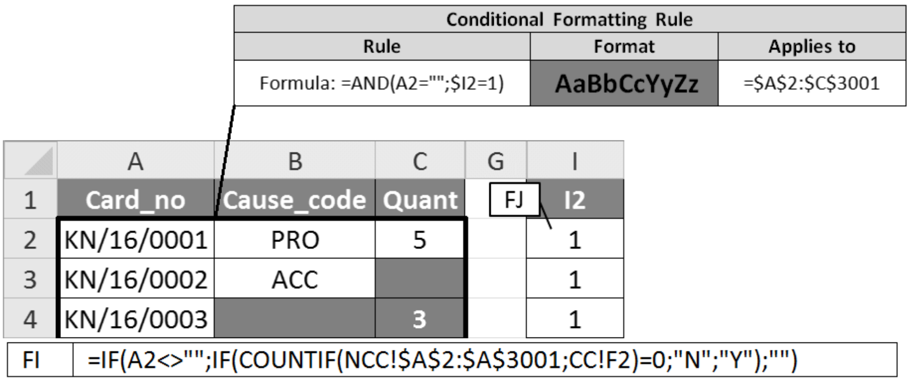

Fig. 8. Model solution verifying whether all attributes have been entered

Conclusion

The solutions proposed in the paper have illustrated that, in the tool developed in accordance with the concept of Microtools Based on the Relational Data Model to support the nonconformity analysis process, data integrity may be successfully achieved by means of built-in spreadsheet components and functions. For purposes of the tool subject to analysis, the following conditions have been identified as those which must be met while data are entered:

- values of foreign keys must conform with values of simple primary keys contained in source tables,
- values of foreign keys must conform with values of composite primary keys contained in source tables,
- values of primary keys must be unique,
- value of the "Quant" attribute assigned to a specific cause and card in table "CC" must conform with the sum of values of the "Quant" attribute assigned to different decisions for the given cause and card in table "DCC",
- each combination of values of attributes "Card_no" and "Cause_code" contained in table "CC" must appear at least once in table "NCC",
- values of all other attributes must be present in records where a value of any of the attributes has been entered.

The components of the MS Excel spreadsheet which have been used to create solutions that verify if the above conditions are met include data validation, name manager and conditional formatting. Some standard built-in function have also been applied, including: OFFSET(), COUNTA(), COUNTIF IF(); CONCATENATE(), LEFT(), SEARCH(), MID(), LEN(), COUNTIFS(), SUMIF(), MATCH() and INDEX(). The data validation solutions discussed above make it significantly easier to use the tool analysed in the paper.

PŘEKRAČUJEME HRANICE
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ARKUSZ KALKULACYJNY WE WSPOMAGANIU ANALZY NIEZGODOŚCI – INTEGRALNOŚĆ DANYCH W NARZĘDZIU

Abstract: Przedstawione w artykule narzędzie stanowi przykład narzędzia stworzonego zgodnie z autorską koncepcją Mikronarzędzi Bazujących na Relacyjnym Modelu Danych do wspomagania procesu analizy niezgodności wyrobów walcowanych. W opracowaniu w pierwszej kolejności omówiono zastosowaną w narzędziu strukturę danych. Zidentyfikowane encje, ich atrybuty, oraz występujące pomiędzy nimi powiązania przedstawiono za pomocą odpowiedniego diagramu związków encji. Przedstawiono także sposób implementacji struktury danych w narzędziu w arkuszu kalkulacyjnym. W dalszej części publikacji opisano rozwiązania zastosowane w celu zapewnienia integralności przechowywanych w narzędziu danych. Określono warunki, jakie dane te muszą spełniać oraz opisano mechanizmy stworzone w celu ich weryfikacji. Wszystkie zaproponowane rozwiązania zostały stworzone przy wykorzystaniu wbudowanych funkcji i elementów arkusza kalkulacyjnego bez konieczności tworzenia kodu w jakimkolwiek języku programowania.

Słowa kluczowe: procesy informacyjne, doskonalenie procesów, relacyjny model danych, MiRel

Globally variational forms on the Möbius strip: Examples

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Abstract: Examples of globally variational source forms (differential equations) defined on the open Möbius strip are studied by means of the Vainberg–Tonti construction. Our choice of the underlying space follows from the variational sequence theory on fibered manifolds, which guarantees global variationality over topological spaces with trivial the second de Rham cohomology group.

Keywords: Möbius strip, Helmholtz conditions, Vainberg–Tonti Lagrangian, variational sequence, global variationality.

1 Introduction

In this paper we study simple examples of variational differential forms on the open Möbius strip, a representative of smooth manifolds possessing *trivial* the second de Rham cohomology group. This topological property of the underlying space assures that locally variational forms are automatically *globally variational,* which is the important result of the *variational sequence* theory over fibered manifolds, the main tool for study the local and global properties of the Euler-Lagrange mapping in the calculus of variations (cf. Krupka [5,7], Takens [11], Urban and Krupka [14], Volná and Urban [16]). Although the existence of a global variational principle is guaranteed by the theory, there is no general construction of a *global Lagrangian* for given differential equations (source forms), defined on this class of underlying manifolds. In the concrete examples, we applied the Vainberg–Tonti construction and obtained the corresponding globally defined Lagrange functions. Nevertheless, the general theory requires further research.

Basic concepts of the geometric theory of second-order variational differential equations are recalled in a slightly simplified setting. For the general theory of global variational principles on fibered manifolds we refer to Krupka [7], and references therein; see also Anderson and Duchamp [1], Brajerčík and Krupka [2], Krupka, Urban, and Volná [8], Krupková and Prince [9].

Throughout, Y denotes a fibered manifold with base X and projection π. The r-jet prolongation of Y is denoted by J^rY, and $\pi^r : J^rY \to X$, $\pi^{r,0} : J^rY \to Y$ are the canonical jet projections. For an open subset $W \subset Y$, we put $W^r = (\pi^{r,0})^{-1}(W)$. The ring of functions on W^r is denoted by $\Omega_0^r W$, and the $\Omega_0^r W$-module of differential k-forms on W^r is denoted by $\Omega_k^r W$. $M_{r,a}$ denotes the open Möbius strip (that is, without boundary) of radius r and width $2a$, where $0 < a < r$.

2 Variational equations and the Vainberg–Tonti Lagrangian

Let W be an open subset of a fibered manifold Y over 1-dimensional base X ("fibered mechanics"). Consider a *source form* $\varepsilon \in \Omega_{2,Y}^2 W$ (also called a *dynamical form* in Lagrangian mechanics), which is by definition a 1-contact, $\pi^{2,0}$-horizontal 2-form, defined on an open subset $W^2 \subset J^2 Y$. In a fibered chart (V, ψ), $\psi = (t, x^i)$, ε is expressed by

$$\varepsilon = \varepsilon_i \omega^i \wedge dt, \tag{2.1}$$

where

$$\omega^i = dx^i - \dot{x}^i dt \tag{2.2}$$

are *contact* 1-forms on V^1, and the coefficients $\varepsilon_i = \varepsilon_i(t, x^j, \dot{x}^j, \ddot{x}^j)$ are real-valued functions on V^2. Every *Lagrangian* $\lambda \in \Omega_{1,X}^1 W$, by definition a π^1-horizontal 1-form on $W^1 \subset J^1 Y$, induces a source form E_λ, expressed in a fibered chart (V, ψ), $\psi = (t, x^i)$, by

$$E_\lambda = \mathscr{E}_i(L) \omega^i \wedge dt,$$

where $\lambda = L dt$, and

$$\mathscr{E}_i(L) = \frac{\partial L}{\partial x^i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}^i} = \frac{\partial L}{\partial x^i} - \frac{\partial^2 L}{\partial t \partial \dot{x}^i} - \frac{\partial^2 L}{\partial x^j \partial \dot{x}^i}\dot{x}^j - \frac{\partial^2 L}{\partial \dot{x}^j \partial \dot{x}^i}\ddot{x}^j$$

are the *Euler–Lagrange expressions* associated to L.

ε is called *locally variational*, if there is a family of Lagrangians $(\lambda_\iota)_{\iota \in I}$, $\lambda_\iota \in \Omega_{1,X}^1 V_\iota$, defined on an open covering $(V_\iota)_{\iota \in I}$ of Y such that

$$\varepsilon|_{V_\iota} = E_{\lambda_\iota}. \tag{2.3}$$

In a fibered chart (V, ψ), $\psi = (t, x^i)$, a Lagrangian has an expression $\lambda = L dt$, and condition (2.3) means that the coefficients ε_i of ε coincide with the Euler–Lagrange expressions of a Lagrange function $L = L(t, x^j, \dot{x}^j)$, that is

$$\varepsilon_i = \mathscr{E}_i(L).$$

ε is called *globally variational* (or simply *variational*), if there exists a Lagrangian $\lambda \in \Omega_{1,X}^1 W$ such that $\varepsilon = E_\lambda$.

Remark 1. Clearly, this concept of (local) variationality transfers to systems of m second-order ordinary differential equations. In a chart (V, ψ), $\psi = (t, x^i)$, we have a system

$$\varepsilon_i(t, x^j, \dot{x}^j, \ddot{x}^j) = 0, \tag{2.4}$$

where $i, j = 1, 2, \dots, m$ (the number of equations and dependent variables are equal). Solutions of (2.4) are differentiable mappings γ defined on an open interval in $\mathbb{R}$ with values in $\mathbb{R}^m$, $\zeta(t) = (x^1\zeta(t), x^2\zeta(t), \dots, x^m\zeta(t))$, which satisfy (2.4). System (2.4) is called *locally variational*, if (2.4) coincides with the Euler–Lagrange equations for some Lagrange functions $L = L(t, x^j, \dot{x}^j)$.

Remark 2. For the purpose of this paper, we shall work with a Cartesian product $Y = \mathbb{R} \times M$ fibered over $\mathbb{R}$ (endowed with a canonical global coordinate), where M is a submanifold of a Euclidean space, the *Möbius strip.* In case of this trivial fibration, the notion of globally variational source form reduces to existence of a globally defined Lagrange function on the corresponding underlying set.

The following theorem describes necessary and sufficient conditions for ε (2.1) to be locally variational.

Theorem 3 (Helmholtz conditions)**.** *Let* (V, ψ), $\psi = (t, x^i)$, *be a fibered chart on* $W \subset Y$, *and* $\varepsilon \in \Omega^2_{2,Y} W$ *be a source form with the expression* (2.1). *The following two conditions are equivalent:*

(a) ε *is locally variational.*

(b) *The functions* ε_i *satisfy the system*

$$\frac{\partial \varepsilon_i}{\partial \ddot{x}^j} - \frac{\partial \varepsilon_j}{\partial \ddot{x}^i} = 0, \tag{2.5}$$

$$\frac{\partial \varepsilon_i}{\partial \dot{x}^j} + \frac{\partial \varepsilon_j}{\partial \dot{x}^i} - \frac{d}{dt}\left(\frac{\partial \varepsilon_i}{\partial \ddot{x}^j} + \frac{\partial \varepsilon_j}{\partial \ddot{x}^i}\right) = 0, \tag{2.6}$$

$$\frac{\partial \varepsilon_i}{\partial x^j} - \frac{\partial \varepsilon_j}{\partial x^i} - \frac{1}{2}\frac{d}{dt}\left(\frac{\partial \varepsilon_i}{\partial \dot{x}^j} - \frac{\partial \varepsilon_j}{\partial \dot{x}^i}\right) = 0. \tag{2.7}$$

Proof. The Helmholtz conditions (2.5)–(2.7) were obtained by von Helmholtz [13]; for the proof see e.g. Havas [3]. Generalized conditions for higher-order partial differential equations can be found in Krupka [4]. □

Remark 4. It is straightforward that conditions (2.5) and (2.6) imply linearity of ε_i in the second derivatives, i.e. $\varepsilon_i = A_i + B_{ij}\ddot{x}^j$, and the property $B_{ij} = \partial C_i / \partial \dot{x}^j = \partial C_j / \partial \dot{x}^i = B_{ji}$ for some functions $C_i = C_i\,(t, x^j, \dot{x}^j)$. Hence the Helmholtz conditions (2.5)–(2.7) for ε_i can be equivalently reformulated for first-order functions A_i, B_{ij} (cf. Sarlet [10]).

Another standard result is a construction of a Lagrangian for locally variational source form.

Theorem 5 (Vainberg–Tonti)**.** *Let* (V, ψ), $\psi = (t, x^i)$, *be a fibered chart on* $W \subset Y$ *such that* $\psi(V)$ *is star-shaped, and* $\varepsilon \in \Omega^2_{2,Y} W$ *be a source form with the expression* (2.1). *If* ε *is locally variational, then* $\varepsilon|_V = E_\lambda$, *where* $\lambda \in \Omega^2_{1,X} V$, $\lambda = Ldt$, *and*

$$L\left(t, x^i, \dot{x}^i, \ddot{x}^i\right) = x^i \int_0^1 \varepsilon_i\left(t, sx^i, s\dot{x}^i, s\ddot{x}^i\right) ds. \tag{2.8}$$

Proof. We refer to Tonti [12]; see also Krupka [6]. □

Remark 6. Note that in the context of Theorem 5, the *Vainberg–Tonti Lagrangian* $\lambda \in \Omega^2_{1,X} V$, given by (2.8), can always be reduced to a *first-order* Lagrangian by means of deleting some total derivative terms.

3 Variational sequence theory in fibered mechanics

We now very briefly recall a sheaf-theoretic concept in the variational calculus on fibered manifolds, the variational sequence theory and its consequences for global variationality; our main reference is Krupka [5], see also Urban and Krupka [14]. The construction can be described rather simply: the de Rham sequence of differential forms on the corresponding underlying manifold is factored through its contact subsequence. It turns out, in particular, that one of the quotient morphisms coincides with the Euler-Lagrange mapping, assigning to a Lagrangian its Euler–Lagrange form. The quotient sheaf sequence, the *variational sequence*, then can be used to study the local and global properties of the Euler-Lagrange mapping. We have the commutative diagram

$$\begin{tikzcd}
 & & & 0 & 0 & 0 & \\
 & & & \Omega_1^r/\Theta_1^r \arrow[u] \arrow[r] & \Omega_2^r/\Theta_2^r \arrow[u] \arrow[r] & \Omega_3^r/\Theta_3^r \arrow[u] \arrow[r] & \cdots \\
0 \arrow[r] & \mathbb{R} \arrow[r] & \Omega_0^r \arrow[r] \arrow[ur] & \Omega_1^r \arrow[u] \arrow[r] & \Omega_2^r \arrow[u] \arrow[r] & \Omega_3^r \arrow[u] \arrow[r] & \cdots \\
 & & 0 \arrow[u] \arrow[r] & \Theta_1^r \arrow[u] \arrow[r] & \Theta_2^r \arrow[u] \arrow[r] & \Theta_3^r \arrow[u] \arrow[r] & \cdots \\
 & & & 0 \arrow[u] & 0 \arrow[u] & 0 \arrow[u] &
\end{tikzcd}$$

Theorem 7. *The variational sequence of order r over Y is an acyclic resolution of the constant sheaf $\mathbb{R}_Y$ over Y.*

Proof. See Krupka [5]. □

From Theorem 7 and the well-known Abstract de Rham theorem (cf. Wells [17]), we get the next result.

Corollary 8. *The cohomology of the complex of global sections of the variational sequence and the de Rham cohomology of Y coincide,*

$$H^k(\Gamma \mathcal{V}^r Y) = H^k_{deR} Y, \quad k \geq 0.$$

The following assertion follows from Corollary 8 and properties of the variational sequence.

Corollary 9. *Suppose ε be a source form on $J^r Y$. If ε is locally variational and the de Rham cohomology group $H^2_{deR} Y$ is trivial, then ε is globally variational.*

4 Smooth atlas adapted to fibered Möbius strip

For the main purpose of this work, the study of examples of globally variational forms on the Möbius strip, we give its smooth manifold structure. Consider the open subset

$$W = \mathbb{R} \times \{\mathbb{R}^3 \backslash \{(0,0,z)\}\}$$

in the Euclidean space $\mathbb{R}^4$, endowed with its open submanifold structure. The global Cartesian coordinates on W are denoted by (t, x, y, z). We introduce an atlas on W adapted to the fibered Möbius strip $\mathbb{R} \times M_{r,a}$ as follows. Let V and $\bar{V}$ be an open covering of W, where

$$V = \mathbb{R} \times \mathbb{R}^3 \backslash \left((-\infty, 0] \times \{0\} \times \mathbb{R} \right), \quad \bar{V} = \mathbb{R} \times \mathbb{R}^3 \backslash \left([0, \infty) \times \{0\} \times \mathbb{R} \right),$$

and define coordinate functions $(t, \varphi, \tau, \kappa)$ on V by $t = t$,

$$\begin{aligned}
\varphi &= \operatorname{atan2}(y, x), \\
\tau &= \frac{1}{\sqrt{2}} \left(\sqrt{x^2 + y^2} - r \right) \sqrt{1 + \frac{x}{\sqrt{x^2 + y^2}}} + \frac{1}{\sqrt{2}} \operatorname{sgn}(y)\, z \sqrt{1 - \frac{x}{\sqrt{x^2 + y^2}}}, \\
\kappa &= -\frac{1}{\sqrt{2}} \left(\sqrt{x^2 + y^2} - r \right) \operatorname{sgn}(y) \sqrt{1 - \frac{x}{\sqrt{x^2 + y^2}}} + \frac{1}{\sqrt{2}} z \sqrt{1 + \frac{x}{\sqrt{x^2 + y^2}}},
\end{aligned}$$

and $(\bar{t}, \bar{\varphi}, \bar{\tau}, \bar{\kappa})$ on $\bar{V}$ by $\bar{t} = t$, $\bar{\tau} = -\tau$, $\bar{\kappa} = -\kappa$, and

$$\bar{\varphi} = \begin{cases} \operatorname{atan2}(y, x), & y \geq 0, \\ \operatorname{atan2}(y, x) + 2\pi, & y < 0, \end{cases}$$

where $\operatorname{atan2}(y, x)$ is the arctangent function with two arguments.

It is easy to check that the pairs (V, Ψ), $\Psi = (t, \varphi, \tau, \kappa)$, and $(\bar{V}, \bar{\Psi})$, $\bar{\Psi} = (\bar{t}, \bar{\varphi}, \bar{\tau}, \bar{\kappa})$, are charts on W *adapted* to $\mathbb{R} \times M_{r,a}$, which form a smooth atlas on W (see Urban and Volná [15]). On the intersection $V \cap \bar{V} = \mathbb{R} \times \mathbb{R}^3 \setminus (\mathbb{R} \times \{0\} \times \mathbb{R})$, the chart transformations between (V, Ψ) and $(\bar{V}, \bar{\Psi})$ are expressed by

$$\Psi \circ \bar{\Psi}^{-1} : \bar{\Psi}(\bar{V}) \backslash \{\bar{\varphi} = \pi\} \to \Psi(V) \backslash \{\varphi = 0\},$$

$$\Psi \circ \bar{\Psi}^{-1}(\bar{t}, \bar{\varphi}, \bar{\tau}, \bar{\kappa}) = \begin{cases} (\bar{t}, \bar{\varphi}, \bar{\tau}, \bar{\kappa}), & \bar{\varphi} \in (0, \pi), \\ (\bar{t}, \bar{\varphi} - 2\pi, -\bar{\tau}, -\bar{\kappa}), & \bar{\varphi} \in (\pi, 2\pi), \end{cases} \tag{4.1}$$

and

$$\bar{\Psi} \circ \Psi^{-1} : \Psi(V) \backslash \{\varphi = 0\} \to \bar{\Psi}(\bar{V}) \backslash \{\bar{\varphi} = \pi\},$$

$$\bar{\Psi} \circ \Psi^{-1}(t, \varphi, \tau, \kappa) = \begin{cases} (t, \varphi, \tau, \kappa), & \varphi \in (0, \pi), \\ (t, \varphi + 2\pi, -\tau, -\kappa), & \varphi \in (-\pi, 0). \end{cases} \tag{4.2}$$

Note that in the chart (V, Ψ) (resp. $(\bar{V}, \bar{\Psi})$), $\mathbb{R} \times M_{r,a}$ has the equation $\kappa = 0$ with $-a < \tau < a$ (resp. $\bar{\kappa} = 0$ with $-a < \bar{\tau} < a$). The associated smooth atlas on $\mathbb{R} \times M_{r,a}$ is defined by the charts $(V \cap (\mathbb{R} \times M_{r,a}), \Psi|_{V \cap (\mathbb{R} \times M_{r,a})})$ and $(\bar{V} \cap (\mathbb{R} \times M_{r,a}), \bar{\Psi}|_{\bar{V} \cap (\mathbb{R} \times M_{r,a})})$, and we denote the associated coordinates by the same letters as $\Psi = (t, \varphi, \tau)$ and $\bar{\Psi} = (\bar{t}, \bar{\varphi}, \bar{\tau})$, if no misunderstanding may arise.

5 Globally variational forms: Examples

We now apply the variational sequence theory over fibered manifold $Y = \mathbb{R} \times M_{r,a}$. Since

$$H^2_{deR} M_{r,a} = 0, \tag{5.1}$$

Corollary 9 implies that every locally variational source form ε on $J^2(\mathbb{R}\times M_{r,a})$ is also *globally* variational. In order words, condition (5.1) assures existence of a Lagrange function defined on $J^1(\mathbb{R}\times M_{r,a})$, for which the corresponding Euler–Lagrange expressions coincide with ε (cf. Remark 2). We give examples of source forms on $J^2(\mathbb{R}\times M_{r,a})$, illustrating this sheaf-theoretic result.

5.1 The kinetic Lagrangian

Consider the canonical embedding $\iota:\mathbb{R}\times M_{r,a}\to\mathbb{R}\times\mathbb{R}^3$ and its jet prolongations $J^r\iota: J^r(\mathbb{R}\times M_{r,a})\to J^r(\mathbb{R}\times\mathbb{R}^3)$. Denote by (t,x,y,z) the canonical coordinates on $\mathbb{R}\times\mathbb{R}^3$, and by $(t,x,y,z,\dot{x},\dot{y},\dot{z},\ddot{x},\ddot{y},\ddot{z})$ the associated coordinates on $J^2(\mathbb{R}\times\mathbb{R}^3)$. The source form

$$\varepsilon=\varepsilon_x\omega^x\wedge dt+\varepsilon_y\omega^y\wedge dt+\varepsilon_z\omega^z\wedge dt,$$

where ω^x, ω^y, and ω^z are contact 1-forms (2.2), and $\varepsilon_x=-\ddot{x}$, $\varepsilon_y=-\ddot{y}$, $\varepsilon_z=-\ddot{z}$, is variational and possesses a global Lagrangian, the *kinetic energy Lagrangian* $\lambda=L_{kin}dt$ on $J^1(\mathbb{R}\times\mathbb{R}^3)$, where

$$L_{kin}=\frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right). \tag{5.2}$$

The induced source form $J^2\iota^*\varepsilon$ on $J^2(\mathbb{R}\times M_{r,a})$ is globally variational. Indeed, if $J^2\iota^*\varepsilon$ is expressed in the chart (V,Ψ), $\Psi=(t,\varphi,\tau,\kappa)$, as introduced in Sec. 4, we have $J^2\iota^*\varepsilon|_V=\varepsilon_\varphi\omega^\varphi\wedge dt+\varepsilon_\tau\omega^\tau\wedge dt$, where

$$\begin{aligned}\varepsilon_\varphi&=\frac{1}{2}\dot{\varphi}^2\tau\sin\frac{\varphi}{2}\left(r+\tau\cos\frac{\varphi}{2}\right)-\frac{1}{2}\dot{\varphi}\dot{\tau}\left(4\cos\frac{\varphi}{2}\left(r+\tau\cos\frac{\varphi}{2}\right)+\tau\right)\\&\quad-\left(\left(r+\tau\cos\frac{\varphi}{2}\right)^2+\frac{\tau^2}{4}\right)\ddot{\varphi},\\\varepsilon_\tau&=\frac{1}{4}\dot{\varphi}^2\left(4\cos\frac{\varphi}{2}\left(r+\tau\cos\frac{\varphi}{2}\right)+\tau\right)-\ddot{\tau}.\end{aligned}$$

The Vainberg–Tonti Lagrangian (2.8) associated with $J^2\iota^*\varepsilon|_V$ is of second order, and it can be reduced to the first-order Lagrangian, which coincides with $J^1\iota^*\lambda=(L_{kin}\circ J^1\iota)\,dt$ on $J^1(\mathbb{R}\times M_{r,a})$, where

$$L_{kin}\circ J^1\iota\,(\varphi,\tau,\dot{\varphi},\dot{\tau})=\frac{1}{2}\left(\dot{\tau}^2+\left(\left(r+\tau\cos\frac{\varphi}{2}\right)^2+\frac{\tau^2}{4}\right)\dot{\varphi}^2\right). \tag{5.3}$$

Using the chart transformations (4.1), (4.2), it is also easy to verify that formula (5.3) defines a global function on $J^1(\mathbb{R}\times M_{r,a})$.

5.2 The Vainberg–Tonti Lagrangian need not be global

We give another simple example of a globally defined source form ε on $J^1(\mathbb{R}\times M_{r,a})$, which is locally hence also globally variational. But contrary to the previous example, it shows that the direct use of the Vainberg–Tonti construction does *not* lead to the global Lagrangian.

Let ε be a source form defined on $J^1(\mathbb{R}\times M_{r,a})$ such that

$$\varepsilon|_V=\varepsilon_\varphi\eta^\varphi\wedge dt+\varepsilon_\tau\eta^\tau\wedge dt,\quad \varepsilon|_{\bar{V}}=\varepsilon_{\bar{\varphi}}\eta^{\bar{\varphi}}\wedge d\bar{t}+\varepsilon_{\bar{\tau}}\eta^{\bar{\tau}}\wedge d\bar{t},$$

where $\varepsilon_\varphi=1=\varepsilon_{\bar{\varphi}}$, $\varepsilon_\tau=0=\varepsilon_{\bar{\tau}}$, and $\eta^\varphi=d\varphi-\dot{\varphi}dt$, $\eta^\tau=d\tau-\dot{\tau}dt$, $\eta^{\bar{\varphi}}=d\bar{\varphi}-\dot{\bar{\varphi}}dt$, $\eta^{\bar{\tau}}=d\bar{\tau}-\dot{\bar{\tau}}dt$. Clearly, ε is locally variational (cf. Theorem 3). The Vainberg–Tonti Lagrangian

(2.8) associated with $\varepsilon|_V$, resp. $\varepsilon|_{\bar{V}}$, reads $L = \varphi$, $\varphi \in (-\pi, \pi)$, resp. $\bar{L} = \bar{\varphi}$, $\bar{\varphi} \in (0, 2\pi)$. These local Lagrange functions, however, do *not* define a global Lagrange function for given ε. Nevertheless, ε is globally variational, and it possesses a global Lagrange function, defined by

$$\mathscr{L}\left(t, \varphi, \dot{\varphi}\right) = \begin{cases} \varphi + \pi\left(1 + \cos\varphi\right) - \pi t \dot{\varphi} \sin\varphi, & \varphi \in (0, \pi), \\ \varphi + 2\pi, & \varphi \in (-\pi, 0], \end{cases}$$

and

$$\bar{\mathscr{L}}\left(\bar{t}, \bar{\varphi}, \dot{\bar{\varphi}}\right) = \begin{cases} \bar{\varphi} + \pi\left(1 + \cos\bar{\varphi}\right) - \pi \bar{t} \dot{\bar{\varphi}} \sin\bar{\varphi}, & \bar{\varphi} \in (0, \pi), \\ \bar{\varphi}, & \bar{\varphi} \in [\pi, 2\pi). \end{cases}$$

Clearly, $\bar{\mathscr{L}} \circ \bar{\Psi} \circ \Psi^{-1} = \mathscr{L}$ on $V \cap \bar{V}$.

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Globálně variační formy na Möbiově pásce: Příklady

Abstrakt: S využitím Vainberg–Tontiho konstrukce jsou studovány příklady globálně variačních zdrojových forem (diferenciálních rovnic) na otevřené Möbiově pásce. Podkladová varieta byla zvolena s ohledem na teorii variační posloupnosti, která zaručuje globální variačnost na topologických prostorech s triviální druhou de Rhamovou kohomologickou grupou.

Klíčová slova: Möbiova páska, Helmholtzovy podmínky, Vainberg–Tonti lagrangián,variační posloupnost, globální variačnost.

Numerical Approaches for Beams on Nonlinear Foundation - Part 1 (Theory)

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Abstract: Our work presents theory and numerical approaches suitable for the solutions of straight plane beams rested on elastic foundations (i.e. nonlinear modified bilateral and unilateral Winkler's models). The nonlinear boundary value problems of 4th-order are solved via finite element method with semi-smooth Newton's method (which discretize the weak formulation of the problem) and central difference method with classical Newton's method (which discretize directly the differential equation). Reaction forces in foundation are defined via nonlinear dependencies based on previous experiments.

Keywords: unilateral and bilateral elastic foundation, nonlinear foundation, beam, Finite Element Method, semi-smooth Newton's method, Central Difference Method.

1 Introduction

There are beams on elastic foundations which are frequently used in the engineering practice; for example see Fig. 1 and 2 and references [2], [3] and [4]. The first theory for the bending of beams on an elastic foundation was proposed by E. Winkler in the Prague in 1867; see [9]. The basic analysis of the bending of beams on an elastic foundation is based on the first assumption that the strains (i.e. deformations) are small.

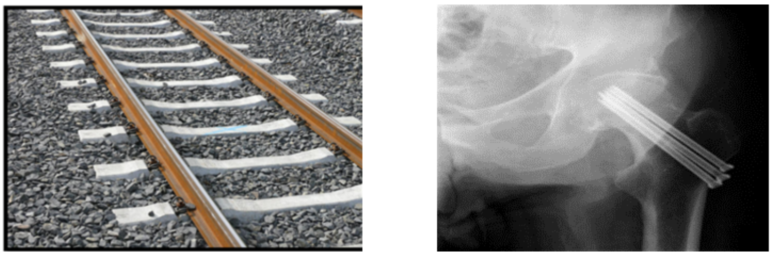

Figure 1: *Examples of beams on elastic foundations (a) Railroads (b) Femoral screws in femur - rtg snapshot.*

In classical problems of engineering/mechanics, the deflection $v = v(x)$ [m] of the straight beam without any volume loads is described by linear/nonlinear differential equation

$$EJ_{ZT}\frac{d^4 v}{dx^4} + q_R = 0,$$

where E [Pa] is the modulus of elasticity of the beam, $J_{ZT} = \int_A y^2 \, \mathrm{d}A \, [\mathrm{m}^4]$ is the major principal second moment of the beam cross-section $A \, [\mathrm{m}^2]$ and $q_R = q_R(x, v, \ldots) \, [\mathrm{Nm}^{-1}]$ corresponds to the linear/nonlinear reaction of the foundation; see Fig. 2. The beam is loaded by force F [N].

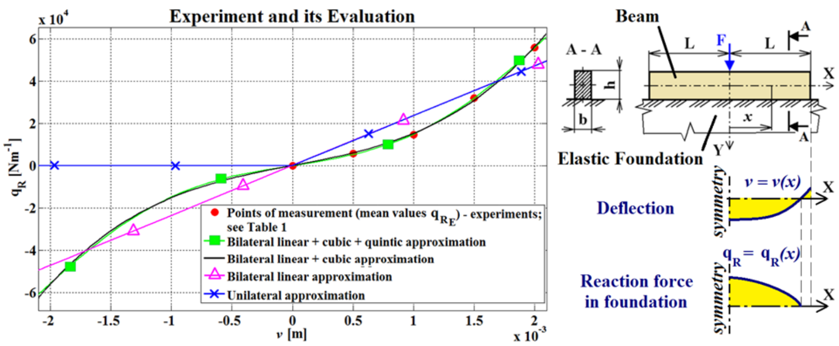

Figure 2: *(a) Dependence of reaction force on deflection (i.e. foundation load-settlement behaviour for a sand, experiment and suitable linear and nonlinear approximations) (b) Beam with cross-section $b \times h$ and length* 2L *is resting on elastic unilateral and bilateral foundation, see [2], [3], [6].*

Our work focuses on the numerical approaches for the solution of straight plane (2D) beams of length 2L on an elastic foundation with nonlinear unilateral or bilateral behaviour (linear Bernouli's beam, small deformations in the beam, Finite element Method, Central Difference Method); see Fig. 2. The methodology of the elastic foundation measuring applied in this paper is based on the pressing of a beam into the foundation; see Fig. 2, Tab. 1 and references [2], [3] and [5]. Hence, in this article, the theory and numerical nonlinear approaches are explained.

Table 1: Elastic foundation - Experiments and their evaluation

Description:	**Expression; see Fig. 2 (a)**	
Experiments	q_{R_E}	measurements (mean values)
Bilateral linear	$q_{R_1} = k_1 v = 2.3587 \times 10^7 v$	$\frac{d^4v}{dx^4} + \frac{k_1 v}{EJ_{ZT}} = 0$
Bilateral linear + cubic	$q_{R_{1,3}} = k_1 v + k_3 v^3 = 1.094 \times 10^7 v + 4.2869 \times 10^{12} v^3$	$\frac{d^4v}{dx^4} + \frac{k_1 v + k_3 v^3}{EJ_{ZT}} = 0$
Bilateral linear + cubic + quintic	$q_{R_{1,3,5}} = k_1 v + k_3 v^3 + k_5 v^5 = 8.8597 \times 10^6 v + 6.4373 \times 10^{12} v^3 - 4.1846 \times 10^{17} v^5$	$\frac{d^4v}{dx^4} + \frac{k_1 v + k_3 v^3 + k_5 v^5}{EJ_{ZT}} = 0$
Unilateral	$q_{\lvert R_1 \rvert} = \frac{k_1}{2}(v + \lvert v \rvert) = \frac{2.3587 \times 10^7}{2}(v + \lvert v \rvert)$	$\frac{d^4v}{dx^4} + \frac{k_1}{2EJ_{ZT}}(v + \lvert v \rvert) = 0$

2 Finite Element Method (FEM) Approach to Unilateral Elastic Foundation

Let us suppose that the solved beam has symmetry. Therefore it is sufficient to solve the differential equation for a half of the beam, i.e. $x \in \langle 0; \mathrm{L} \rangle$.

Hence, the deflection of the beam is described by the equation

$$EJ_{ZT}\frac{d^4 v}{dx^4} + kv^+ = 0 \qquad \text{on} \quad x \in (0, \mathrm{L})$$

with following boundary conditions prescribed in points $x = 0$ and $x = \mathrm{L}$

$$\begin{aligned} \frac{dv(x=0)}{dx} &= 0, \qquad & M_o(x = \mathrm{L}) &= 0, \\ T(x=0) &= \frac{\mathrm{F}}{2}, \qquad & T(x = \mathrm{L}) &= 0. \end{aligned} \tag{1}$$

2.1 Weak Formulation and FEM

Lets denote V as the space of virtual displacements then $V = \left\{ w \in H^2\left((0, \mathrm{L})\right) : \frac{dw(x=0)}{dx} = 0 \right\}$.
The weak formulation of the beam defection on the unilateral foundation is following

$$\begin{gathered} \text{find } v \in V \text{ such that} \\ EJ_{ZT}\int_0^{\mathrm{L}} \frac{d^2 v}{dx^2}\frac{d^2 w}{dx^2}\, dx + k\int_0^{\mathrm{L}} v^+ w\, dx = \frac{\mathrm{F}}{2}w(0) \text{ is fulfilled for all } w \in V. \end{gathered} \tag{2}$$

For the sake of solvability of (2) the prescribed external force F must be positive. See (Sysala 2008) for details.

Lets divide the interval $(0, \mathrm{L})$ into n parts of the same length. This equidistant discretization with nodes $x_1 = 0, x_{i+1} = x_i + h$ has the constant step $h = \mathrm{L}/n$.

The discrete approximation of the space V is subspace V_h such that

$$V_h = \left\{ v_h \in C^1\big((0, \mathrm{L})\big) : \ v_h|_{\langle x_i, x_{i+1}\rangle} \in P_3, \ \frac{dv_h(0)}{dx} = 0 \right\}.$$

The discrete form of (2) is following

$$\begin{gathered} \text{find } v_h \in V_h \text{ such that} \\ EJ_{ZT}\int_0^{\mathrm{L}} \frac{d^2 v_h}{dx^2}\frac{d^2 \varphi_i}{dx^2}\, dx + k\int_0^{\mathrm{L}} v_h^+ \varphi_i\, dx = \frac{\mathrm{F}}{2}\varphi_i(0) \text{ for all } i = 1, \dots, 2n+2, \end{gathered} \tag{3}$$

where φ_i, $i = 1, \dots 2n+2$ are piecewise-cubic smooth functions, the base function of space V_h. Because the solution v_h of (3) is element of the space V_h, we can write

$$v_h = \sum_{i=1}^{2n+2} u_i \varphi_i(x). \tag{4}$$

And we will denote the vector $\boldsymbol{u}$

$$\boldsymbol{u} = \Big(\underbrace{v_h(x_1)}_{u_1}, \underbrace{\frac{dv_h(x_1)}{dx}}_{u_2=0}, \underbrace{v_h(x_2)}_{u_3}, \underbrace{\frac{dv_h(x_2)}{dx}}_{u_4}, \dots, \underbrace{v_h(x_{n+1})}_{u_{2n+1}}, \underbrace{\frac{dv_h(x_{n+1})}{dx}}_{u_{2n+2}} \Big)^{\top}.$$

The algebraic FEM representation of the first integral in (3) and the right side of (3) can be set by a standart way. The global stiffness matrix $\boldsymbol{K}$ and the global load vector $\boldsymbol{f}$ corresponding to (3) are shown. ($h = \mathrm{L}/n$ constant).

$$\boldsymbol{K} = \frac{1}{h^3}\begin{pmatrix} 12 & 0 & -12 & 6h & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & h^3 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ -12 & 0 & 24 & 0 & -12 & 6h & \dots & 0 & 0 & 0 & 0 \\ 6h & 0 & 0 & 8h^2 & -6h & 2h^2 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & -12 & -6h & 24 & 0 & \ddots & -12 & 6h & 0 & 0 \\ 0 & 0 & 6h & 2h^2 & 0 & 8h^2 & \ddots & -6h & 2h^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -12 & -6h & \ddots & 24 & 0 & -12 & 6h \\ 0 & 0 & 0 & 0 & 6h & 2h^2 & \ddots & 0 & 8h^2 & -6h & 2h^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & -12 & -6h & 12 & -6h \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 6h & 2h^2 & -6h & 4h^2 \end{pmatrix} \qquad \boldsymbol{f} = \begin{pmatrix} \frac{\mathrm{F}}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

2.2 Semi-Smooth Newton's Method and Numerical Algorithm

We will present here one way how to find a numerical algorithm to the problem (3), which was present in [7] for the case of the thin annular plate. Notice, there is the nonlinear expression

$$v_h^+ = (v_h(x))^+ \;=\; \frac{1}{2}(|v_h(x)| + v_h(x)) \;=\; \frac{1}{2}\left(\left|\sum_{i=1}^{2n+2} u_i\varphi_i(x)\right| + \sum_{i=1}^{2n+2} u_i\varphi_i(x)\right)$$

in the second integral on the left side of the equation. We deal with it in two steps.

The 1st step. We use the well-known trapezoidal rule[1] for approximation of the integral $\int_0^{\mathrm{L}} v_h^+\varphi_i\,dx$ from the left side of the equation in (3). The main reason is that we get the following approximation

$$\int_0^{\mathrm{L}} v_h^+\varphi_i\,dx \approx \begin{cases} \frac{1}{2}\,h\,u_1^+\,, & \text{if } i = 1\,, \\ h\,u_i^+\,, & \text{if } i \text{ is odd, } i \neq 1, i \neq 2n+1\,, \\ \frac{1}{2}\,h\,u_{2n+1}^+\,, & \text{if } i = 2n+1\,, \\ 0\,, & \text{if } i \text{ is even,} \end{cases}$$

which moves the non-linearity $(\,\cdot\,)^+$ from the function v_h from (4) to its finite element components $u_i \in \mathbb{R}$ and therefore the evaluation is easy in any numerical algorithm. Now we get the homogenous equation

$$G(u) = 0 \tag{5}$$

instead of (3) for the non-linear mapping

$$G(u) = EJ_{ZT}\boldsymbol{K}\boldsymbol{u} + k\boldsymbol{B}\boldsymbol{u}^+ - \boldsymbol{f}\,, \tag{6}$$

where the matrix $\boldsymbol{K}$ and the vector $\boldsymbol{f}$ are from the finite element method mentioned above and the matrix $\boldsymbol{B}$ is diagonal, $\boldsymbol{B} = \mathrm{diag}(h/2, 0, h, 0, h, 0\ \dots\ , h, 0, h/2, 0)$.

[1]Exactly, we use the trapezoidal rule with the same grid as is used in the finite element method, see above.

The 2nd step. Because we do not have available any derivative of (6) due to the absolute value in $\boldsymbol{u}^+$, . For this reason, the second step to find the numerical algorithm is the usage of more suitable semi-smooth Newton's method, see [1], which introduces so called slanting function G^o and use it instead of Jacobian in the standard Newton's iterations. We define

$$\boldsymbol{G}^o(u) = EJ_{ZT}\boldsymbol{K} + k\boldsymbol{B}\mathrm{diag}(A(\boldsymbol{u}^+))$$

in our case, where the symbol $(A(\boldsymbol{u}^+))$ stands for the active set of indexes of such nods x_i, in which the elastic beam foundation is active. The resulting iterative equation in the $(k+1)$-th step of the semi-smooth Newton's method is

$$\boldsymbol{u}^{(k+1)} = \boldsymbol{u}^{(k)} - \boldsymbol{G}^o\left(\boldsymbol{u}^{(k)}\right)^{-1} G(\boldsymbol{u}^{(k)})$$

for known solution $\boldsymbol{u}^{(k)}$ from the previous step. This iteration process converges for sufficiently small distance between the initial estimate $\boldsymbol{u}^{(0)}$ and the exact solution of the equation (5), for details see [1].

There must be defined the suitable starting estimate $\boldsymbol{u}^{(0)}$ in our computational algorithm. We use the result deflection of the beam without any foundation. This deflection is solution of equation (3) without the part with v_h^+.

The computational process of the algorithm is repeated until at least one termination condition has been reached:

- either the solution $\boldsymbol{u}^{(k+1)}$ satisfies the criteria of sufficiently small relative error

$$\frac{\|\boldsymbol{u}^{(k+1)} - \boldsymbol{u}^{(k)}\|}{\|\boldsymbol{u}^{(k)}\|}$$

- or the "exact" solution is reached at which the relative reziduum

$$\frac{\|\boldsymbol{G}^o(\boldsymbol{u}^{(k+1)})\,\boldsymbol{u}^{(k+1)} - \boldsymbol{f}\|}{\left(EJ_{ZT}\,\|\boldsymbol{K}\| + k\,\|\boldsymbol{B}\mathrm{diag}\left(A\left((\boldsymbol{u}^{k+1})^+\right)\right)\|\right)\|\boldsymbol{u}^{(k+1)}\| + \|\boldsymbol{f}\|}$$

  vanishes.

3 Central Difference Method (CDM) Approach to Bilateral Elastic Foundation

According to the theory of CDM, the beam and its surroundings can be divided into $n+5$ nodes "i" with step $\Delta = \frac{L}{n}$; see Fig. 3.

Central differences (CD) at the point "i" can be defined as an approximation of derivatives $v^{(i)} = \frac{d^i v}{dx^i}$. Hence,

$$v^{(1)} \approx \frac{v_{i+1} - v_{i-1}}{2\Delta}, \qquad v^{(2)} \approx \frac{v_{i+1} - 2v_i + v_{i-1}}{\Delta^2},$$
$$v^{(3)} \approx \frac{v_{i+2} - 2v_{i+1} + 2v_{i-1} - v_{i-2}}{2\Delta^3}, \qquad v^{(4)} \approx \frac{v_{i+2} - 4v_{i+1} + 6v_i - 4v_{i-1} + v_{i-2}}{\Delta^4}$$

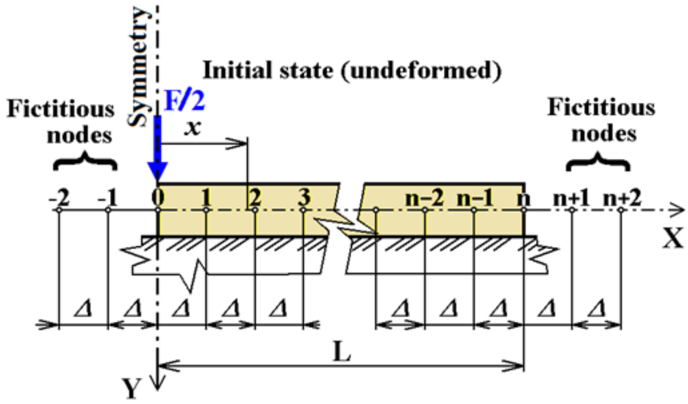

Figure 3: *Beam of length* 2L *resting on an elastic foundation and loaded by force* F *and divisions of the beam (CDM – one half of the beam).*

For example, differential equation

$$EJ_{ZT}\frac{d^4v}{dx^4} + k_1v + k_3v^3 + k_5v^5 = 0$$

can be approximated via CD as

$$v_{i-2} - 4v_{i-1} + (6+a_1)v_i - 4v_{i+1} + v_{i+2} + a_3v_i^3 + a_5v_i^5 = 0 \quad \text{for } i = 0, 1, 2, \ldots, n,$$

where $a_1 = \frac{k_1\Delta^4}{EJ_{ZT}}$, $a_3 = \frac{k_3\Delta^4}{EJ_{ZT}}$, $a_5 = \frac{k_5\Delta^4}{EJ_{ZT}}$, $c = 6 + a_1$ and $b = \frac{\mathrm{F}\Delta^3}{EJ_{ZT}}$. The variables v_{-2}, v_{-1}, v_{n+1} and v_{n+2} (i.e. connection with fictitious nodes) can be expressed from boundary conditions. Hence, $v_{-1} = v_1$, $v_{-2} = v_2 - b$, $v_{n+1} = 2v_n - v_{n-1}$ and $v_{n+2} = 4v_n - 4v_{n-1} + v_{n-2}$. For more information see [2], [3], [8] and [6] . This leads to a system of $n+1$ nonlinear equations

$$\boldsymbol{M}\boldsymbol{v} + a_3\boldsymbol{v}^3 + a_5\boldsymbol{v}^5 - \boldsymbol{b} = \boldsymbol{o}, \tag{7}$$

where

$$\boldsymbol{M} = \begin{pmatrix} c & -8 & 2 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -4 & 7+a_1 & -4 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & -4 & c & -4 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -4 & c & -4 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & 0 \\ 0 & \cdots & 0 & 1 & -4 & c & -4 & 1 & 0 \\ 0 & \cdots & 0 & 0 & 1 & -4 & c & -4 & 1 \\ 0 & \cdots & 0 & 0 & 0 & 1 & -4 & 5+a_1 & -2 \\ 0 & \cdots & 0 & 0 & 0 & 0 & 2 & -4 & 2+a_1 \end{pmatrix},$$

$$\boldsymbol{b} = \begin{pmatrix} b \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \boldsymbol{v} = \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_n \end{pmatrix}, \quad \boldsymbol{v}^3 = \begin{pmatrix} v_0^3 \\ v_1^3 \\ \vdots \\ v_n^3 \end{pmatrix}, \quad \boldsymbol{v}^5 = \begin{pmatrix} v_0^5 \\ v_1^5 \\ \vdots \\ v_n^5 \end{pmatrix}.$$

The nonlinear equation (7) is solved by well-known Newton's method (also known as the Newton–Raphson method) with the following termination condition

$$\|\boldsymbol{v}^{(k+1)} - \boldsymbol{v}^{(k)}\| < \varepsilon_{\text{tol}}\,,$$

which means that the distance of two last iteration solutions $\boldsymbol{v}^{(k+1)}$ and $\boldsymbol{v}^{(k)}$ is sufficiently small.

Conclusion

There is considered the boundary value problems describing the deflection of the straight beam rested on two classes of nonlinear elastic foundations in our paper. The definitions of the reaction forces in the foundations are based on previous experiments described in the previous papers listed in the references. The first class is case of beam on unilateral foundation. We have described the derivation of the finite element method formulation of the problem and then we have suggested the computational algorithm via semi-smooth Newton's method. On the other hand we have used the central difference method in the case of bilateral foundations, which is the second class of nonlinear elastic foundations. And we use the classical Newton's method to solve the resulting equation. Both approaches lead to computational algorithms through which we are able to get the numerical solutions which are comparable with analytical solutions with good results.

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Numerické řešení ohybu nosníku v nelineárním prostředí - část 1 (teorie)

Abstrakt: V článku se zabýváme numerickým řešením úlohy, která popisuje ohyb rovinného nosníku uloženého v různých typech elastického prostředí (t.j. nelineární modifikované bilaterální a unilaterální Winklerova typu). Jsou popsány dva způsoby řešení okrajové úlohy s nelineární diferenciální rovnicí čtvrtého řádu. První je pomocí metody konečných prvků s využitím nehladké Newtonovy metody a druhý je založený na metodě centrálních diferencí a použití klasické Newtonovy metody. Reakční síly v podloží jsou definovány nelineárními zobrazeními, jejichž tvar vychází z předchozích experimentů.

Klíčová slova: jednostranné a oboustranné elastické podloží, nelineární podloží, nosník, metoda konečných prvků, nehladká Newtonova metoda, metoda konečných diferencí.

Numerical Approaches for Beams on Nonlinear Foundation - Part 2 (Applications)

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Abstract: This work presents numerical approaches and applications for the solutions of straight plane beams rested on elastic foundations. There are linear/nonlinear modified bilateral and unilateral Winkler's models (i.e. suitable approximations for dependencies of distributed reaction forces on deflection in the foundation). At first, for solutions of bilateral foundation, the Central Difference Method is applied in combination with the Newton's Method. At second, for solutions of unilateral foundation, the Finite Element Method is applied in combination with the Semi-Smooth Newton's method. The results acquired by linear/nonlinear approaches are evaluated and compared.

Keywords: unilateral and bilateral elastic foundation, nonlinear foundation, beam, Finite Element Method, Semi-Smooth Newton's method, Central Difference Method, programming.

1 Introduction

Beams on elastic foundations are often used in civil, mechanical and miming engineering, biomechanics etc.; see [4], [5], [7], [9]. This work is a continuation of our previous work where the theory is explained, see [9].

Hence, the symmetrical 2D Bernouli's beam of length 2L $= 2 \times 12.045$ m or 2L $= 2 \times 6$ m with cross-section $b \times h$ ($b = 0.2$ m, $h = 0.4$ m) is resting on an elastic foundation, see Fig. 1. The beam is loaded by force F $= 7 \times 10^6$ N. The modulus of elasticity of the beam is $E = 2 \times 10^{11}$ Pa, the principal quadratic moment of the beam cross-section is $J_{ZT} = \frac{bh^3}{12}$ and the differential equation of this problem is $EJ_{ZT}\frac{d^4v}{dx^4}+q_R = 0$, where $q_R = q_R(x, v, \ldots)$ [Nm^{-1}] are the linear/nonlinear descriptions for reactions in the foundation and $v = v(x)$ [m] is deflection (i.e. vertical displacement).

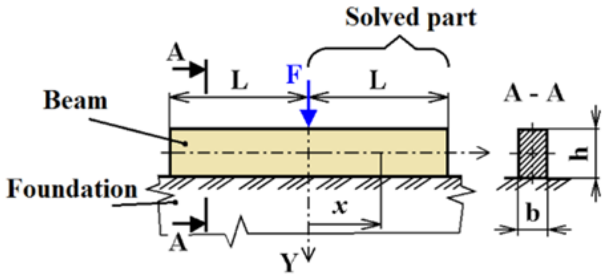

Figure 1: *Solved beam of length 2L resting on an elastic foundation.*

From the boundary conditions prescribed in points $x = 0$ m and $x = \mathrm{L}$ follow equations

$$\begin{aligned} &\frac{dv(x=0)}{dx} = 0 && M_o(x=\mathrm{L}) = 0 \quad \Rightarrow \quad \frac{d^2v(x=\mathrm{L})}{dx^2} = 0, \\ &T(x=0) = \frac{\mathrm{F}}{2} \quad \Rightarrow \quad \frac{d^3v(x=0)}{dx^3} = \frac{\mathrm{F}}{2EJ_{ZT}} && T(x=\mathrm{L}) = 0 \quad \Rightarrow \quad \frac{d^3v(x=\mathrm{L})}{dx^3} = 0. \end{aligned} \tag{1}$$

where $T(x)$ [N] is shearing force and M_0 [Nm] is bending moment.

In practical applications, the Central Difference Method (CDM) in combination with the Newton's Method and the Finite Element Method (FEM) in combination with the Semi-Smooth Newton's Method are applied.

2 Bilateral Elastic Foundation — Application of CDM in Combination with Newton's Method

In this case, the reaction in elastic bilateral foundation is described by three chosen functions q_{R_1}, $q_{R_{1,3}}$ and $q_{R_{1,3,5}}$ (i.e. linear approximation of experiments $\frac{d^4v}{dx^4} + \frac{k_1 v}{EJ_{ZT}} = 0$ with $k_1 = 2.3587 \times 10^7 \mathrm{Nm}^{-2}$, linear + cubic approximation of experiments $\frac{d^4v}{dx^4} + \frac{k_1 v + k_3 v^3}{EJ_{ZT}} = 0$ with $k_1 = 1.094 \times 10^7$ Nm^{-2}, $k_3 = 4.2869 \times 10^{12}$ Nm^{-4}, and (i.e. linear + cubic + quintic approximation of experiments, $\frac{d^4v}{dx^4} + \frac{k_1 v + k_3 v^3 + k_5 v^5}{EJ_{ZT}} = 0$ with $k_1 = 8.8597 \times 10^6$ Nm^{-2}, $k_3 = 6.4373 \times 10^{12}$ Nm^{-4}, $k_5 = -4.1846 \times 10^{17}$ Nm^{-6}); see [9].

According to CDM, the beam and its surroundings can be divided into $n+5$ nodes "i" with step $\Delta = \frac{\mathrm{L}}{n}$ for $x \in (0, \mathrm{L})$. For example, the differential equation $\frac{d^4v}{dx^4} + \frac{k_1 v + k_3 v^3 + k_5 v^5}{EJ_{ZT}} = 0$ can be approximated by CDM as

$$v_{i-2} - 4v_{i-1} + (6 + a_1)v_i - 4v_{i+1} + v_{i+2} + a_3 v_i^3 + a_5 v_i^5 = 0 \quad \text{for } i = 0, 1, 2, \ldots, n \tag{2}$$

where $a_1 = \frac{k_1 \Delta^4}{EJ_{ZT}}$, $a_3 = \frac{k_3 \Delta^4}{EJ_{ZT}}$, $a_5 = \frac{k_5 \Delta^4}{EJ_{ZT}}$, $c = 6 + a_1$ and $b = \frac{\mathrm{F}\Delta^3}{EJ_{ZT}}$, see [1], [2], [9].

From the boundary conditions (1) follow

$$\begin{aligned} &\frac{v_{i+1} - v_{i-1}}{2\Delta} = 0 \quad \Rightarrow \quad v_1 - v_{-1} = 0, \\ &\frac{v_{i+2} - 2v_i + 2v_{i-1} - v_{i-2}}{2\Delta^3} = \frac{\mathrm{F}}{2EJ_{ZT}} \quad \Rightarrow \quad v_2 - 2v_1 + 2v_{-1} - v_{-2} = b, \\ &\frac{v_{i+1} - 2v_i + v_{i-1}}{\Delta^2} = 0 \quad \Rightarrow \quad v_{n+1} - 2v_n + v_{n-1} = 0, \\ &\frac{v_{i+2} - 2v_i + 2v_{i-1} - v_{i-2}}{2\Delta^3} = 0 \quad \Rightarrow \quad v_{n+2} - 2v_{n+1} + 2v_{n-1} - v_{n-2} = 0. \end{aligned} \tag{3}$$

The variables v_{-2}, v_{-1}, v_{n+1} and v_{n+2} (i.e. results in fictitious nodes -2, -1, $n+1$ and $n+2$, see [1], [2], [9]) can be expressed from boundary conditions (3). Hence, the set of nonlinear equations can be written in the matrix form as

$$\boldsymbol{M}\boldsymbol{v} + a_3\boldsymbol{v}^3 + a_5\boldsymbol{v}^5 - \boldsymbol{b} = \boldsymbol{o}, \tag{4}$$

$$\boldsymbol{b} = \begin{pmatrix} b \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \boldsymbol{v} = \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_n \end{pmatrix}, \quad \boldsymbol{v}^3 = \begin{pmatrix} v_0^3 \\ v_1^3 \\ \vdots \\ v_n^3 \end{pmatrix}, \quad \boldsymbol{v}^5 = \begin{pmatrix} v_0^5 \\ v_1^5 \\ \vdots \\ v_n^5 \end{pmatrix}.$$

$$\boldsymbol{M} = \begin{pmatrix} c & -8 & 2 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -4 & 7+a_1 & -4 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & -4 & c & -4 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -4 & c & -4 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & 0 \\ 0 & \cdots & 0 & 1 & -4 & c & -4 & 1 & 0 \\ 0 & \cdots & 0 & 0 & 1 & -4 & c & -4 & 1 \\ 0 & \cdots & 0 & 0 & 0 & 1 & -4 & 5+a_1 & -2 \\ 0 & \cdots & 0 & 0 & 0 & 0 & 2 & -4 & 2+a_1 \end{pmatrix},$$

For more information see Part 1 of this article [9].

The system of coupled nonlinear equations (4) can be solved iteratively via Newton's (Newton–Raphson) Method as

$$\boldsymbol{v}^{(j+1)} = \boldsymbol{v}^{(j)} - \left(\boldsymbol{J}^{(j)}\right)^{-1} \left(\boldsymbol{M}\boldsymbol{v}^{(j)} + a_3\left(\boldsymbol{v}^{(j)}\right)^3 + a_5\left(\boldsymbol{v}^{(j)}\right)^5 + \boldsymbol{b}\right), \tag{5}$$

where vectors $\boldsymbol{v}^{(j)} = (v_0^{(j)}, v_1^{(j)}, \ldots, v_n^{(j)})^\top$ and $\boldsymbol{v}^{(j+1)} = (v_0^{(j+1)}, v_1^{(j+1)}, \ldots, v_n^{(j+1)})^\top$ are old and new iterations and $\boldsymbol{J}^{(j)} = \left(\frac{\partial f_i}{\partial v_k^{(j)}}\right)_{i,k=0,1,2,\ldots,n}$ is the matrix of derivatives (i.e. the Jacobian matrix) defined by

$$\boldsymbol{J}^{(j)} = \begin{pmatrix} J_{0,0} & -8 & 2 & 0 & 0 & \cdots & 0 \\ -4 & J_{1,1} & -4 & 1 & 0 & \cdots & 0 \\ 1 & -4 & J_{2,2} & -4 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & -4 & J_{n-2,n-2} & -4 & 1 \\ 0 & \cdots & 0 & 1 & -4 & J_{n-1,n-1} & -2 \\ 0 & \cdots & 0 & 0 & 2 & -4 & J_{n,n} \end{pmatrix}, \tag{6}$$

where diagonal matrix elements $J_{0,0} = c + 3a_3(v_0^{(j)})^2 + 5a_5(v_0^{(j)})^4$, $J_{1,1} = 7 + a_1 + 3a_3(v_1^{(j)})^2 + 5a_5(v_2^{(j)})^4$, $J_{i,i} = c + 3a_3(v_i^{(j)})^2 + 5a_5(v_i^{(j)})^4$, for $i = 2, \ldots, n-2$, $J_{n-1,n-1} = 5 + a_1 + 3a_3(v_{n-1}^{(j)})^2 + 5a_5(v_{n-1}^{(j)})^4$ and $J_{n,n} = 2 + 3a_3(v_n^{(j)})^2 + 5a_5(v_n^{(j)})^4$.

Table 1: *Programming in MATLAB software*

```
% MATLAB (solution of a beam on an elastic nonlinear bilateral foundation)
% INPUTS; see previous text:
L=12.045; F=1e5; E=2e11; Jzt=(0.2*0.4^3)/12; v0=1e-4;
k1=8.8597e6; k3=6.4373e12;  k5=-4.1846e17;
n=500; accuracy=1e-5; delta=L/n; x=0:delta:L;
% PARAMETERS FOR SET OF EQUATIONS:
b1=(F*delta^3)/(E*Jzt); a1=(k1*delta^4)/(E*Jzt);
a3=(k3*delta^4)/(E*Jzt); c=6+a1; a5=(k5*delta^4)/(E*Jzt);
% BUILD OF BASIC MATRICES AND VECTORS:
Z=v0*ones(n+1,1); % initial deflection
Z3=Z.^3;  Z5=Z.^5;
M=[c  -8  2  0  0  0; -4  7+a1  -4  1  0  0];
M(n,n-2)=1; M(n,n-1)=-4; M(n,n)=5+a1; M(n,n+1)=-2;
M(n+1,n-2)=0; M(n+1,n-1)=2; M(n+1,n)=-4;  M(n+1,n+1)=2+a1;
for i=3:n-1
  M(i,i-2)=1; M(i,i-1)=-4; M(i,i)=c; M(i,i+1)=-4; M(i,i+2)=1;
end
b(1,1)=b1; b(n+1,1)=0;
% BUILD OF JACOBIAN:
for i=1:n+1, D(i,i)=3*a3*Z(i,1)^2+a5*Z(i,1)^4; end
J=M+D;                   % Jacobian matrix
% 1st ITERATION:
F=M*Z+a3*Z3+a5*Z5-b;
V=Z-J\F;                 % deflection (first iteration)
error_of_deflection(1)=norm(V-Z);
% NEXT ITERATIONS:
G=1;                     % Number of iterations
while error_of_deflection(G) >= accuracy
    Z=V;  G=G+1;
    for i=1:n+1, Z3(i,1)=Z(i,1)^3;   Z5(i,1)=Z(i,1)^5; end
    F=M*Z+a3*Z3+a5*Z5-b;
    for i=1:n+1, D(i,i)=3*a3*Z(i,1)^2+a5*Z(i,1)^4; end
    J=M+D;               % Jacobian matrix
    V=Z-J\F;             % deflection (new iteration)
    error_of_deflection(G)=norm(V-Z);
end
% SLOPE dV:
dV(1,1)=0;  dV(n+1,1)=(2*V(n+1,1)-2*V(n,1))/(2*delta);
for i=2:n, dV(i,1)=(V(i+1,1)-V(i-1,1))/(2*delta); end
% BENDING MOMENT Mo:
Mo(1,1)=(-E*Jzt/delta^2)*(2*V(2,1)-2*V(1,1));  Mo(n+1,1)=0;
for i=2:n, Mo(i,1)=(-E*Jzt/delta^2)*(V(i+1,1)-2*V(i,1)+V(i-1,1)); end
% SHEARING FORCE T:
T(1,1)=(-E*Jzt/(2*delta^3))*b1;
T(2,1)=(-E*Jzt/(2*delta^3))*(V(4,1)-2*V(3,1)+2*V(1,1)-V(2,1));
T(n,1)=(-E*Jzt/(2*delta^3))*(-V(n,1)+2*V(n-1,1)-V(n-2,1));
T(n+1,1)=0;
for i=3:n-1
  T(i,1)=(-E*Jzt/(2*delta^3))*(V(i+2,1)-2*V(i+1,1)+2*V(i-1,1)-V(i-2,1));
end
```

The solution (iterative loops) was performed via MATLAB software; see the MATLAB script in Table 1. The value 10^{-5} m was chosen for the "accuracy". Just a few iterative loops were enough to generate accurate results.

Some basic results for the long beam (i.e. total beam length $2L = 2 \times 12.045$ m) are presented in Fig. 2 to 5 (i.e. dependencies for deflection, slope, bending moment and shearing force on coordinate x for different types of linear and nonlinear foundation). The distinctions between each type of foundation are evident. Deflection and slope (see Fig. 2 and 3) show the biggest differences between linear and nonlinear solutions. However, nonlinear approximations of foundations $q_{R_{1,3}}$ and $q_{R_{1,3,5}}$ give nearly the same results; see Fig. 2 to 5.

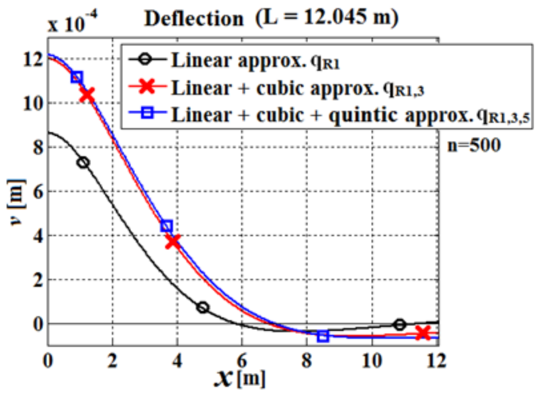

Figure 2: *Dependence for deflection on coordinate x of the beam for different types of approximations (solved example, bilateral foundation - results acquired by CDM).*

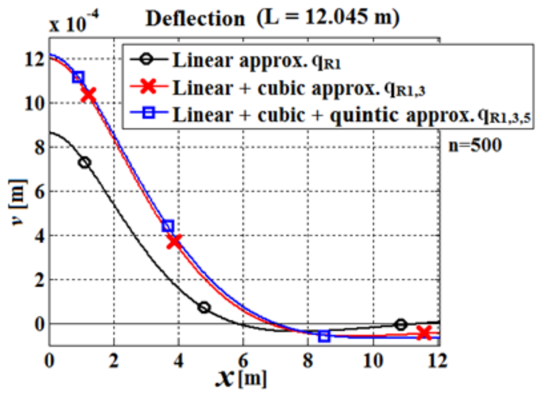

Figure 3: *Dependence for slope on coordinate x of the beam for different types of approximations (solved example, bilateral foundation - results acquired by CDM).*

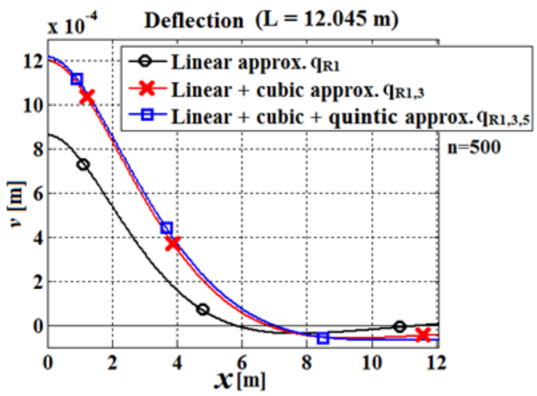

Figure 4: *Dependence for bending moment on coordinate x of the beam for different types of approximations (solved example, bilateral foundation - results acquired by CDM).*

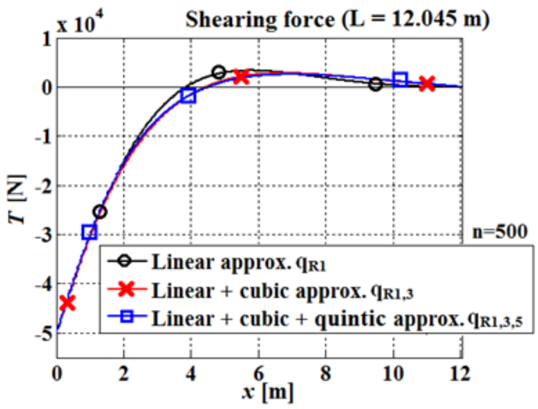

Figure 5: *Dependence for shearing force on coordinate x of the beam for different types of approximations (solved example, bilateral foundation - results acquired by CDM).*

Hence, from the results, the importance of the correct choice with regard to foundation behaviour is evident.

3 Unilateral Elastic Foundation – Application of FEM in Combination with Semi-Smooth Newton's Method

Let us suppose that the solved beam has symmetry. Therefore it is sufficient to solve the differential equation for a half of the beam, i.e. $x \in \langle 0; \mathrm{L}\rangle$.

Hence, the deflection of the beam is described by the equation

$$EJ_{ZT}\frac{d^4v}{dx^4} + kv^+ = 0 \qquad \text{on} \quad x \in (0, \mathrm{L})$$

with following boundary conditions prescribed in (1).

Lets divide the interval $(0, \mathrm{L})$ into n parts (elements) of the same length. This equidistant discretization with nodes $x_1 = 0, x_{i+1} = x_i + h$ has the constant step $h = \mathrm{L}/n$.

The discrete form of the weak formulation is following, see [9]

$$\begin{aligned} &\text{find } v_h \in V_h \text{ such that} \\ &EJ_{ZT}\int_0^{\mathrm{L}} \frac{d^2v_h}{dx^2}\frac{d^2\varphi_i}{dx^2}\,dx + k\int_0^{\mathrm{L}} v_h^+\varphi_i\,dx = \frac{\mathrm{F}}{2}\varphi_i(0) \text{ for all } i = 1,\dots,2n+2, \end{aligned} \tag{7}$$

where φ_i, $i = 1, \dots 2n+2$ are piecewise-cubic smooth functions, the base function of space V_h. Because the solution v_h of (7) is element of the space $V_h = \left\{v_h \in C^1\big((0,\mathrm{L})\big) : \ v_h|_{\langle x_i, x_{i+1}\rangle} \in P_3, \ \frac{dv_h(0)}{dx} = 0\right\}$, we can write

$$v_h = \sum_{i=1}^{2n+2} u_i\varphi_i(x). \tag{8}$$

And we will denote the vector $\boldsymbol{u}$

$$\boldsymbol{u} = \left(v_h(x_1), \frac{dv_h(x_1)}{dx}, v_h(x_2), \frac{dv_h(x_2)}{dx}, \dots, v_h(x_{n+1}), \frac{dv_h(x_{n+1})}{dx}\right)^{\top}.$$

The algebraic FEM representation of the first integral in (7) and the right side of (7) can be set by a standart way. The global stiffness matrix $\boldsymbol{K}$ and the global load vector $\boldsymbol{f}$ corresponding to (7) are shown. ($h = \mathrm{L}/n$ constant).

$$\boldsymbol{K} = \frac{1}{h^3}\begin{pmatrix} 12 & 0 & -12 & 6h & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & h^3 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ -12 & 0 & 24 & 0 & -12 & 6h & \dots & 0 & 0 & 0 & 0 \\ 6h & 0 & 0 & 8h^2 & -6h & 2h^2 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & -12 & -6h & 24 & 0 & \ddots & -12 & 6h & 0 & 0 \\ 0 & 0 & 6h & 2h^2 & 0 & 8h^2 & \ddots & -6h & 2h^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -12 & -6h & \ddots & 24 & 0 & -12 & 6h \\ 0 & 0 & 0 & 0 & 6h & 2h^2 & \ddots & 0 & 8h^2 & -6h & 2h^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & -12 & -6h & 12 & -6h \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 6h & 2h^2 & -6h & 4h^2 \end{pmatrix} \qquad \boldsymbol{f} = \begin{pmatrix} \frac{\mathrm{F}}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

For more information see Part 1 of this article [5], [9].

The acquired FEM results (for L = 6 m, F = 10^5 N, $k = 2.3587 \times 10^7$ Nm^{-2}) were compared with the Central Difference Method with good agreement. There is a comparison of unilateral and bilateral linear approaches of elastic foundation. For example $v_{MAX,bilateral} = 9.0607 \times 10^{-4}$ m, $v_{MAX,unilateral} = 9.4242 \times 10^{-4}$ m, $M_{o\ MAX,bilateral} = 6.2070 \times 10^4$ Nm, $M_{o\ MAX,unilateral} = 6.6701 \times 10^4$ Nm. The differencies between unilateral and bilateral foundation are evident, see Fig. 6 to 10.

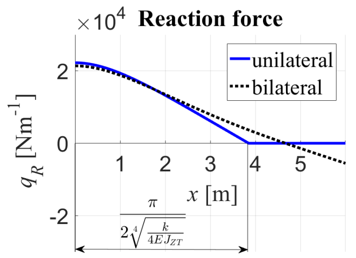

Figure 6: *Dependence for reaction force on coordinate x of the beam: Beam on unilateral (results acquired by FEM) and bilateral elastic foundation.*

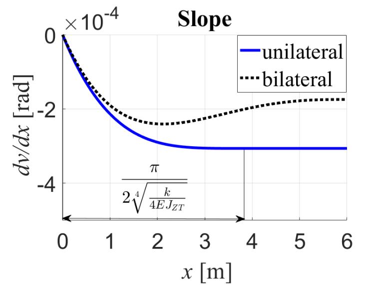

Figure 7: *Dependence for slopes on coordinate x of the beam: Beam on unilateral (results acquired by FEM) and bilateral elastic foundation.*

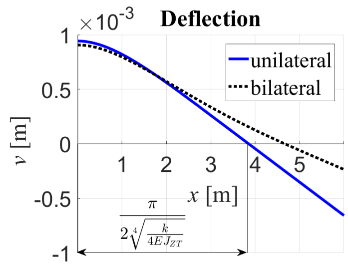

Figure 8: *Dependence for deflection on coordinate x of the beam: Beam on unilateral (results acquired by FEM) and bilateral elastic foundation.*

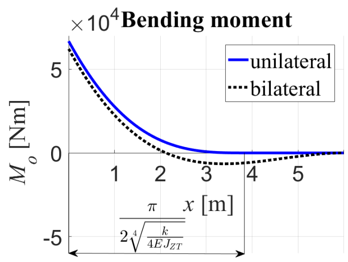

Figure 9: *Dependence for bending moment on coordinate x of the beam: Beam on unilateral (results acquired by FEM) and bilateral elastic foundation.*

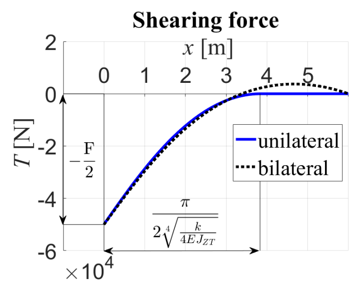

Figure 10: *Dependence for shearing force on coordinate x of the beam: Beam on unilateral (results acquired by FEM) and bilateral elastic foundation.*

Conclusion

This work is a continuation of our previous work (see Part 1 of this article [9]) presenting the theory and practice of beams on elastic linear/nonlinear foundations. Two applications (i.e. beam on bilateral foundation and beam on unilateral foundation) are presented. he CDM with Newton's method and FEM with semi-smooth Newton's method were used.

Acknowledgement

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Numerické řešení ohybu nosníku v nelineárním prostředí - část 1 (aplikace)

Abstrakt: V článku se zabýváme numerickým řešením úlohy, která popisuje ohyb rovinného nosníku uloženého v různých typech elastického prostředí, a to lineární/nelineární modifikované bilaterální a unilaterální Winklerova typu. Jsou popsány dva způsoby řešení této úlohy. První je založený na metodě centrálních diferencí a použití klasické Newtonovy metody a druhý je pomocí metody konečných prvků s využitím nehladké Newtonovy metody. V článku jsou uvedeny výsledky a srovnání těchto výpočetních postupů.

Klíčová slova: jednostranné a oboustranné elastické podloží, nelineární podloží, nosník, metoda konečných prvků, nehladká Newtonova metoda, metoda konečných diferencí, programování.

Solids and surfaces of revolution in GeoGebra

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Abstract: We present the application tool for demonstration of the Riemannian definite integral based on decomposition of a given solid onto the elementary cylinders and their surfaces of revolution in GeoGebra. The tool is presented in the form of worksheets with detailed commentary for the reader. This tool may support students' imagination.

Keywords: GeoGebra, Riemannian definite integral, solids of revolution, surfaces of revolution.

1 Introduction

GeoGebra is one of the best free software and supports education process in mathematics and descriptive geometry. GeoGebra also improves students' imagination and simplify mathematical concepts.

We use GeoGebra in many different ways during education of mathematics. Student can meet GeoGebra during lectures, they also work with GeoGebra during tutorials and they use GeoGebra at their homes for elaboration of homeworks and their own preparation for maths.

GeoGebra can cover many important topics, namely: differential and integral calculus of functions depending on one or two real variables; linear algebra - matrices, systems of linear equations, vectors; analytical geometry - lines, planes, conic sections, polygons; solids - spheres, cylinders, cones, pyramids, prisms, Platonic solids; probability theory and statistical methods; geometric mappings and planimetric and stereometric problems.

In the paper we present an applet for demonstration of the volume and the surface of solids of revolution which arise by rotations of graphs of some functions. This applet is in the form of worksheets which enable the reader to follow every single step necessary for the creation of this tool, c.f. [1–4].

2 Concept of the applet

We divide the applet into two parts. The first part called control panel enable the user to input arbitrary function and set its lower and upper limits. Then GeoGebra calculate the volume of the appropriate solid of revolution which arises by the rotation of area under the graph of the given function around x-axis, Fig. 1.

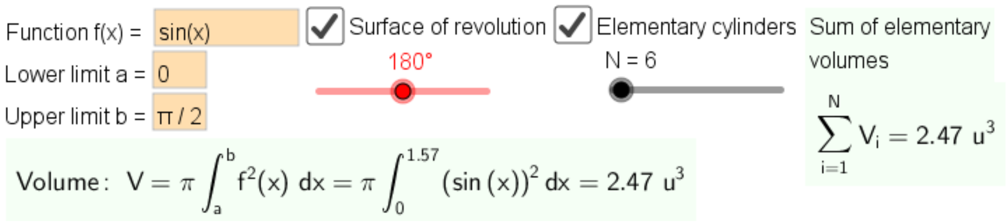

Figure 1: Control panel

The second part of the applet contains visual representation of the problem and a decomposition of the solid onto elementary cylinders, Fig. 2.

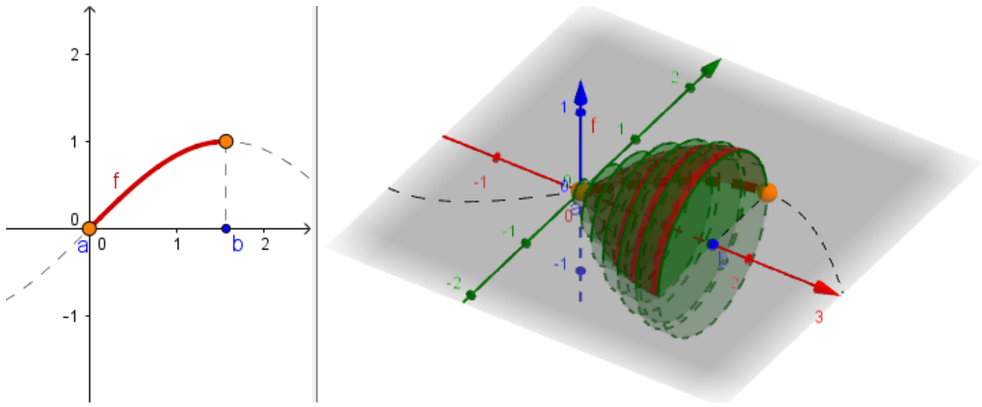

Figure 2: Representation of volume

The orange points can be dynamically changed to set new range of the integration.

3 Worksheets

The following tables contain sequences of commands for GeoGebra applet. At first we open in GeoGebra the following windows: Graphics, Graphics2 and 3D Graphics. We place 2D objects into Graphics, the control panel into Graphics2 and 3D objects into 3D Graphics window.

1.	Input:	Graphics. Create Function `g` by entering `g(x)=sin(x)` into the *Input Bar*, set Line Style to dashed line, Hide Label.
2.	Input:	Create Number `a` by entering `a=0` and create Number `b` by entering `b=`π`/2` into the *Input Bar*.
3.	Input:	Create bounded function `f` by entering `f(x)=Function[g,a,b]` into the *Input Bar*, change color to red, change Line Thickness to 7.
4.	a = 1	Graphics2. Create *Input Box* with Caption `Function f(x)=` and Linked Object g(x). Similarly create *Input Box* with Caption `Lower limit a=` and Linked Object `a`. and create *Input Box* with Caption `Upper limit b=` and Linked Object `b`

5.	Input:	Graphics2. Create *Check Box* `sur` by entering `sur=CheckBox["Surface of revolution"]` into the *Input Bar*. Create *Check Box* `cyl` by entering `cyl=CheckBox["Elementary cylinders"]` into the *Input Bar*.
6.	a=2	Graphics2. Insert *Slider* `angle` from 0° to 360° with increment 1°, change color to red, change width to 120 px. Set value to `180°`.
7.	a=2	Graphics2. Insert *Slider* `N` from 1 to 100, with increment 1, change width to 120 px. Set value to `6`.
8.	Input:	Create auxiliary value `WidthX=(b - a) / N`.
9.	Input:	Create auxiliary object `DividingPoints= Sequence[a + i WidthX, i, 0, N]`.
10.	Input:	Create auxiliary object `ReprPoints=Sequence[DividingPoints(i) + WidthX / 2, i, 1, N]`.

We use the preceding objects for creation of elementary cylinders. These cylinders decompose given solid of revolution and provide us with an approximation of the volume of the solid. The volume of the solid is also calculated using definite integral to obtain the exact value of the volume. Finally, one can compare both values of the volume, the approximated and the exact ones.

11.	Input:	3D Graphics. Create elementary cylinders `ListCylinders= Sequence[Cylinder [(DividingPoints (i), 0, 0), (DividingPoints(i + 1), 0, 0), g(ReprPoints(i))], i, 1, N]`.
12.	Input:	Calculate volume of cylinders `VolumeCylinders=Sum[ListCylinders]`.
13.	Input:	Calculate volume of solid of revolution `VolumeSolid=Integral[pi f(x)^2, a, b]`. Hide object.
14.	Input:	Graphics. Calculate area of region under function `f`, `Areaf=Integral[ f(x), a, b]`. Hide object.
15.	Input:	Graphics. Create first point on graph `A=Point[g]`. Open Object Properties, Scripting, On Update and put `SetValue[a,x(A)]`.
16.	Input:	Graphics. Create second point on graph `B=Point[g]`. Open its Object Properties, Scripting, On Update and put `SetValue[b,x(B)]`.
17.		Open Object Properties of `InputBox1`, set Scripting, On Update, put commands `SetActiveView[1]; CenterView[((a+b)/2,Areaf/(2*(a+b)))];`.
18.		Open Object Properties of `InputBox2`, set Scripting, On Update by commands `SetActiveView[1]; CenterView[((a+b)/2,Areaf/(2*(a+b)))]; SetValue[A, (a,g(a))];`.
19.		Open Object Properties of `InputBox3`, set Scripting, On Update by commands `SetActiveView[1]; CenterView[((a+b)/2,Areaf/(2*(a+b)))]; SetValue[B, (b,g(b))];`.
20.	Input:	Graphics. Create auxiliary point `AuxA=(x(A),0)` on x-axis and put Caption "a".
21.	Input:	Graphics. Create auxiliary point `AuxB=(x(B),0)` on x-axis and put Caption "b".
22.		Graphics. Create auxiliary segment `Segm1=Segment[AuxA,A]` open Object Properties, Hide Label and set dashed line.
23.		Graphics. Create auxiliary segment `Segm2=Segment[AuxB,B]` open Object Properties, Hide Label and set dashed line.
24.	ABC	Graphics2. Create new text `text1` containing "Sum of elementary volumes".

25.	ABC	Graphics2. Create new text `text2` containing `"\sum_{i=1}^N V_i= VolumeCylinders u^3"`, and mark LaTeX. The highlighted object `VolumeCylinders` must be selected from the list of Objects.
26.	ABC	Graphics2. Create new text `text3` containing `"\mbox{Volume\!: }V=\pi\int_a^b \!\! f^2(x) \ dx=\pi\int_{ a }^{ b }\!\!\left( g \right)^2 dx= VolumeSolid u^3"`, and mark LaTeX. The highlighted objects `a`, `b`, `g` and `VolumeSolid` must be selected from the list of Objects.
27.	Input:	3DGraphics. Create surface of revolution `RotSurface=Surface[f, angle, xAxis]`.
28.		For objects `RotSurface` and `angle` open Object Properties, Advanced, Condition to Show Objects and put `sur`.
29.		For objects `ListCylinders`, `N`, `text1` and `text2` open Object Properties, Advanced, Condition to Show Objects and put `cyl`.

Every user can individually manage position of graphics windows, colors, styles, etc. Note that we use two 2D graphics views, namely Graphics and Graphics2. It is necessary to click into appropriate window if you want to create an object in concrete graphics view. If you cannot see an object you have created, it is possible that the object is placed in wrong graphics view. One can change visibility or the location of an object in Object Properties, Advanced, Location.

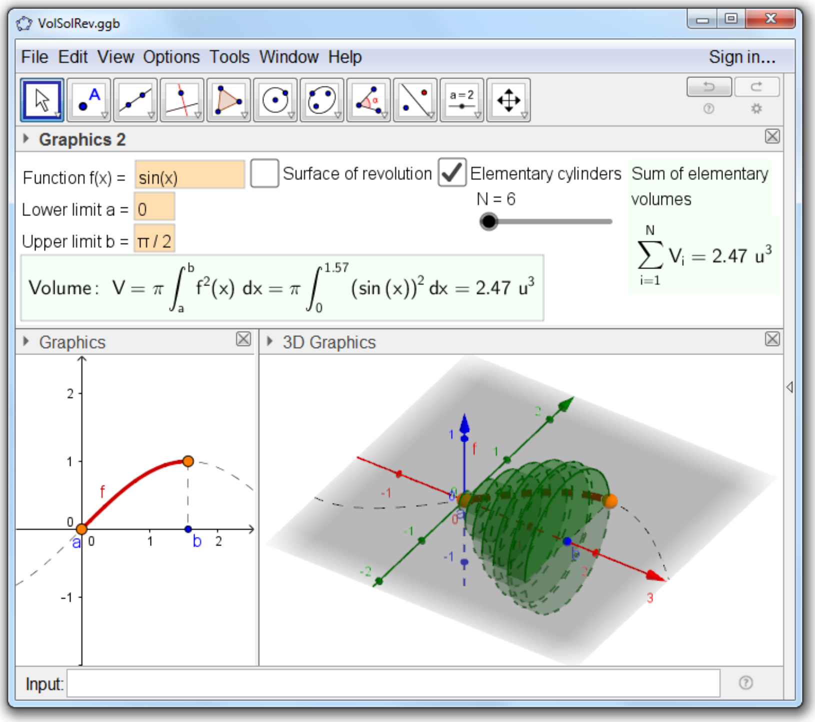

Figure 3: Applet

4 Conclusion

The applet for demonstration of the decomposition of a given solid of revolution onto elementary cylinders can help students to gain better understanding of integral calculus of functions depending on one real variable. The applet can be used during the lecture of mathematics but it is also available for students. GeoGebra helps us to improve education of mathematics. Creation of such applets is one of the activities of our GeoGebra Institute of Ostrava.

Acknowledgments

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Rotační tělesa a jejich pláště v GeoGebře

Abstrakt: Prezentujeme nástroj pro demonstraci Riemannova určitého integrálu založený na rozkladu daného rotačního tělesa na elementární válce v GeoGebře. Nástroj je prezentován ve formě pracovních listů s detailním komentářem pro čtenáře. Tento nástroj by mohl sloužit k podpoře rozvoje představivosti studentů.

Klíčová slova: GeoGebra, Riemannův určitý integrál, rotační těleso, plášť rotačního tělesa.

ANALYSIS OF THE POSSIBILITY OF GAS-FUELED MICRO-COGENERATION APPLICATION IN SINGLE-FAMILY DWELLING BUILDINGS IN POLAND

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Abstract: Conditionings related to application of high-efficiency gas micro-cogeneration in single-family dwelling buildings were presented in this paper. Analyses of power and heat demand profiles in this type of buildings were shown and a selection process of micro-cogeneration system was conducted in two variants – devices with a Stirling type engine (1 kW electric and 3-5,8 kW heating capacity) and a piston internal combustion type engine (2,5-6 kW electric and 8-13 kW heating capacity). For both the variants technical and economical analyses of optimal application of this technology were performed. Low values of base-load consumption of heat and power in this type of buildings in a scale of a year, and hence a possibility of application of the smallest micro-cogeneration units available on the market only, significantly negatively influence an amount of running-cost savings obtained. Key-importance conditionings which should be fulfilled in the aim to obtain acceptable running-cost savings and return on investment periods were pointed.

Keywords: Gas micro-cogeneration, dwelling buildings, technical conditions, running-cost savings, return on investment period, prosumer energy system.

1 Introduction

Gas-fueled micro-cogeneration is a highly efficient technology of simultaneous production of power and heat in a single process and single device. Gas fuel (natural gas or LPG) feeds an internal combustion engine, which directly transfers mechanical energy to a drive of electric power generator. Internal combustion engine and power generator are cooled during their operation and heat gained in this way is transported with use of a liquid medium (water or glycol solution) and in this way transferred for utilization. The power generator produces alternating current electricity in a single or three-phase system. Two streams of energy are obtained simultaneously in this way. Hence we can speak of a combined, highly efficient energy production (Fig. 1).

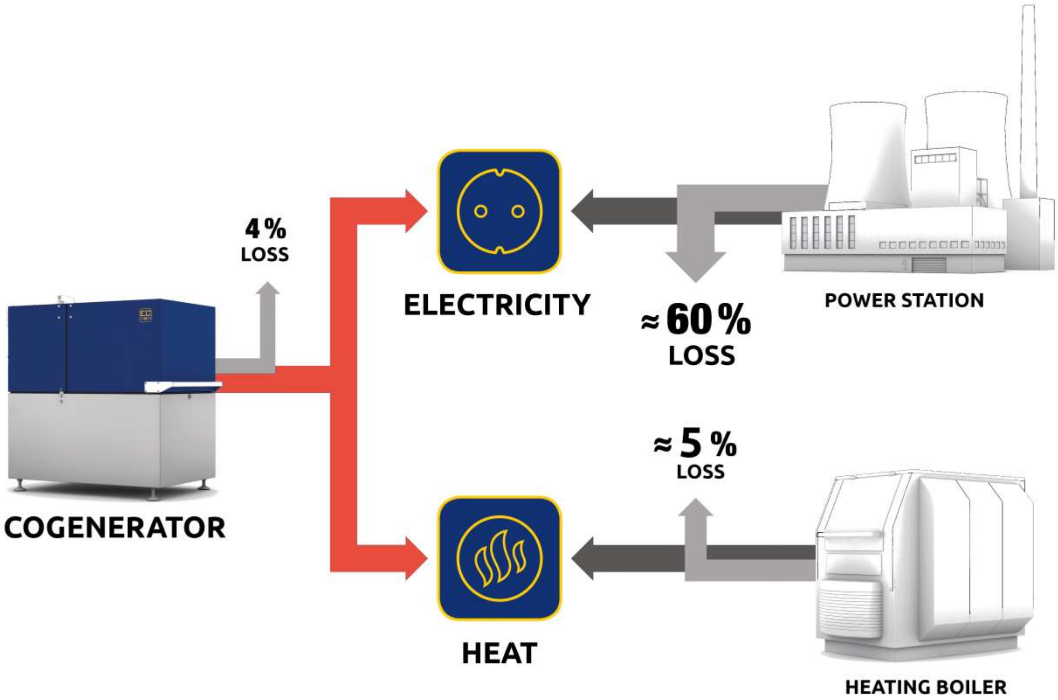

Fig. 1. Differences in loss occurrence in a separate and combined system of heat and power production

Description "micro" in the name of micro-cogeneration means that it is a cogenerating system in which up to 40 kW of electric power and up to 70 kW of heat is produced in a single device [1]. The devices are characterized by small dimensions, which allow for their installation in every boiler room or technical space of existing or newly-designed buildings. It means a possibility of power and heat production directly in a place of their utilization, with no transmission losses and hence with an additional increase of system efficiency and running-cost savings. Usage of gas fuels is an element which allows for reduction of disadvantageous environmental emissions, and the high efficiency of the process contributes to reduction of primary resources consumption in comparison to separate production in traditional technologies.

All these elements mean that application of highly-efficient gas micro-cogeneration is beneficial both from economical and environmental aspects, however in a large extent the final effect will depend on proper selection of a cogeneration system capacity in relation to energy demand in buildings with differentiated power and heat consumption profiles [2]. For a cogeneration system installed, to bring expected savings, in the first place it is necessary that both streams of energy produced are constantly consumed. Such a permanent and simultaneous consumption of heat and power is necessary for the device to undertake and maintain its work. Every object is characterized with different power and heat consumption, which additionally varies in the scale of natural day and summer/winter seasons. This ensues a necessity of conducting an energy demand analysis for a building in subject and proper selection of cogeneration unit(s) capacity. Because of that, not every object meets the condition of simultaneousness of heat and power demand on a certain level, proper for the cogeneration system.

2 Energy performance of a single-family dwelling

A typical single-family dwelling in Poland is an object of around 100 m^2 square footage [3]. Most frequently such buildings are supplied with electricity from the grid through a three-phase electric installation. Thermal energy for heating is usually assured by a building central heating system with a heating boiler fueled with natural gas or hard coal. Most frequently the central heating system is a two-function system and at the same time hot domestic water is produced in it.

2.1 Power demand

From the point of view of cogeneration system application in dwelling buildings, one of the elements which has to be thoroughly analyzed is the building profile of power consumption. Figure 2 shows an example of electric power consumption in a single-family dwelling, registered during winter period.

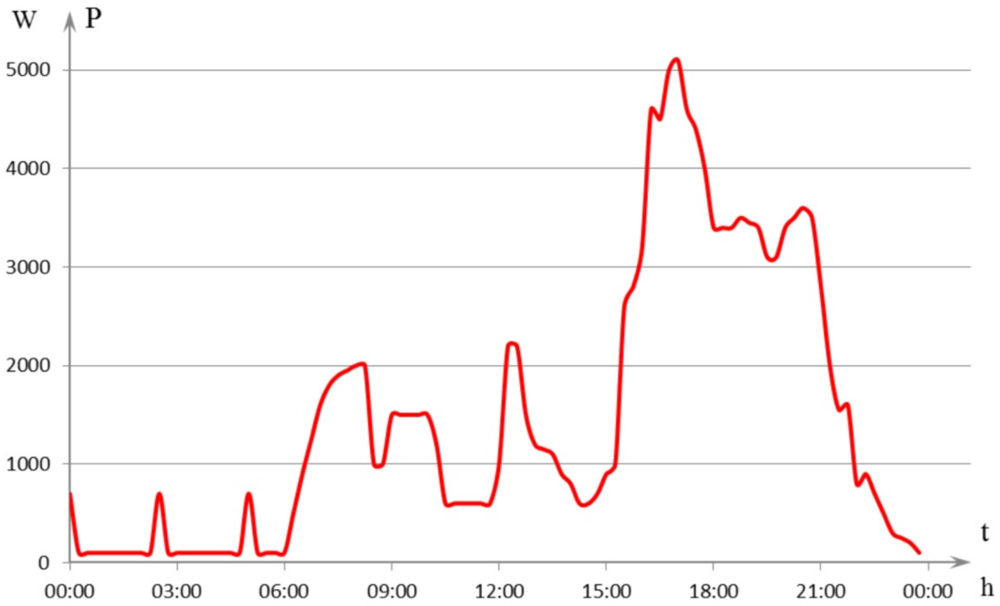

Fig. 2. Electric power consumption in a single-family dwelling
Source: [4]

The power consumption showed in Fig. 2 presents a summary consumption which occurs all-in on all three phases of electric installation of the building at a certain time. So it is possible that a situation occurs when phase load is non-evenly distributed – e.g. 5 kW on the first and 1 kW on the second and 1 kW on the third phase. Such the unevenness negatively influences the cogeneration system operation and one should aim to equal the load on phases at least by application of symmetrizing units [5]. In case when after application of a proper symmetrization device the phase load is equalized, only then we can start determining a base-load of power consumption, which means a value below which the power demand in building never decreases in the scale of a year through a certain time. The most optimal from the point of view of shortening the return on investment time is determining the base-load which occurs a whole year long, i.e. 8760 h, however not in every type of buildings it is possible – in great extent it depends on a type and number of electricity receivers operating in continuous mode.

2.2 Heat demand

Heat demand in single-family dwellings results from heat demands of buildings (during transition periods and during winter), and from the need of hot domestic water preparation (whole year long). Buildings of this type are mainly equipped with gravitational ventilation systems and no need of ventilating air heating occurs.

Depending on materials applied in the building construction and wall insulation, the heating demand will be located in the range of 10-20 kW. It is the heat source capacity which is supposed to assure heating for the building at computational temperatures used by designers, which in case of Poland mean -16 to -24 ºC, depending on geographical location. Hence it is a peak-demand estimated source capacity. Taking into consideration the thermal inertia of building, this capacity can be reduced in some degree while selecting a cogeneration system.

Demand for hot domestic water is determined on the basis of proper standards and regulations [6-8]. Heating for the need of hot domestic water production in winter period and partially during transition periods comes from building's central heating system which works in a two-function mode. So from the point of view of co-generation system, the heating capacity required for hot domestic water production in summer period will be crucial. The amount of hot domestic water used in dwelling does not depend on its cubic measure nor the way of construction – it depends on the number of dwellers only. Considering a 3-5 person family, the heating capacity of 1-2 kW will be sufficient when a proper volume hot domestic water tank is provided (200-500 dm^3).

3 Selection of the micro-cogeneration system

On the basis of analyses presented in Chap. 2, it is possible to estimate a capacity of micro-cogeneration system which could be possible to be applied in a single-family dwelling building. The base-load demand for electric power will reach 2 kW for ~5000 h in the scale of year, and herein 3.5 kW in the period of ~2000 h.

As for heat consumption in the building, the co-generation system should provide heating capacity around 10 kW and should be equipped with a heat storage tank(s) of 2-3 thous. dm^3 volume. Additionally, the heating installation should be completed with an independent system of electric heaters (as a backup and for the needs of antibacterial heating).

These values of heat and power consumption are low even when referred to micro-cogeneration unit series available on the market [9]. Because of this, for the need of further analyses, a micro-cogeneration unit based on Stirling engine with electric output of 1 kW (model Dachs Stirling) and a microcogeneration unit based on piston internal combustion engine with electric power modulated in the range of 2.5-6 kW (model EC Power XRGI 6). Because of the low values of base-load demands for heat and power, only single micro-cogeneration units can be considered for application in single-family dwellings, which though will not operate in a part of year or will work with modulation (reduction) of output capacity.

4 Analysis of running-cost savings and return on investment periods

When analyzing worthwhileness of cogeneration system application in dwelling buildings, including micro-cogeneration systems, there always appear a situation in which from one side cogeneration unit capacity must be reduced to conform to low values of base-load power consumption (whole year period) and heat consumption (in summer period), and from the other side reduction of the micro-cogeneration system capacity will result in lower running-cost savings and longer return on investment periods. Nevertheless, in this case adopting to the power and heat base-load consumption is overriding, because unduly oversizing of the micro-cogeneration unit capacity could lead to a situation in which power/heat consumed by the building is too low and the cogeneration unit does not start at all. Such the situation takes place in case when the heat/power output received from the co-generator is less than 50% of its nominal capacity. A control system stops the micro-cogenerator in this situation because its work with such low load on internal combustion engine would be economically unjustified and too much of fuel would be consumed for too small energy production.

A micro-cogeneration unit based on Stirling engine (with 1 kW no-modulation output of power and 3-5.8 kW modulated heat output) applied in a single-family dwelling could possibly work ~5000 h per year. It means that running-cost savings can be obtained on the basis of avoided costs of electricity purchased from the grid on the level of 5000 kWh, which at the price of 0.5 PLN/kWh gives savings of 2500 PLN/year. Additionally, an owner of the cogeneration system can obtain a subsidy related to power production in highly efficient cogeneration system (so called yellow certificates) on the level of ~0.1 PLN/kWh. It means additional savings in amount of 500 PLN/year. At a price of ~60 thous. PLN for the micro-cogeneration system it means the return on investment time on the level of 20 years.

A micro-cogeneration unit based on a piston internal combustion engine (with 2.5-6 kW modulated power output and 8-13 kW modulated heat output) applied in a single-family dwelling could possibly work with modulation (on the level of about 3.5 kW) for 2000 h only (this limitation comes from the power consumption profile). It means that running-cost savings can be obtained on the basis of avoided costs of electricity purchased from the grid on the level of 7000 kWh, which at the price of 0.5 PLN/kWh means savings of 3500 PLN/year. Additional savings resulting from yellow certificates will amount 700 PLN/year. At a price of ~120 thous. PLN of the micro-cogeneration system it means the return on investment time on the level of 28 years. [2]

It is necessary to consider also maintenance and service costs here, which will increase the return on investment periods by extra ~5%. So it can be seen that in case of Stirling engine (1 kW el.) the device is of too low output capacity to be able to provide significantly high savings, and in turn, in case of internal combustion engine (6 kW el.) the device works with a partial load in a part of year and hence is also not able to generate optimal savings and short return on investment time. Similar results were presented for micro-cogenerators of different ranges of power and heat output capacities [9]. It means that in case of micro-cogeneration system working for own needs of a single-family dwelling building, it is not an optimal solution because of limited and non-permanent consumption of heat and power.

5 Possibilities of system effectiveness improvement

To achieve optimal application of micro-cogeneration systems in single-family dwelling buildings, some activities aiming to elimination of barriers created by power and heat consumption profiles of this type buildings should be undertaken.

The barrier created by electricity demand can be eliminated by transferring the excess of electricity produced to the grid. It is a technically possible solution, however it is difficult to be executed from a formal point of view and does not bring such great savings as in case of avoiding cost of power purchase from the grid. The Energy Law [10] foresees some simplifications in connecting and operation so called micro-installations to the grid, however the term “micro-installation” is defined as a renewable energy source up to 40 kW electric power, and the gas micro-cogeneration, besides a proper range of output power, cannot be qualified so because natural gas or LPG are not treated as renewable fuels. The avoided costs of power purchase from the grid mean savings of ~0.5 PLN/kWh, and while selling electricity to the grid the building’s owner can obtain

~0.15 PLN/kWh only. Nonetheless this solution allows for significant increasing of cogeneration system operation time, and thereby increase of running-cost savings.

The fact of delivering electricity to the grid with no limitations will increase the operating time of the system with Stirling engine (1 kW el.) to 8760 h, so the system will operate 3760 h longer with selling during this time 3760 kWh of electricity to the grid at the price of ~0.15 PLN/kWh. Along with the basic time of 5000 h (when savings amount 0.5 PLN/kWh) and yellow certificates it gives total running-cost savings on a level of 3940 PLN/year. Return on investment time is reduced from 20 to 15 years.

In turn, in case of the co-generation system with piston internal combustion engine (2.5-6 kW el.) the operation time could also be increased to 8760 h in a year, but in this case the profile of building heat demand becomes a limitation. Even when the unit works with modulation on the lowest output level, the stream of heat is too big for a permanent receiving by the building. Heat storage tanks will be used, but the operating time of the system during a year will be reduced anyway. In this case it is possible to gain 2000 h of operating hours in year with full nominal output, and because of the heat demand profile in summer period and in parts of transitional periods the system can work next 3000 h with modulation. Along with yellow certificates, running-cost savings of total 9125 PLN/year will be gained and it means shortening of return on investment time from 28 to 13 years.

Only in case of complete elimination of barriers also on the side of heat receiving, which could be possible by connecting a greater number of dwelling buildings into a common, internal shared heating network, the micro-cogeneration system could work uninterruptedly whole year with full nominal output. This would mean savings on a level of 16.6 thous. PLN/year, which would result in return on investment time on the level of 7.2 years.

Conclusion

The conditionings presented in this paper, related to application of high-efficiency gas micro-cogeneration in single-family dwelling buildings, point to too large discrepancy between levels of electricity/power consumed in such buildings in the scale of a year and a required load of cogeneration system which may assure its optimal work in relation to running-cost savings and a short return on investment time. For the cogeneration system to be able to operate efficiently and bring benefits for the object it is necessary to have a full simultaneous reception of output energy produced, both heat and power. The base-load demand of heat and power in this type buildings is of such low level that eventual application of smallest micro-cogeneration units from series available on the market is the only possibility, and additionally, the units have to operate in modulation mode with reduction of output energy. This dependence has a negative influence on running-cost savings possible to obtain because the lower capacity of the co-generation system is, less power can be produced in it and hence a ratio between electricity purchased from the grid and produced in the building changes in an unfavorable way. The highest is the part produced by the building itself, the savings are more favorable.

Small capacities of micro-cogeneration units which are possible to be applied in this type of buildings, their operation in a modulated mode, periodical breaks in the scale of a year, all of it causes that return on investment time for such installations in single-family dwellings exceeds 20 years. Elimination of the barrier related to low electric power consumption in a way of transferring its exceed to the grid will result in shortening of the return on investment time to around 13 years and it is still a relatively long period. For a comparison, an analysis of return on investment time of micro-cogeneration in objects with higher and permanent power and heat demand can be quoted, as in case of sport and recreational objects, hospitals, swimming pools and hotels, where the return on investment time reaches 4-5 years [11].

Among all the improvement possibilities mentioned in Chap. 5, which could increase worthwhileness of micro-cogeneration system application in single-family dwellings, only the possibility of selling the power exceed to the grid is a solution which is practically achievable with no significant rebuilding of building installations needed. This solution, besides a standard declaration of connection of a generating unit to the grid, will require an agreement of energy selling and installation of a bi-directional energy meter with safety and telemetric hardware. More difficult in implementation is the increase of heat reception from the cogeneration unit by connecting of neighboring buildings in a common, shared heating network – both because of formal issues, investment cost division and in a later stage division of maintenance costs and running-cost savings. More simple from the point of view of formalities and clearances solution seems to be a construction of underground tanks for heat storage on the premises of a relevant dwelling building, but in situation when a co-generator while operating produces heat in amount of 8-13 kW all year long, storage of its exceeding part (the part which is not used by the building) makes no sense because when the cogenerator operates permanently, there would be no periods in a scale of a year in which the heat stored could be utilized.

A building micro-cogeneration as an element of a prosumer energy system, to be able to optimally function in a frame of dissipated prosumer energy system must be extended at least by the possibility of transferring the produced power surpluses to the grid. For such the system to be able to function more commonly, facilitations in law regulations should be introduced in relation to connecting of energy sources to the grid and power selling also for micro-installations (devices up to 40 kW of electric power) fueled with gas fuels like natural gas or LPG. Also increase of prices of selling electricity to the grid for private prosumers and upholding the system of yellow certificates for cogeneration together with increasing their price will be factors favoring development of this area of prosumer energy system in Poland.

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ANALIZA MOŻLIWOŚCI ZASTOSOWANIA MIKROKOGENERACJI GAZOWEJ W JEDNORODZINNYCH BUDYNKACH MIESZKALNYCH W POLSCE

Abstrakt (Streszczenie): W artykule przedstawiono uwarunkowania dotyczące aplikacji wysokosprawnej mikrokogeneracji gazowej w jednorodzinnych budynkach mieszkalnych. Przedstawiono analizę profili zapotrzebowania na ciepło i energię elektryczną w tego typu budynkach i przeprowadzono dobór układu mikrokogeneracji w dwóch wariantach – urządzeń z silnikiem Stirlinga (1 kW mocy elektrycznej i 3-5,8 mocy grzewczej) oraz z tłokowym silnikiem spalinowym (2,5-6 kW mocy elektrycznej i 8-13 kW mocy grzewczej). Dla obu wariantów przeprowadzono analizę techniczną i ekonomiczną optymalnego zastosowania tej technologii. Niskie wartości podstaw poboru energii elektrycznej i ciepła w tego typu budynkach w skali roku i co za tym idzie możliwość stosowania jedynie najmniejszych urządzeń mikrokogeneracyjnych spośród typoszeregu dostępnych na rynku jednostek znacznie negatywnie wpływają na wielkość uzyskiwanych oszczędności eksploatacyjnych. Wskazano na kluczowe uwarunkowania jakie powinny być spełnione w celu uzyskania akceptowalnych oszczędności eksploatacyjnych oraz okresów zwrotu nakładów inwestycyjnych.

Klíčová slova (Słowa kluczowe): Gazowa mikrokogeneracja, budynki mieszkalne, uwarunkowania techniczne, korzyści eksploatacyjne, czas zwrotu nakładów inwestycyjnych, energetyka prosumencka.

THE IMPACT OF THE IMPLEMENTATION OF THE NEW PRODUCTION LINE ON THE ORDER PROCESSING PROCESS

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Abstract: The purpose of the publication is to present an analysis of the impact of the implementation of a new production line on the process of order execution on the example of an industrial enterprise. In order to analyse the process, it was divided into detailed steps and a detailed dimensioning of the activity was defined - the minimum and maximum time for each activity. Analysing the activities before and after the introduction of a new production line determines the effects of the company's actions on the process itself and on the customer.

Keywords: production management, production process, process map, activity time analysis, quality management, process management, processes.

1 Introduction

Over the past several decades, the importance of innovation in the company has been recognized as an impotent factor of the company development. Manufacturers around the world are striving to streamline the manufacturing process [1, 5, 6, 9, 11, 22, 24, 25]. The main goals of such pursuits may be different, for example, reducing production costs, improving product quality, or extending the range of goods offered. This usually requires changes in the production process.

The purpose of the publication is to present an analysis of the impact of the implementation of a new production line on the process of order execution on the example of an industrial enterprise.

2 The concept of process

The definition of the term "process" refers to all activities and situations that last for some time, which can happen unsettled, one after the other, or simultaneously. In addition, they can interact with one another, leading to the goal being achieved [13].

ISO standard defines "process" as a single or collective action, transforming input resources into outputs. Processes can be subdivided into sub processes, which together make up the whole.

Each of them performs other functions that change the input objects in the process to the desired output state [8, 23].

On the other hand, M. Hammer and J.Champa stated that the process is a group of activities with the required inputs and outputs that produce value for the customer. Also, processes in the process are carried out by a group of employees, not by a single person.

Processes are classified into [10]:

- Main - adding value to the company, are related to the main business, affect the financial result.
- Supporting - a source of cost for the enterprise, however, the main processes could not function without them.
- General - These include management processes that allow for problem-free business operations.

One of the types of processes are business processes, which were referred to in the article by J. Rutkowska. According to her, the business process is: "A whole consisting of successively performed operations to achieve predetermined results. The process uses input resources (information, raw materials, semi-finished products) that are processed and transferred to the next process or to the final customer [12, 14, 15, 16].

With the concept of process, the process approach is closely linked, which in quality management is part of the eight basic principles. They are based on the collection of measurement of results from the operation of processes, and on continuous improvement, which is to solve problems. In the process approach, in order to develop and improve a company, the focus is most heavily on [13]:

- resources,
- methods,
- materials.

3 Measures of process efficiency

Efficiency of the process may concern economic, financial, social, spiritual, moral and ecological values. It can be defined as the relationship between the results obtained and the resources used. Due to the measurement of processes in the enterprise, financial, economic and operational efficiency is distinguished. The first is the monetary relationship between the results obtained and the means used. On the other hand, operational efficiency refers to the organization of processes and the reduction in the use of means of production per unit of product. Economic efficiency is the link between the specific effect and the factors of production or the group. In practice, different performance measures are used, depending on what is defined as the effect and the effort [7, 17, 18, 19, 20, 21]. From the example relationship you can associate the ratio of the obtained effects to the spent effort [3, 4].

The first is it [2]:

- added value,
- production,
- profit,
- income.

Performance indicators use quantitative indicator methods that use synthetic or partial indicators or meters to identify, measure and evaluate economic or non-economic effects.

4 Process Analysis

Process map in general outlines processes and important sub processes. A sub process is a separate part of a process that can be treated as a separate, smaller process, for its own sake and distinct from other parts (for example, in the process of recruiting a large company, sub processes of employee adaptation can be separated). There is no single mapping standard. It most often shows information or material flows between processes.

In order to present the changes that occurred in the examined enterprise after the implementation of the change, the process of order realization was analyzed. A map of the process is used, which shows all the major activities taking place in each area of the surveyed enterprise. Graphical presentation of the order of the process fulfillment gives you the possibility to specify each activity from receipt of the order until delivery to the customer. All of these steps in the order to fulfillment the process were numbered sequentially and then characterized in table number 1. After analyzing the duration of each stage, the problem faced by the company before the implementation of the innovative solution emerges.

Table 1. Actions in the process of order execution before implementation of the change

Action No	Description of activities	Minimum time	Maximum time
1	The company by telephone, electronic or in person receives a product inquiry - its availability and price	1 min.	15 min.
2	In the sales department is checked the stock status in the electronic database, or send a query about the availability of goods to the warehouse.	2 min.	10 min.
3	Where the product is available, information about availability and price of the product is sent to the customer.	1 min.	5 min.
4	In the absence of goods availability, the production department receives a request from the warehouse for the possibility of producing the goods.	5 min.	20 min.
5A	In the case of the possibility of producing the ordered goods, the technical department sends the information to the sales department.	5 min.	120 min.
5B	When there is no possibility to produce the ordered item, the sales department sends the request to the supplier and awaits the reply.	5 min.	24h
6	The sales department draws up and provides the customer with an offer containing the price and delivery date.	5 min.	60 min.
7,8	The customer's decision to place an order or resignation awaits.	1 min.	3 months
9A	Where the order relates to own goods, the inventory is re-checked.	2 min.	10 min.
9B	When an order is for an item to be purchased from an outside supplier, the sales department orders the product. There is waiting for order fulfillment and delivery.	24h	14 days
10	Where the goods are in stock, the department	10 min.	120 min.

	prepares the goods according to the way of receipt.		
11	If the order relates to one's own product that is not available in stock, the sales department receives information on the need to start the production process.	2 min.	10 min.
12	Production order is issued by the sales department.	5 min.	20 min.
13	Production of ordered goods is started.	30 min.	Unpredictable
14	The produced goods are subjected to quality control.	10 min.	60 min.
15	The tested products go to the warehouse where they are prepared for sale.	5 min.	Unpredictable
16	It defines how the goods are delivered to the recipient and the preparation of the delivery documents. Documents are uploaded to the magazine.	10 min.	30 min.
17A	Waiting for pick up.	Unpredictable	
17B	Loading a company car.	5 min.	15 min.
17C	There is a selection of the carrier and the shipping of the goods.	5 min.	48h
18	Pick up a personal item by the customer.	1 min.	10 min.
19	The goods are transported to the customer by company or carrier transport.	10 min.	48h
20	The sales department stores archives related to the sale of goods. An electronic inventory database is being updated. The order processing process is closed.	5 min.	15 min.

Source: On basis [7].

Tables with a list of activities in the realization of the process contain situations where the maximum time is difficult to specify. For steps 13 and 15, the maximum duration depends on the size of the order (orders for larger quantities can be divided into lots) or the decision to continue production to increase stock. At point 18, the maximum time for picking up is dependent on the customer's decision. The company stores the product as long as the customer wishes.

In the case of waiting for a client's decision to place an order, the maximum time is the validity period of the offer, for example in the case of mine auctions. The customer after receiving the tender offer has a maximum of 3 months to decide on the order. After this period, the offer becomes obsolete and you must submit a re-inquiry.

By measuring the times of individual situations, you can make the following conclusions:

1. When the goods are in stock, the time from the acceptance of the order to the time of leaving the company does not exceed 1 hour.
2. In case ordered items have to be produced, the minimum time is prolonged by approximately 1 hour. It depends on how many pieces you have to make. For larger orders, the time is relatively long, but it is possible to determine precisely.
3. Minimum order execution time for rolled sealing orders that must be ordered from another manufacturer is drastically increased and due to the necessary shipping of the product is at least 1 day. This time is completely dependent on the external provider and is difficult to determine accurately.

By analyzing the times of different stages of the order processing process, the time of order fulfillment by the external supplier is considered. If your order is received by a company for a small particulate seal other than that produced by the injection molding method, the company is awarded to an outside supplier. It is often the case that the declared lead time of the order discourages the customer from ordering this type of seal at Kreon. Long waiting times can be caused by several factors. The external seal supplier can have a long queue of orders waiting. This may also be caused by a production line failure or lack of raw material. Long waiting time is one of the main reasons for taking action to install your own production line.

Eliminating the problem of not being able to produce atypical seals for the mining industry makes it possible to become independent from the external suppliers of these products. Therefore, the time of order fulfillment is significantly shortened. The additional benefit of the ability to produce seals for each type of actuator is the predictability of the contract completion date. Existing suppliers of rolled seals often had delays in relation to the declared deadline. Due to the fact that organization Kreon was entirely dependent on the suppliers, it could not afford to give up their services and had to accept such situations. The result of the delays was a clear dissatisfaction of customers and even a resignation from further cooperation with Kreon. The installation of the new production line has helped to eliminate this problem. After the change was made, the order processing process was again analyzed.

Conclusion

Analyzing of the process map to show the process of order execution after the installation of the new production line, it can be seen as a simplification compared to the previous situation. The company is now able to produce any type of sealed order, so it is not dependent on external suppliers. As a result, there is considerable shortening and predictability of order fulfillment time. The shortest possible lead time for this type of product is now close to the time of ordering seals produced on injection molding machines. The positive effects of this change are numerous. By improving the customer service process, the company can retain existing customers and acquire customers specializing in other industries. After implementing the new method of sealing, the company undertook marketing activities, informing both current and new customers about the expansion of production capacities. As a result of these efforts, the number of customers increases, which translates into significantly increased orders and improved financial condition of the company.

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WPŁYW WPROWADZENIA NOWEJ LINII PRODUKCYJNEJ NA PROCES REALIZACJI ZAMÓWIENIA

Streszczenie: Celem publikacji jest przedstawienie analizy wpływu wdrożenia nowej linii produkcyjnej na proces realizacji zamówienia na przykładzie przedsiębiorstwa przemysłowego. Aby dokonać analizy procesu podzielono go na szczegółowe etapy i dokonano szczegółowego zwymiarowania czynności – określenia czasów minimalnych i maksymalnych dla każdej czynności. Analizując poszczególne czynności przed i po wprowadzeniu nowej linii produkcyjnej określono skutki działań firmy dla samego przebiegu procesu produkcyjnego jak i dla klienta.

Słowa kluczowe: zarządzanie produkcją, proces produkcji, mapa procesu, analiza czasów czynności, zarządzanie jakością, zarządzanie procesami, procesy.

ANALYSIS OF THE RELIABILITY OF SELECTED MACHINES IN A PRODUCTION ENTERPRISE – CASE STUDY

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Abstract: The article presents the results of analyses which were conducted on the basis of data collected in one of production companies manufacturing pre-insulated pipes for the heat engineering industry. The data from the facility maintenance system included among others duration of failures, duration of breakdown removal and the type of failures for the major machines in the enterprise, i.e. polyethylene pipes extruders. The conducted analysis allowed determining the reliability indices. The obtained values and information on the type of breakdowns allowed developing recommendations for facility maintenance service teams regarding the organisation of technicians' work, management of human and material resources as well as the planning of inspections and overhauls.

Keywords: reliability, facility maintenance, MTBF (Mean Time Between Failures), failure rate, breakdown

1 Introduction

Ensuring the total production capacity of a company requires having a very well organised and effectively managed facility maintenance system. The efficiently working fleet of machines will largely determine the company's position on the market by providing its clients with goods and services in the required quality and quantity, within a required term. Failure to keep delivery terms or worsening the goods' quality may lead to losing the clients' trust and, in consequence, will result in losing the sales market for the company's goods [1]. Facility maintenance is usually one of the biggest items in the company's operational costs. For this reason, endeavours are constantly made to find more effective methods of work and management of this area of the company's activity [2]. The increasing share of direct costs of facility maintenance department in the company's changeable costs as well as the competitive situation currently faced by companies force continued searches for cost reduction possibilities [3]. The analysis of various approaches to facility maintenance versus time enables determining three periods, which overlap in the process of development [4].

1) Reactive maintenance – overhauls after a damage
2) Preventive maintenance – preventive overhauls

3) Predictive/proactive maintenance – preventive inspections, technical condition monitoring, participation of machine operators in facility maintenance

Despite minimizing the risk, it is impossible to avoid breakdowns of machinery fleet elements. In case of breakdown, service teams responsible for maintaining the machinery park in good conditions should try to remove the breakdown as soon as possible, shortening the duration of downtime to minimum. In terms of the effects caused by a breakdown in the production process we can make a division presented in Fig. 1.

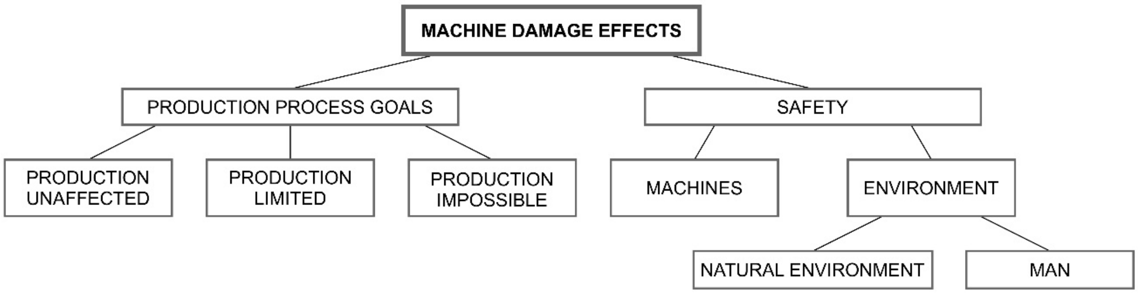


Fig. 1. Breakdown effects [5]

The possible effects caused by a breakdown of a machine taking part in the production process include: impossibility to continue production, reduced yield – delays in production, threat to operators or natural environment, increased risk of not keeping the delivery terms or worse quality of the products [1]. There are also failures which do not affect the production process – however, these are rare occurrences and concern mainly:

- machines which support but do not participate directly in the production process,
- machines that can be replaced (the company has surplus machinery),
- machines which do not have to take part in the production at a given time (there is a surplus of half-products manufactured with these machines – they are stored in the warehouse).

Current modern concepts of facility maintenance management to be implemented require considerable resources. Not every organisation can afford such sacrifices; also, the invested expenditure will not return a profit in every company. Changes can also be made to a limited extent, where it is absolutely necessary. To identify such areas, one must first of all consider the types of failures and major machines from the point of view of production process goals and safety. Based on the analysis of the indices of their failure rate, the management of their operation should be reorganized.

In this article single-screw extruders applied in the production of pipes used for installing heat pipelines have been studied. An example of this type of machine has been presented in Fig. 2.

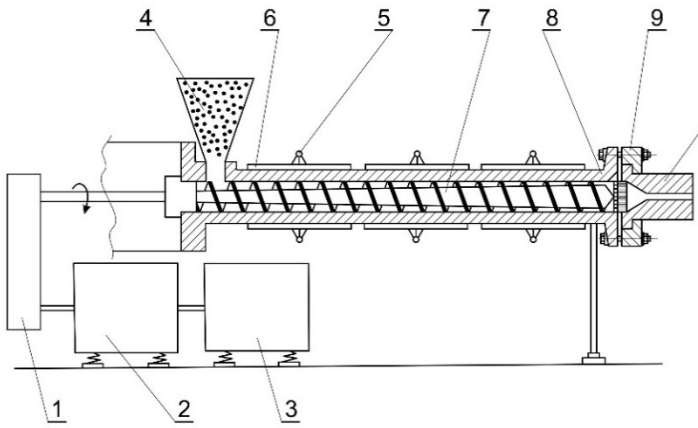


Fig. 2. Extruder diagram [6, 7]

Basic elements of a typical extruder are (Fig. 2):
1) electric motor driving the extruder screw,
2) transmission system for regulating the screw rotation speed,
3) drivetrain,
4) feed hopper for polyethylene granulate connected with a silos for granulate by means of an intake pipe,
5) extruder cylinder cooling system,
6) system for heating the plastic in the cylinder,
7) screw feeding the plastic to the head,
8) extruder body,
9) head installation,
10) head body.

Other necessary elements include the extruder parameters control system as well as a supply system with all indispensable connections and sensors.

Technological lines providing a basis for their production have been presented in Fig. 3.

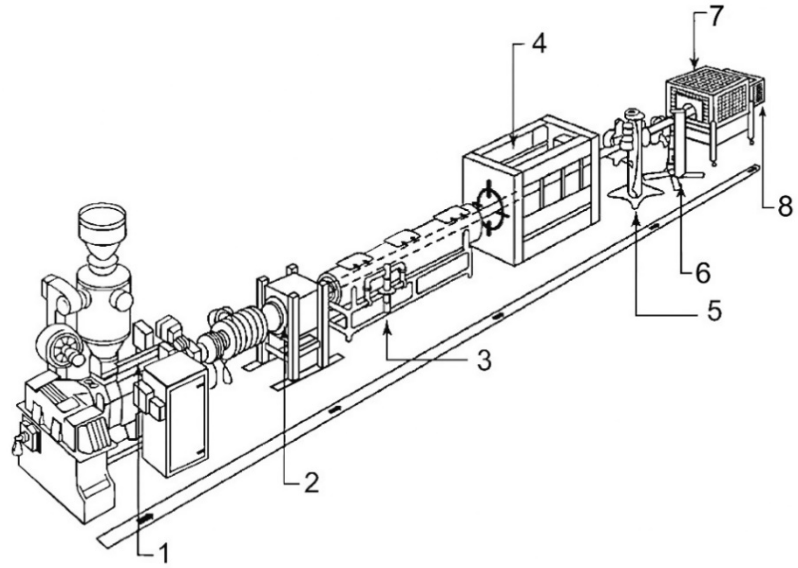


Fig. 3. Diagram of a pipe production line [8]

The basic machine in the pipe production line (Fig. 3) is an extruder (1), in which plastic is plasticized and a pipe is extruded. Next, the pipe, which is still plastic, gets to a vacuum calibrator (2), which is placed directly behind the extruder – it is here where the final dimension and shape of the product are determined. Behind the calibrator there is a cooling bath (3), where the pipe dimensions and shape are solidified. In the cooling bath the product moves owing to the rollers placed in the bath. Another element of the line is the caterpillar capstan (4), which ensures the pipe's movement through the production line. Behind the capstan there is a pipe stamping machine (5), which allows placing inscriptions on the product. Behind the stamping machine there is a device for plasma crowning, i.e. a pipe internal surface activator. At the end of the line there is a planetary saw (6), which cuts the pipes to the set length as well as an ejector (7), which puts ready products on the transporting trolley.

As can be seen in the enclosed diagram (Fig. 3), machines in the extruder line are placed in a serial configuration. For this reason, a breakdown of one of the line elements disables the whole production process.

2 Research results

The data on the machine failure rate was generated from the system supporting the facility maintenance management, and its processing was based on a spreadsheet. The data covered a period of 30 months and included information about: duration of machine downtimes, facility maintenance working time and a description of the breakdown given by the reporting person.

The initial analysis of the obtained results was aimed at identifying particular failures (according to reported descriptions) and assigning them to one of the following groups:

- mechanical breakdown: a mechanical failure of a part or some parts due to their wear or improper use, which must be removed by a mechanic;
- electric breakdown: an electric failure (of control or supply), which must be removed by an electrician having a licence;
- automation breakdown: a failure of the machine's control system, which must be removed by an automation specialist;
- anomaly: abnormal work of a machine resulting from its improper use or human error, which requires a specialist's intervention (e.g. inadvertent machine stoppage).

The results of the initial analysis have been presented in Fig. 4.

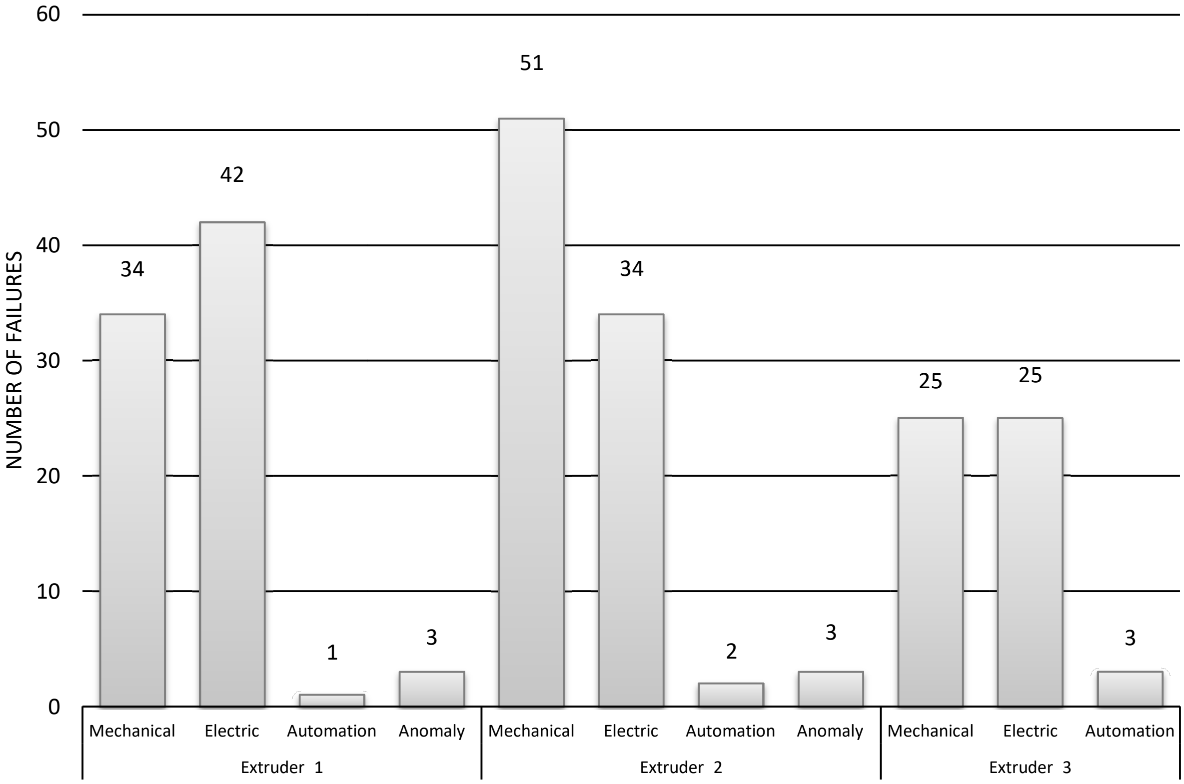


Fig. 4. Number of breakdowns by the type and machine

It was observed that irrespective of the type of machine, most breakdowns were mechanical and electric – accounting for nearly 95% of all the identified failures. Therefore, further analysis was focused on these two types of breakdowns.

The next step was to analyse mean time between failures for particular types of machines. MTBF indices for particular machines and types of failure were calculated from the following formula:

$$MTBF_m; MTBF_e = \frac{working\ time + tim\ \ of\ downtime}{n}$$

where:

MTBFm – mean time between mechanical failures,

MTBFe – mean time between electric failures,

n – number of failures.

The results of calculations have been presented in Fig. 5.

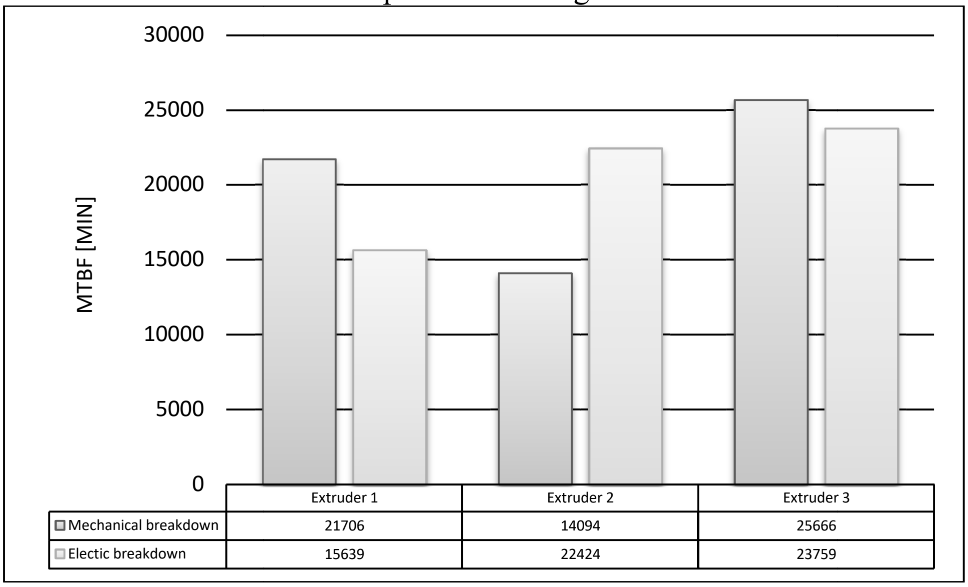


Fig. 5. MTBF for particular types of failures and machines

Based on the data resulting from calculations of MTBF indices for particular types of failures, the mean values for particular machines were calculated ($MTBF_s$). As the types of failures should be treated like a serial configuration, the formula has the following form:

$$MTBF_s = \left(\frac{1}{MTBF_m} + \frac{1}{MTBF_e}\right)^{-1}$$

$MTBF_s$ for Extruder 1 was 9090 minutes, for Extruder 2 – 8654 minutes, and for Extruder 3 – 12338 minutes. In the case of Extruder 1 it is electric failures that determined the low values of MTBF, while for Extruder 2 it was mechanical failures; in the case of Extruder 3 the results were comparable, but it had the lowest number of failures of all the extruders.

The MTBF index is determined by the working time and downtime, which depends on the failure removal duration and the time of waiting for spare parts. The results regarding the mean downtime components have been presented in Fig. 6, 7 and 8.

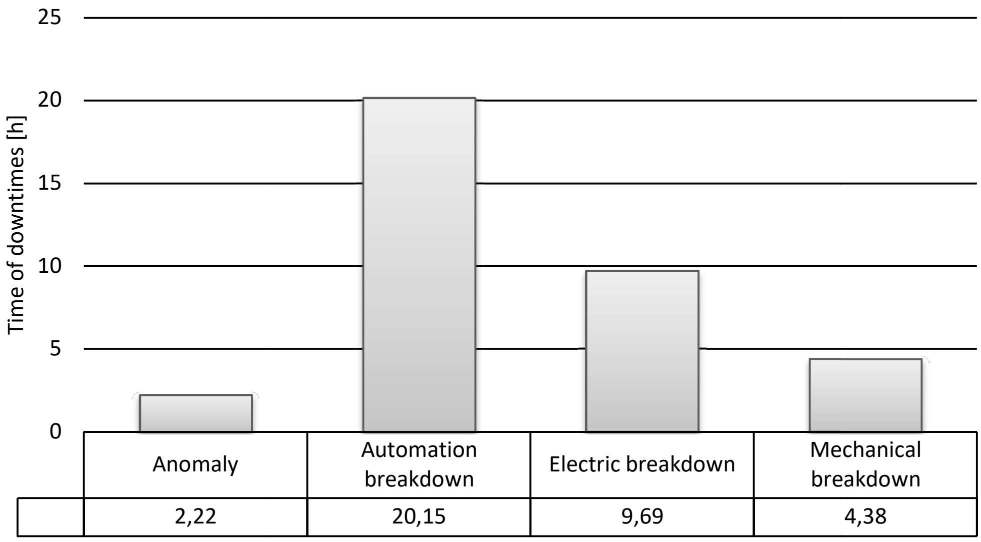


Fig. 6. Downtimes of machines

The longest downtime is observed in the case of automation failures – 20.2 hours; this value is twice as high compared to electric failures – 9.6 hours. The shortest downtimes are caused by anomalies – an average of 2.2 hours and mechanical failures – 4.4 hours.

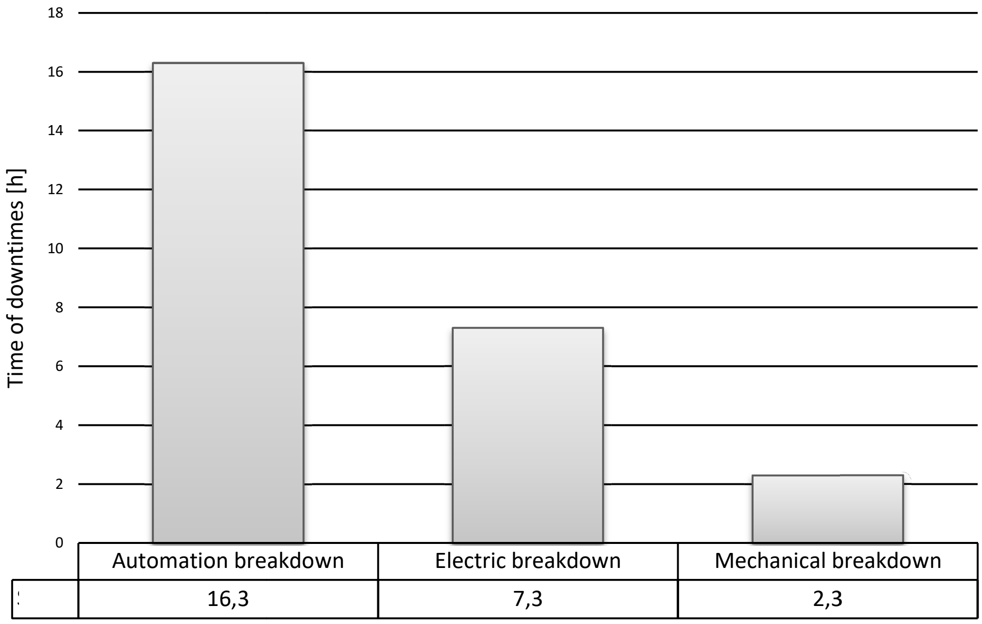


Fig. 7. Time of waiting for spare parts

As shown in Fig. 7, most of the time of downtimes caused by automation and electric failures is connected with waiting for spare parts. Waiting accounts for nearly 80% of the whole duration of downtime, as opposed to mechanical failures, in the case of which the waiting time is approximately 50% of the downtime duration.

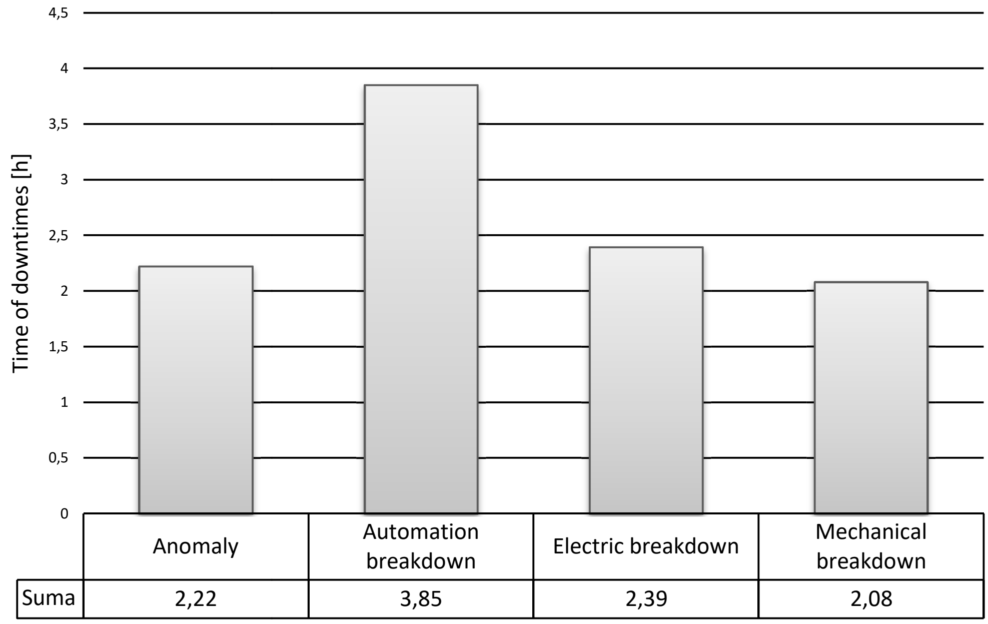
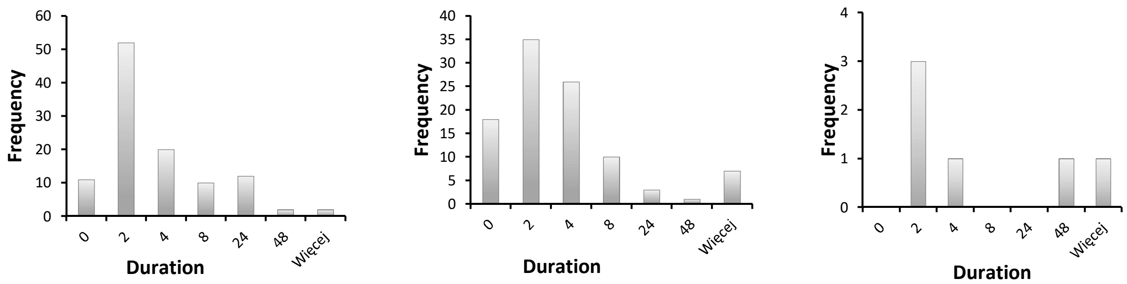


Fig. 8. Mean time of work related to failure removal

The results regarding the time of maintenance service teams' work contained in Fig. 6 demonstrate that the mean time of removing mechanical breakdowns, electric failures and anomalies is the same and reaches more than 2.2 hours, except for automation failures, in which case failure removal takes an average of nearly 4 hours.

The frequency of failures versus their duration has been shown in histograms placed in Fig. 9.



mechanical failures **electric failures** **automation failures**

Fig. 7. Histograms of failures duration

By far the biggest group among the most numerous mechanical and electric failures are breakdowns lasting no more than 2 – 4 hours; longer failures occur sporadically. The exception are automation failures, which occur sporadically, but last much longer.

Conclusion

The conducted analysis allows concluding as follows:

1. The most frequent are mechanical and electric failures – automation failures are very rare.
2. The most reliable machine is Extruder 3 – the failure rate of the other two machines is comparable.
3. The longest downtimes are caused by automation failures – followed by numerous electric failures.

4. Long downtimes result mainly from long waiting for spare parts.
5. The most numerous failures are the ones lasting maximum 2 hours.
6. The above conclusions allow formulating the following recommendation:
   – first the availability of parts for electric systems should be improved by increasing the availability of parts in the warehouse and establishing good contacts with suppliers,
   – as automation failures do not often occur and spare parts storage costs are high, it is necessary to establish very good relations with automation system suppliers and implement predictive/proactive actions,
   – to limit the duration of removing small mechanical and electric failures to minimum, it is necessary to increase the availability of the most frequently used parts in storerooms in the production hall and implement innovative technologies to support the repair process (time-sheets, augmented reality or virtual reality),
   – the number of anomalies should be minimized by providing information and organizing trainings which consolidate good practices in machine operation
   – due to a higher failure rate of Extruders 1 and 2, it is necessary to increase monitoring of these machines' technical condition and engage their operators in the process,
   – due to a very high number of mechanical and electric failures, employment in facility maintenance teams should be increased.

The results of the analysis of the machinery stock allow finding weak points in the facility maintenance process and enable developing improvement actions in work organization and warehouse management, which contributes to shortening the duration of downtimes caused by failures.

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ANALIZA NIEZAWODNOŚCI WYBRANYCH MASZYN W PRZEDSIĘBIORSTWIE PRODUKCYJNYM – STUDIUM PRZYPADKU

Abstrakt: Artykuł prezentuje wyniki analiz, jakich dokonano na podstawie danych zebranych w jednym z przedsiębiorstw produkujących rury preizolowane dla przemysłu ciepłowniczego. Dane pochodzące z systemu utrzymania ruchu zawierały m. in. długość czasu trwania awarii, czas usuwania awarii i rodzaj awarii dla najważniejszych urządzeń w przedsiębiorstwie jakimi są wytłaczarki rur z polietylenu. Przeprowadzona analiza pozwoliła na wyznaczenie wskaźników niezawodności. Otrzymane wielkości wraz z informacjami na temat rodzaju awarii umożliwiło wyciągnięcie wniosków zawierających zalecenia dla służb utrzymania ruchu w zakresie organizacji pracy techników, zarządzania zasobami ludzkimi i materialnymi oraz planowania przeglądów i remontów.

Słowa kluczowe: niezawodność, utrzymanie ruchu, organizacja, MTBF, awaryjność, awaria

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