



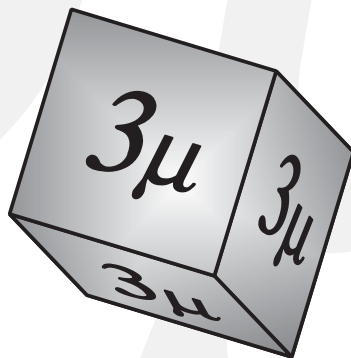
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Vážené kolegyně, vážení kolegové,

třináctý ročník mezinárodního semináře Moderní matematické metody v inženýrství, česko-polský seminář, se konal začátkem letošního června v Horní Lomné u Jablunkova. Ostravská pobočka Jednoty českých matematiků a fyziků a Katedra matematiky a deskriptivní geometrie VŠB - TU Ostrava již tradičně zajistily organizaci semináře. Do hotelu Excelsior přijelo více než 60 účastníků, z toho 30 zahraničních (28 z Polska a 2 ze Slovenska). Mezi účastníky byli opět v menšině ti dříve narození, organizátory velmi těší, že o seminář ve stále větší míře projevují zájem mladí pedagogové a vědci.

Pan doc. RNDr. Jindřich Bečvář, CSc. z Matematicko-fyzikální fakulty Univerzity Karlovy v Praze věnoval letos cyklus tří plenárních přednášek historii matematiky ve starověkém Řecku, konkrétně Archimédovi. V přednáškách Archimédés a číslo π , Archimédés a odmocniny, Archimédés a úloha o dobytku seznámil účastníky se zajímavými informacemi o jednom z nejvýznamnějších matematiků starověku.

Bylo předneseno celkem 45 referátů a komentováno 16 posterů. Odborně zaměřené příspěvky se týkaly matematického modelování, simulace, kódování, statistiky, aplikací matematiky v geologii, geodézii a ekonomii. Metodicky zaměřené příspěvky seznamovaly s analýzou náročnosti studia matematiky, přijímacími testy nebo úrovní znalostí studentů technických vysokých škol.

S velkým zájmem se opět setkal workshop GeoGebra, jehož pokračování se předpokládá i v dalších ročnících. Pokračoval rovněž kulatý stůl na téma Výuka matematiky na technických vysokých školách s výměnou názorů na obsah a rozsah vysokoškolské matematiky a zkušenosti s metodikou její výuky. Diskuse se také zabývala neustále klesající úrovní studentů, nastupujících na vysoké školy, plošným testováním na základních školách a státní maturitou z matematiky.

Na financování semináře byly použity prostředky EU z Fondu mikroprojektů v Euroregionu Silesia (CZ.3.22/3.3.04/13.03561). Umožnilo to účast širokému okruhu zájemců, včetně doktorandů a zejména výrazně vyšší účast polských kolegů z příhraničních vysokých škol.

Výstupem ze semináře jsou dva sborníky. Sborník všech příspěvků je vydán na CD, zatímco příspěvky v anglickém jazyce jsou publikovány v tradiční tištěné podobě. Příspěvky byly dodány ve formě camera ready, neprošly tedy odbornou ani jazykovou úpravou. Vybrané příspěvky vyjdou v časopisu AEEE (Advances in Electrical and Electronic Engineering) s tím, že jejich výběr provádí ediční rada AEEE s ohledem na odborné zaměření.

Na závěr si Vás dovoluujeme pozvat na příští, již 24. ročník semináře, který proběhne v termínu 1. – 3. června 2015 opět v Horském hotelu Excelsior v Horní Lomné.

Říjen 2014

Za organizační a programový výbor Jarmila Doležalová

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Inverse problems with application to linear ordinary differential equations

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Abstract: Linear ordinary differential equation with constant coefficients and with a noisy right hand side was considered. This type of problem arises in applications when the right hand side is a result of some measurement. We suggested a solution method based on the Laplace transform and the inverse problems theory. The influence of noise level on the performance of the proposed method was studied and the obtained results were compared with the efficiency of Euler's method.

1 Introduction

We consider the initial value problem for the linear ordinary differential equation with constant coefficients

$$\sum_{j=0}^N a_j y^{(j)}(t) = \tilde{g}(t), \quad (1)$$

$$y^{(j)}(0) = \tilde{y}_j, \quad \forall j = 0, \dots, N-1, \quad (2)$$

where $a_j \in \mathbb{R}$, $\forall j = 0, \dots, N$, $a_N \neq 0$. The values of function $\tilde{g}(t)$ and the initial conditions (2) are results of some measurement. It means, that the right hand side is given only on the finite set $0 \leq t_1 < t_2 < \dots < t_k < +\infty$, $k \in \mathbb{N}$, and the values $\{\tilde{g}(t_i)\}_{i=1}^k$ and $\{\tilde{y}_j\}_{j=0}^{N-1}$ are noisy. Furthermore, the number of measurements k cannot be arbitrarily high, because any measurement costs time and money. Due to this fact, it is important to keep k small.

We denote

$$\begin{aligned} \tilde{g}(t_i) &= g(t_i) + \varepsilon_i^g, \quad \forall i = 1, \dots, k, \\ \tilde{y}_j &= y_j + \varepsilon_j^y, \quad \forall j = 0, \dots, N-1, \end{aligned}$$

where $g(t_i)$, $i = 1, \dots, k$, and y_j , $j = 0, \dots, N-1$, are unknown exact values. The vectors $\varepsilon^g = (\varepsilon_1^g, \dots, \varepsilon_k^g)$ and $\varepsilon^y = (\varepsilon_0^y, \dots, \varepsilon_{N-1}^y)$ are the realizations of the measurement noise. We suppose that ε^g and ε^y are randomly drawn from a normal distribution with a mean of zero and a given standard deviation $\sigma^g > 0$ and $\sigma^y > 0$ respectively. The task is to reconstruct the solution of the following exact problem

$$\sum_{j=0}^N a_j y^{(j)}(t) = g(t), \quad (3)$$

$$y^{(j)}(0) = y_j, \quad \forall j = 0, \dots, N-1, \quad (4)$$

from the knowledge of $\{\tilde{g}(t_i)\}_{i=1}^k$ and $\{\tilde{y}_j\}_{j=0}^{N-1}$.

2 Laplace transform

The method based on the Laplace transform is a classical technique for solving linear ordinary differential equations with constant coefficients (see [6]). Let $Y(s) = L\{y\}(s)$ and $G(s) = L\{\tilde{g}\}(s)$. Because we know only the values $\{\tilde{g}(t_i)\}_{i=1}^k$, we cannot express the Laplace transform of $\tilde{g}$ analytically. However, the function $G(s)$ can be calculated numerically using, for example, the trapezoidal rule

$$G(s) \approx \frac{t_k}{2k} (e^{-st_1} \tilde{g}(t_1) + 2e^{-st_2} \tilde{g}(t_2) + \dots + e^{-st_k} \tilde{g}(t_k)). \quad (5)$$

We choose a set $0 < s_1 < s_2 < \dots < s_n < +\infty$. The values $G(s_i)$, $i = 1, \dots, n$, are calculated using the formula (5). The Laplace transform of problem (1) is

$$q(s)Y(s) - p(s) = G(s),$$

where

$$q(s) = \sum_{j=0}^N a_j s^{N-j}, \quad p(s) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-i-1} a_j s^{N-j-i-1}$$

in according to the properties of the Laplace transform (see [1] or [4]). Recently, we need to use the inverse Laplace transform to reconstruct $y(t)$ from the equation

$$Y(s) = \frac{G(s) + p(s)}{q(s)}. \quad (6)$$

The inversion of the Laplace transform is hard to calculate even for an analytically given function, because there is no universal method of the inverse Laplace transform (see [2]). In our case, we can compute only $\{Y(s_i)\}_{i=1}^n$ from the equation (6). According to [2], the knowledge of the values $\{Y(s_i)\}_{i=1}^n$ is enough to reconstruct an approximation $\hat{y}$ of function y . Therefore, we can use a numerical method to find the values $\{\hat{y}(T_i)\}_{i=1}^K$, where $0 \leq T_1 < T_2 < \dots < T_K < +\infty$.

2.1 Numerical inversion of the Laplace transform

The numerical inverse Laplace transform is a severely ill-posed problem and values $\{Y(s_i)\}_{i=1}^n$ are affected by an error due to the noisy right hand side of the original

problem (1). We approximate the Laplace transform by the use of the trapezoidal rule as in (5) by the following formula

$$Y(s) \approx \frac{T_K}{2K} (e^{-sT_1}y(T_1) + 2e^{-sT_2}y(T_2) + \dots + e^{-sT_K}y(T_K)). \quad (7)$$

By denoting $\tilde{\mathbf{m}}_i = Y(s_i)$, $i = 1, \dots, n$, and $\mathbf{f}_j = y(T_j)$, $j = 1, \dots, K$, we obtain a linear model

$$\tilde{\mathbf{m}} = A\mathbf{f}, \quad (8)$$

where

$$A = \frac{t_k}{2k} \begin{pmatrix} e^{-s_1T_1} & 2e^{-s_1T_2} & \dots & 2e^{-s_1T_{K-1}} & e^{-s_1T_K} \\ e^{-s_2T_1} & 2e^{-s_2T_2} & \dots & 2e^{-s_2T_{K-1}} & e^{-s_2T_K} \\ \vdots & \vdots & & \vdots & \vdots \\ e^{-s_nT_1} & 2e^{-s_nT_2} & \dots & 2e^{-s_nT_{K-1}} & e^{-s_nT_K} \end{pmatrix}. \quad (9)$$

The linear model (8) is severely ill-posed due to extremely small values (in comparison to the largest value), that appears in matrix (9). Hence, a regularization technique is needed to solve the problem (8).

3 Inverse problems

Inverse problems typically come from incomplete and indirect physical measurements. Theoretical breakthroughs and computational advances in the field of inverse problems are inspired by practical tasks. Inverse problems are extremely sensitive to errors and measurement noise. There are many important inverse problems which are ill-posed. For example a deconvolution problem, backward heat propagation, the inverse Laplace transform, image deblurring, X-ray tomography and electrical impedance tomography (see [3]).

We consider a finite-dimensional linear inverse problem in the following form

$$\mathbf{m} = A\mathbf{f} + \varepsilon, \quad (10)$$

where $A \in \mathbb{R}^{k \times n}$ is a matrix, $\mathbf{m} \in \mathbb{R}^k$ and $\mathbf{f} \in \mathbb{R}^n$. The measurement error $\varepsilon \in \mathbb{R}^k$ has bounded norm, i. e. $\|\varepsilon\| \leq \delta$ for some known $\delta > 0$.

The Moore-Penrose pseudoinverse of A is recommended in general to examine the severity of the ill-posedness of a problem. This method uses the singular value decomposition of A . More refined method is Tikhonov regularization. This technique takes into account known properties of the solution and provides smoothing. However, problems like image deblurring and X-ray tomography need a reconstruction with sharp edges. An edge-preserving method is total variation regularization. The listed solution methods are described in detail in [3]. Further information about the methods can be find also in [5].

4 Numerical experiments

We consider the following problem

$$y'' + y = \tilde{g}(t), \quad y(0) = 0, \quad y'(0) = 0,$$

where $\tilde{g}(t_i) = 1 + \varepsilon_i^g$, $i = 1, \dots, 16$, are given. Each component of the vector $\varepsilon^g \in \mathbb{R}^k$ was randomly drawn from a normal distribution having a mean of zero. The standard deviation of the normal distribution was taken as a variable $\sigma > 0$. The influence of $\sigma \in \langle 0\%, 30\% \rangle$ on the relative error (11) was studied and 95% confidence intervals of the relative error were computed by the use of Monte Carlo approach.

The relative error

$$\frac{\|y - \hat{y}\|_2}{\|y\|_2}, \quad (11)$$

where y is an exact solution of the problem, was used to measure the quality of an approximation $\hat{y}$.

Let

$$\begin{aligned} k &= 16, & \Delta t &= \frac{t_k}{k-1}, \\ K &= 128, & \Delta T &= \frac{T_K}{K-1}, \\ n &= 1024, & \Delta s &= \frac{s_n}{n}, \end{aligned}$$

where $t_k = T_K = 2\pi$ and $s_n = 20$. We set the meshes

$$\begin{aligned} t_i &= i\Delta t, & \forall i &= 0, \dots, k-1, \\ T_i &= i\Delta T, & \forall i &= 0, \dots, K-1, \\ s_j &= j\Delta s, & \forall j &= 1, \dots, n. \end{aligned}$$

Regarding Euler's method, the results were unsatisfactory with the relative error around 70% (see Figure 1). It was caused mainly by the low number of the given right hand side values. The proposed method with Tikhonov regularization variant gave the best results with the mean relative error around or lower than 10% (see Figure 1).

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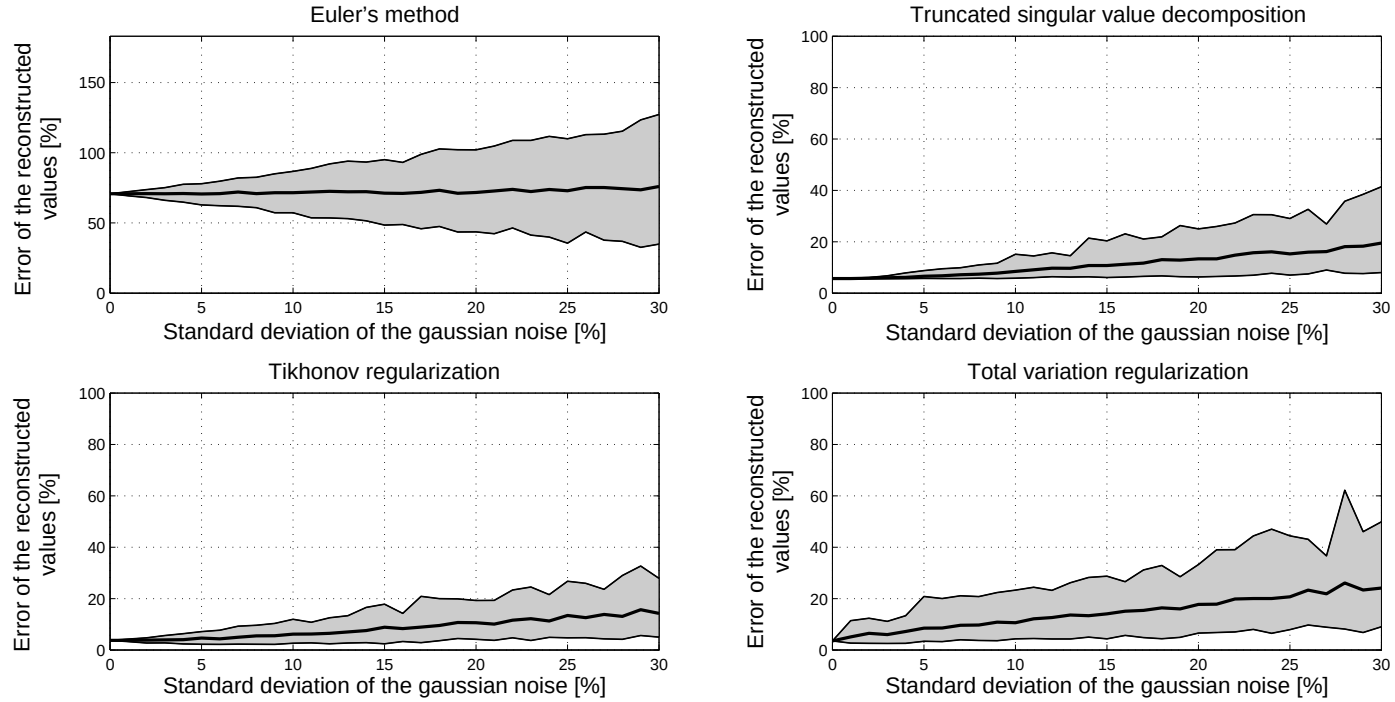


Figure 1: The performance of Euler's method and the proposed method with three different regularization variants depending on the standard deviation of the Gaussian noise. Grey areas represent 95% confidence intervals of the relative error and thick solid lines depict mean values of the error.

PROGNOZY JEDNODNIOWE MAKSYMALNEGO STĘŻENIA METANU NA WYLOCIE Z REJONU ŚCIANY EKSPLOATUJĄCEJ POKŁAD WĘGLA KAMIENNEGO – STUDIUM PRZYPADKU

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The article presents forecast models for the maximum methane concentration at the exit of the ventilation area of a coal extraction wall. These are one-variable models, where the independent variables are the maximum or mean methane concentrations from the previous day. The models were used for the forecasts of the maximum methane concentration at the exit of the B-8 wall area in the coal seam 648, and an analysis of errors in the forecast has been conducted.

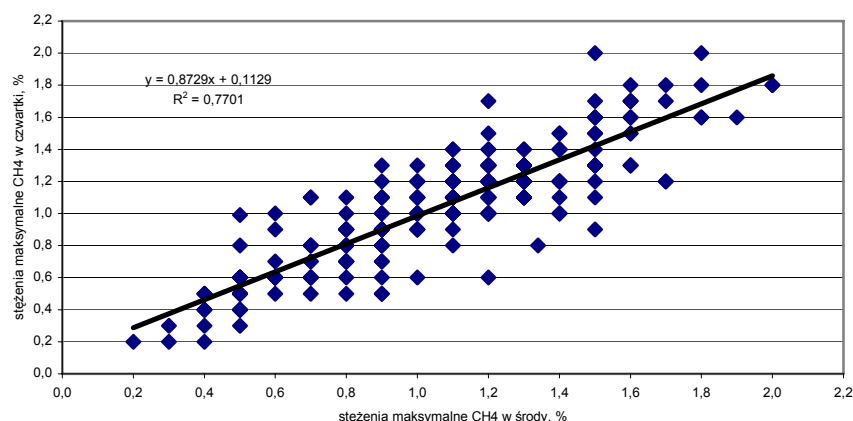
1. Wprowadzenie

Zagrożenie metanowe w polskich kopalniach węgla kamiennego, mimo bardzo dużych postępów w dziedzinie profilaktyki metanowej, stanowi jeden z najbardziej niebezpiecznych czynników w dużej liczbie kopalń. Około 75% wydobywania węgla kamiennego pochodzi z pokładów metanonośnych. Projektowanie odpowiedniej profilaktyki metanowej rozpoczyna się już w fazie projektowania eksploatacji określonej ścianą (wyrobiskiem wybierkowym). Dane wejściowe do prognoz zawierają informacje odnośnie metanonośności pokładu przeznaczonego do eksploatacji, metanonośność warstw i pokładów leżących powyżej i poniżej eksploatowanego pokładu oraz ich odległości od pokładu eksploatowanego. Pozyskanie danych odnośnie metanonośności można uzyskać wykonując otwory wiertnicze do warstw metanonośnych, co jest jednak bardzo kosztowne. Najczęściej dane te uzyskuje się na zasadzie interpolacji pomiędzy wynikami pomiarów w dość znacznie oddalonych od siebie miejscach. Jest to jedną z przyczyn, że prognozy wykonane przed eksploatacją obciążone są dość dużym błędem. Następstwem tego faktu jest konieczność prowadzenia bieżących korekt profilaktyki. Przydatne w tym względzie mogą być prognozy krótkookresowe stężenia metanu w wyrobiskach związanych z konkretną ścianą. W dalszej części artykułu omówiono sposób budowy modeli prognostycznych, z pomocą których wykonano prognozy jednodniowe maksymalnego stężenia metanu na wylocie z rejonu ściany.

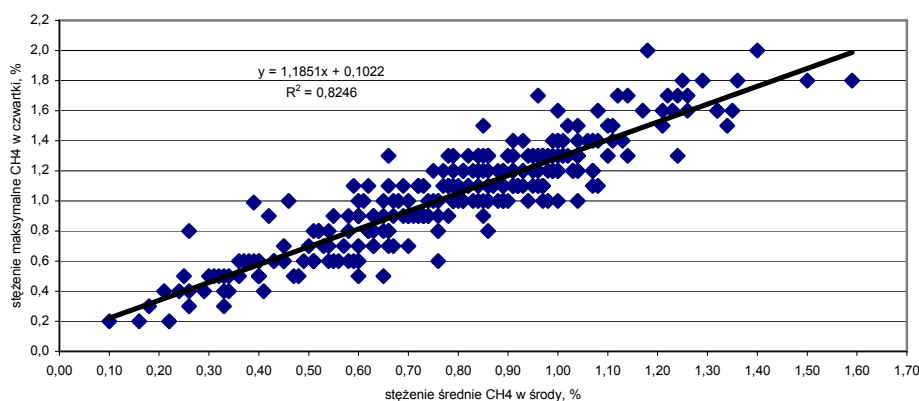
2. Budowa statystycznych modeli prognostycznych

Statystyczne modele prognostyczne zostały zbudowane na podstawie próby liczącej 2239 przypadków, przy czym każdy z nich odnosił się do jednego dnia [1]. Na każdy

przypadek składały się trzy dane, a mianowicie średnie, maksymalne i minimalne stężenie metanu w określonym dniu. Dane pozyskano z pomiarów stężenia metanu, prowadzonych w sposób ciągły przez systemy telemetryczne. Wykorzystano pomiary wykonane na wylocie z rejonów ścianowych, czyli w chodnikach łączących ścianę z innymi wyrobiskami, przez które płynęło powietrze nie przepływające przez ścianę. Ściany eksploatowały pokłady zalegające w różniących się od siebie warunkach naturalnych, były także przewietrzane w różnych układach przewietrzania i różną ilością powietrza. Analiza danych wykazała między innymi, że istotny występuje silna zależność pomiędzy maksymalnym stężeniem metanu w danym dniu a średnim stężeniem metanu w dniu poprzednim oraz że występuje także silna zależność pomiędzy maksymalnym stężeniem w danym dniu a maksymalnym stężeniem w dniu poprzednim. Stwierdzono także, że zależności te różnią się pomiędzy sobą dla różnych dni tygodnia. Przykłady takich zależności przedstawiono na rysunkach 1 i 2.



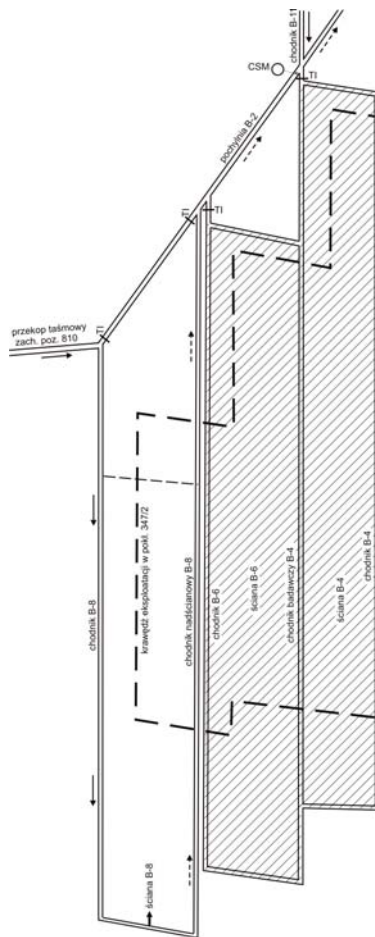
Rys. 2. Zależność maksymalnych stężeń metanu w czwartki od maksymalnych stężeń metanu w środy



Rys. 2. Zależność maksymalnych stężeń metanu w czwartki od średnich dobowych stężeń metanu w środy

Parametry funkcji obliczono dla poszczególnych dni tygodnia. Estymowano je za pomocą klasycznej metody najmniejszych kwadratów. W przypadku estymacji parametrów równań liniowych posiadających jako zmienną niezależną zmienną opóźnioną, zastosowanie klasycznej metody najmniejszych kwadratów może nie być najbardziej efektywne. Problemy związane z powyższym zagadnieniem przedstawiono w pracach [2, 3, 4, 5, 6].

3. Charakterystyka danych pomiarowych



Rys. 3. Schemat ściany B-8 w pokładzie 348

Dane pomiarowe uzyskano z czujnika stężenia metanu usytuowanego na wylocie z rejonu ściany B-8 w pokładzie 348 w jednej z kopalń Jastrzębskiej Spółki Węglowej S.A. Rysunek 3 przedstawia schemat ściany B-8, a położenie czujnika pomiarowego oznaczono kółkiem opisanym jako CSM. Strzałki narysowane linią ciągłą oznaczają przepływ powietrza do ściany (powietrze świeże), a liniami przerywanymi kierunek przepływu powietrza po przewietrzeniu ściany (powietrze zużyte). Powietrze zużyte odprowadza metan, który wydzielił się do wyrobisk rejonu ściany, a jego stężenie w powietrzu mierzy czujnik CSM. Do obliczeń wykorzystano pomiary stężenia metanu w okresie 347 dni. Na ich podstawie obliczono dla każdego dnia średnie i maksymalne stężenie metanu na wylocie z rejonu ściany, czyli w miejscu zainstalowania czujnika CSM.

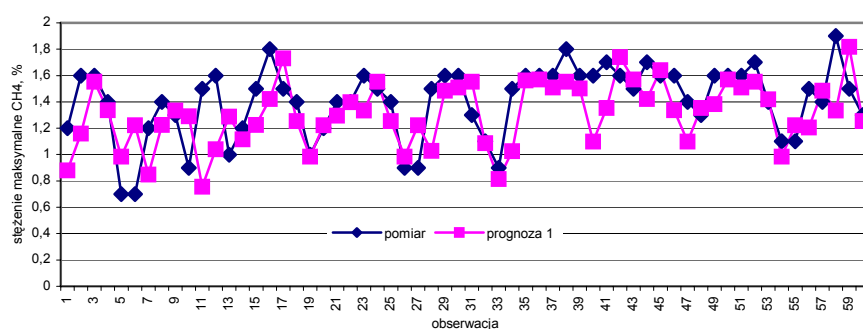
Średnie stężenia metanu w całym okresie obserwacji charakteryzują się następującymi parametrami. Średnia wartość średnich stężeń metanu wynosi 0,77% CH₄, mediana 0,78% CH₄. Maksymalna wartość średniego stężenia metanu wynosiła 1,32% CH₄, a

minimalna 0,21% CH₄. Stężenie średnie nie posiadało rozkładu normalnego ani rozkładu gamma. Rozkład odznaczał się niewielką skośnością i był nieco spłaszczony w stosunku do rozkładu normalnego.

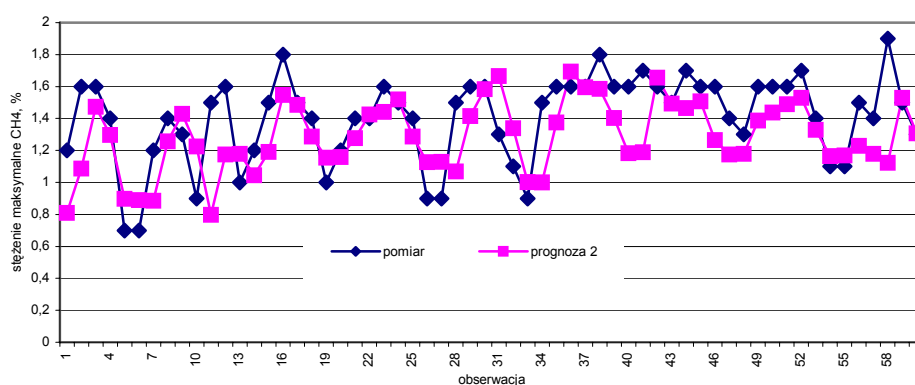
Średnia wartość stężenia maksymalnego wynosiła 1,01% CH₄, a mediana 1,0% CH₄. Kres górny stężenia maksymalnego wynosił 2% CH₄, a kres dolny 0,3% CH₄. Również rozkład maksymalnego stężenia nie był rozkładem normalnym ani rozkładem gamma. Odznaczał się niewielką skośnością i był spłaszczony w stosunku do rozkładu normalnego.

4. Prognozy maksymalnego stężenia metanu na wylocie z rejonu ściany B-8

Wykonano dwa rodzaje prognoz maksymalnego stężenia metanu na wylocie z rejonu ściany B-8. Pierwszy rodzaj prognoz (prognozy 1) wykorzystywał zależności maksymalnego stężenia metanu od maksymalnego stężenia metanu w dniu poprzednim, a drugi rodzaj prognoz (prognozy 2) wykorzystywał zależności maksymalnego stężenia metanu od wartości średniej stężenia metanu w dniu poprzednim. Prognozy wykonano dla 336 dni (za wyjątkiem pierwszego dnia w okresie prowadzenia obserwacji). **Rysunki 4 i 5** przedstawiają wartości pomiarowe i prognozowane w wybranych częściach okresu obserwacji.



Rys. 4. Wartości pomiarowe oraz obliczone wg prognozy 1 w wybranym przedziale okresu obserwacji



Rys. 5. Wartości pomiarowe oraz obliczone wg prognozy 2 w wybranym przedziale okresu obserwacji

Dla poszczególnych dni w okresie prognoz obliczono wartości błędu bezwzględnego i względnego, a ich parametry statystyczne przedstawiono w tabeli 1.

Tabela 1. Parametry charakteryzujące błędy bezwzględne i względne dla wykonanych prognoz maksymalnego stężenia metanu.

Parametr	Błędy prognoz 1		Błędy prognoz 2	
	bezwzględne, %CH ₄	względne, %	bezwzględne, %CH ₄	względne, %
średnia	0,2	20,5	0,2	18,6
pierwszy kwartył	0,1	6,0	0,1	6,2
mediana	0,1	14,1	0,1	13,0
trzeci kwartył	0,2	26,7	0,2	25,4
dziewiąty decyl	0,3	44,5	0,3	39,6
maksimum	1,5	364,7	1,4	122,1

Z analizy parametrów zawartych w tabeli 1 wynika, że zarówno w przypadku prognoz 1 jak i prognoz 2 wartości parametrów charakteryzujących błędy bezwzględne prognoz są prawie identyczne. Połowa wartości błędów bezwzględnych nie przekracza dokładności pomiarowej czujników stężenia metanu, wynoszącej 0,1%CH₄, 25% błędów nie przekracza 0,2%CH₄, a 15% błędów nie przekracza wartości 0,3%CH₄. Na podstawie błędów względnych prognoz można stwierdzić, że nieco dokładniejszą metodą prognozy jest prognoza 2, czyli prognoza wykorzystująca jako zmienną niezależną maksymalne stężenie metanu w dniu poprzednim.

5. Wnioski

Przeprowadzona analiza wyników prognoz maksymalnego stężenia metanu na wylocie z rejonu ściany nasuwa następujące wnioski:

1. średnia wartość błędów bezwzględnych prognoz nie przekracza 0,2 %CH₄,
2. połowa wyników prognoz była wykonana z dokładnością odpowiadającą dokładności pomiarowej czujników stężenia metanu, stosowanych w kopalniach, wynoszącą 0,1 %CH₄,
3. 25% błędów bezwzględnych nie przekracza 0,2 %CH₄, a 15% błędów nie przekracza 0,3 %CH₄,
4. dokładniejsze wyniki prognoz otrzymano stosując jako zmienną niezależną średnie stężenie metanu w dniu poprzednim. Wartość średnia błędu bezwzględnego tych prognoz nie przekracza 19%, a połowa błędów względnych nie przekracza 13% wartości pomiarowych maksymalnego stężenia metanu,
5. analiza potwierdziła, że zaproponowane w pracy [1] prognozy mogą być przydatne do praktycznego wykorzystania w kopalniach węgla kamiennego.

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MATEMATIKA V I. ROČNÍKU FBI VŠB - TU OSTRAVA Z POHLEDU STUDENTŮ

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Abstrakt: Na klesající úroveň matematických znalostí studentů technických vysokých škol si stěžují všichni, koho se tato skutečnost týká. S nedostatečnými vědomostmi z matematiky se potýkají rovněž vyučující odborných předmětů. Příčiny tohoto stavu jsme se pokusili zjistit formou anonymního dotazníku, který měl odrazit pohled studenta i pedagoga.

Abstract: All who may be concerned with the decreasing level of mathematical knowledge of the students of technical higher education institutions complain of this fact. With insufficient knowledge of mathematics, teachers of special subjects wrestle as well. We made an effort to establish the causes of this state by means of an anonymous questionnaire, which was to show the opinions of both students and teachers.

1 Úvod

Na klesající úroveň matematických znalostí studentů technických vysokých škol si stěžují všichni, koho se tato skutečnost týká. S nedostatečnými vědomostmi z matematiky se potýkají rovněž vyučující odborných předmětů.

Příčiny tohoto stavu jsme se pokusili zjistit formou anonymního dotazníku, který vyplnili studenti tří fakult VŠB-TU: Fakulty strojní (FS, 231 student), Fakulty stavební (FAST, 207 studentů) a Fakulty bezpečnostního inženýrství (FBI, 143 studenti). Otázky byly formulovány tak, aby dotazník odrazil také pohled studenta.

Předložený článek se týká Fakulty bezpečnostního inženýrství. Dotazník byl předložen studentům třetího ročníku. Proto je datový soubor menší než v případě jiných fakult a neobsahuje studenty, kteří neuspěli u zkoušky z předmětů Matematika I a Matematika II.

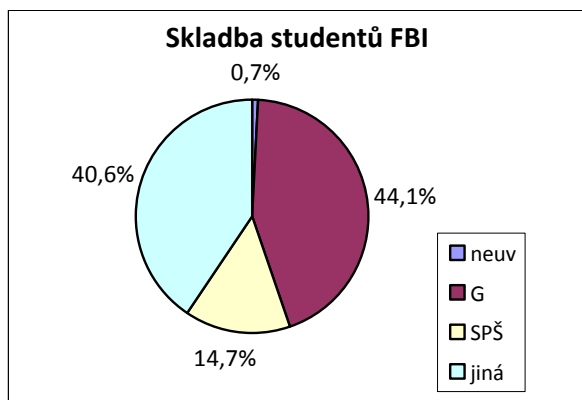
Možné příčiny ?

- Složení studentů
- Zájem o studium
- Absence maturitní zkoušky z matematiky, připravenost z matematiky ze SŠ

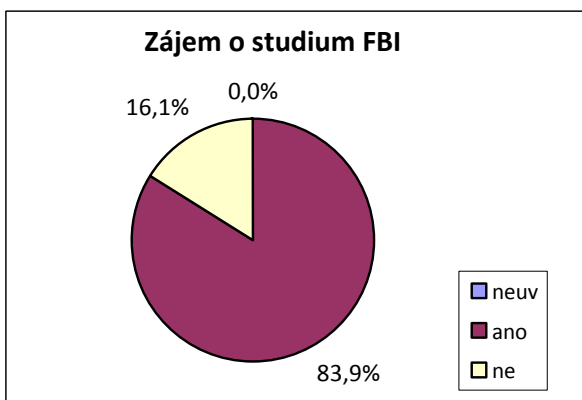
2 Složení studentů, zájem a připravenost na studium oboru

Procento absolventů gymnázií je poměrně velké (44%). Překvapivě velké je procento škol označených jako jiné (41%). Zde je však výsledek bohužel zkreslen, protože do této

kategorie byla zahrnuta i specializovaná Střední odborná škola požární ochrany, která by měla být zařazena spíše do kategorie průmyslových škol (obr. 1).

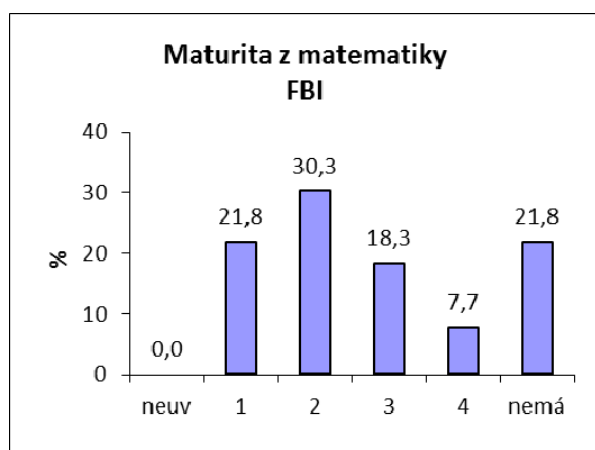


obr. 1

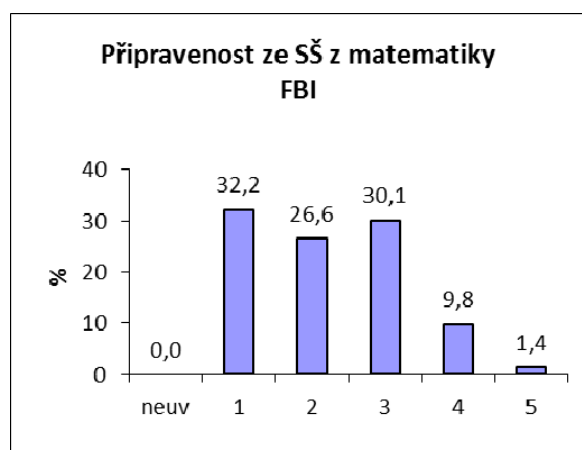


obr. 2

Se složením studentů velmi úzce souvisí zájem o fakultu, na níž studují. Téměř 84% respondentů se na příslušnou fakultu přihlásilo z vlastního zájmu (obr. 2).



obr. 3



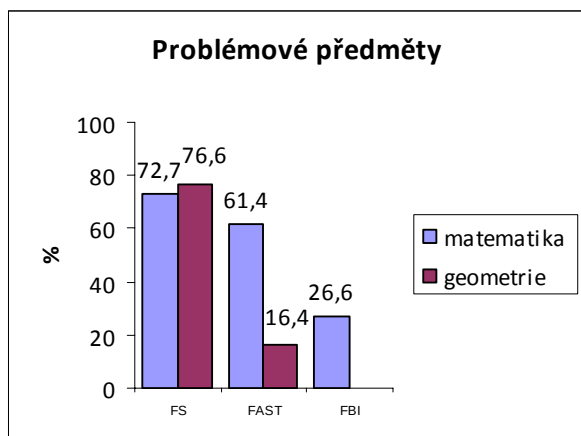
obr. 4

Maturitní zkoušku z matematiky složilo téměř 80% studentů, přitom s výborným výsledkem 22%, s chvalitebným výsledkem 30% (obr. 3). Tato informace je překvapující a výsledky, kterých studenti dosahují v prvním ročníku (viz dále), jí neodpovídají.

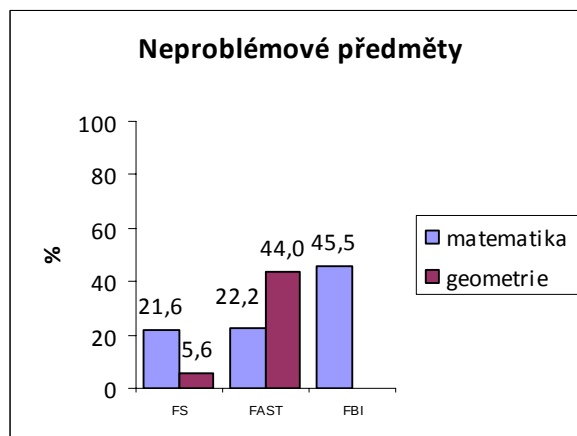
Složení studentů se odráží rovněž v jejich odpovědi na otázku, jak je střední škola připravila z matematiky (obr. 4) pro studium na technické vysoké škole. Téměř 60% respondentů hodnotí svou přípravu z matematiky jako kvalitní (32% výborně, 27% chvalitebně), což je zřejmě dáno zastoupením absolventů gymnázií.

Podobně, podíváme-li se na problémové a neproblémové předměty (z pohledu studentů), je matematika problémovým předmětem pouze pro čtvrtinu studentů FBI (obr. 5, obr. 6) Na obrázcích pro zajímavost uvádíme názory respondentů i z jiných fakult. Pro úplnost je třeba zmínit, že na FBI se geometrie nevyučuje.

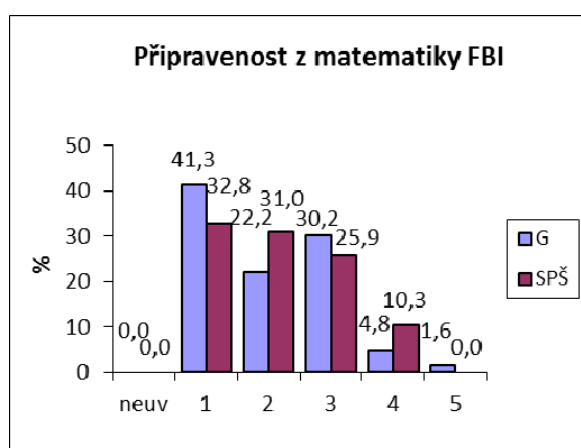
Při podrobnějším zkoumání dat se jeví jako přednost, že studenti gymnázií jsou připraveni z matematiky i geometrie lépe než studenti SPŠ (obr. 7, obr. 8).



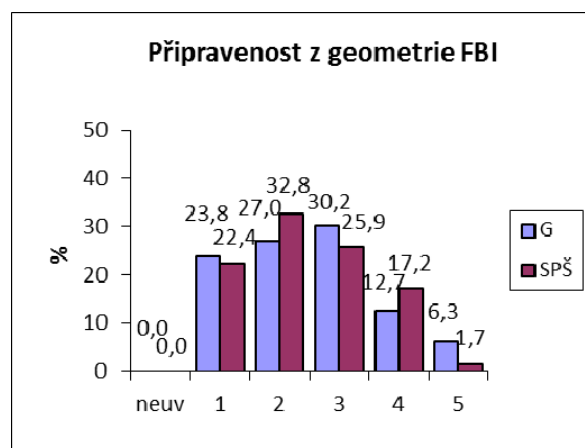
obr. 5



obr. 6



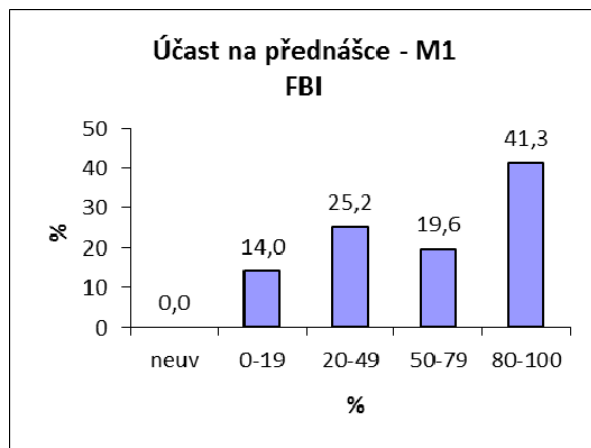
obr. 7



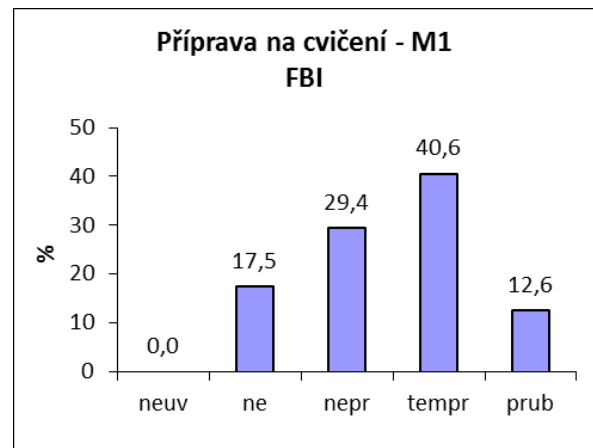
obr. 8

3 Pohled ze strany pedagoga

Bez ohledu na názor studentů je zřejmé, že matematika je Achillovou patou studentů technických vysokých škol. Proto je zarážející nízká účast na přednáškách (39% má menší než poloviční účast), špatná příprava na cvičení (17% se nepřipravuje vůbec, 29% ne průběžně, jen 53% se připravuje průběžně nebo téměř průběžně, viz obr. 9 a obr. 10). I s přihlédnutím ke skutečnosti, že řada studentů je nucena si na studium privydlávat, což však nebude hlavní příčinou tohoto stavu, jde o žalostné zjištění.

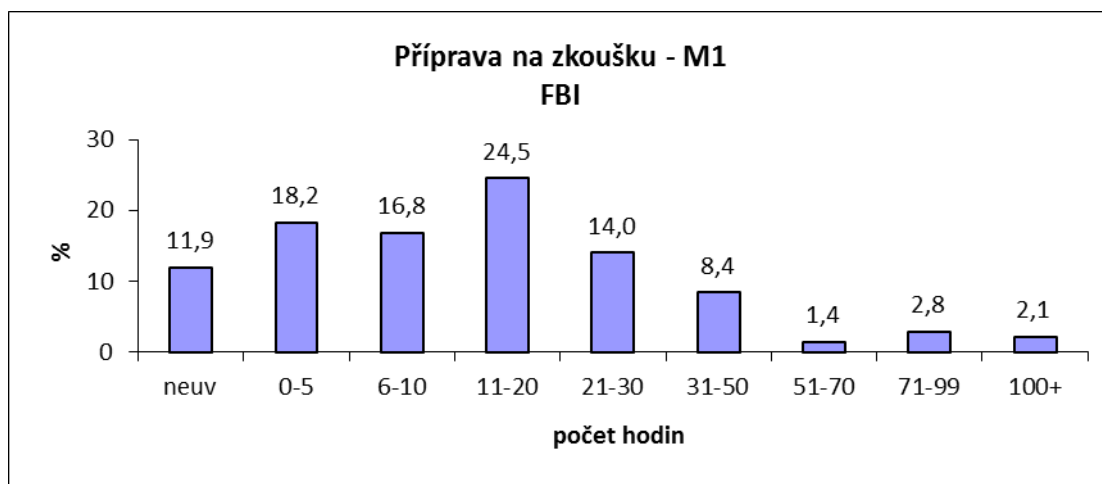


obr. 9

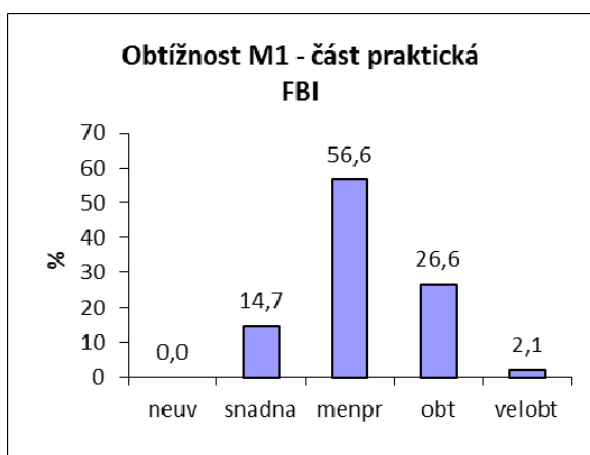


obr. 10

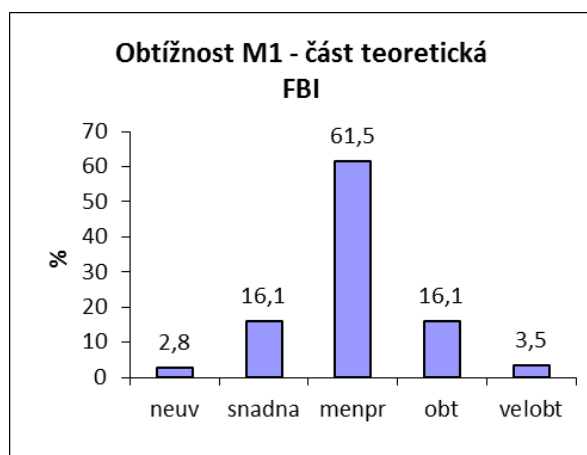
Podobně je tomu s přípravou na zkoušku (obr. 11). Graf na obrázku snad ani nepotřebuje komentáře. Připravit se na zkoušku z matematiky během pěti resp. deseti hodin můžeme považovat spíše za pokus o vtíp. Přitom podle odpovědí studentů jen 15% resp. 16% považuje praktickou (obr. 12) resp. teoretickou část (obr. 13) zkoušky z matematiky za snadnou. Uvedené výsledky se týkají předmětu Matematika 1, v případě předmětu Matematika 2 jsou zjištění podobná.



obr. 11



obr. 12



obr. 13

4 Závěr

K závěrečným steskům je nutno poznamenat, že neutěšeným stavem vzdělávání nejsou vinní studenti. Bez systémových změn ve vzdělávací soustavě je náprava nemožná. Pokud se nezredukuje přebujelé střední školství, kde maturita je cár papíru, který není odrazem znalostí, nicméně je vstupenkou do (rovněž přebujelého) vysokého školství, které podbízivě soutěží o studenty, prodává diplomy za školné a umožňuje absolvování v rekordně krátkých časech (mnohdy samozřejmě legislativně ošetřené), není z propasti, v níž se teď nalézáme, cesty zpět. Snad je v ČR právě odstartovaný projekt Matematika+ zablesknutím na lepší časy.

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Determination of dynamic parameters at impact of freely falling mass

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Abstract: *The one of the case of dynamic load often occurring in practice is the impulse load caused by the freely falling impact mass. Quantities characterizing this phenomenon are time of action of dynamic impulse and maximum value of force acting on the struck body. On this basis one can determine values of dynamic and restitution coefficients. In the article there are presented results of tests based on which, values of these coefficients for elements of support of dog headings used in mining excavation were determined. Also there was presented analysis of dynamic impulse and ways of determination of its characterizing parameters were discussed.*

1. Introduction

Mechanical interaction in a form of impact of freely falling mass is one of the most common sources of dynamic load. During this phenomenon there occurs energy transfer from one body to another, as a result of their direct contact.

Very significant meaning for this exchange has contact time of bodies and its character (elastic, elastic-plastic or plastic). In a case of mechanical impact the forces of mutual interaction between bodies acting in defined time interval, and not continuously as it is in a case of static loading.

Also very significant change of mechanical parameters of impacted body is observed. Together with increasing velocity of impact, the plasticity limit and ultimate strength for compression increase. In a case of plastic deformations also phenomenon of plastic delay is observed [5].

Analysis of impact phenomenon of freely falling mass is the most often carried out with use of principle of conservation of energy [4, 5, 6]. The equation describing an impact has a form:

$$\frac{1}{2} m \cdot v^2 = \int_0^{\varepsilon_{max}} S \cdot l \cdot \sigma \cdot d\varepsilon \quad (1)$$

where:

m – impact mass, kg,

v – velocity of impact, m/s,

S – cross-section area of impacted body, m²,

l – length of impacted body, m,

ε_{max} – maximum unit shortening, m.

From the above equation one can determine maximum value of relative deformation (ε_{max}) and stresses (σ_{max}) of the impacted body.

$$\varepsilon_{max} = v \sqrt{\frac{m}{E \cdot S \cdot l}}; \quad \sigma_{max} = \sqrt{\frac{E \cdot m}{S \cdot l}} \quad (2)$$

Mechanical impact is also the source of wave processes occurred in the colliding bodies. As a result of impact, a disturbance in a form of particles motion in the perpendicular and parallel direction to the axis of impacted body is appeared.

This disturbance displaces with defined velocity depending on mechanical properties of the medium in which it propagates. In a case of continuous medium, that velocity depends on its density and elastic properties.

Wave equation (for longitudinal plane wave) describing the motion of particles of impacted body has a form [4]:

$$\frac{\partial^2 u}{\partial t^2} = v^2 \cdot \frac{\partial^2 u}{\partial x^2} \quad (3)$$

Velocity of propagation of wave in a solid body, one can determine from dependence:

$$v = \sqrt{\frac{E}{\rho}} \quad (4)$$

where: ρ – density of body, kg/m³.

In a case of transverse wave, it is assumed that its velocity of propagation in solid bodies equals about 0.66 velocity of longitudinal wave [5]. In practice for description of impact phenomenon the restitution and dynamic coefficients are used.

Coefficient of restitution (k) defines dependence between normal components of velocity of impact (V_u) and reflection (V_o) of impact mass from the investigated body [1, 2].

$$k = -\frac{V_o}{V_u} \quad (5)$$

Value of this coefficient defines character of impact and is not depending on dimensions of colliding bodies, but properties of materials, of which they are made of.

Dynamic coefficient (k_d) with an impact of freely falling mass from a specified height, there is defined as ratio of maximum force acting in body ($P_{d\max}$) to the value of force acting on this body as a result of its static loading (P_{st}) of the same mass [3]:

$$K_d = \frac{P_{d\max}}{P_{st}} \quad (6)$$

Presented phenomenon of impact is very often a source of dynamic loadings acting on support applied for protection the underground mining headings. Cracking layers of rocks, rocks ripping from the roof and side walls of heading during displacement strike the support, which cause large hazard for its proper operation.

For this reason in the article there is presented results of stand tests of impact, freely hit falling mass of section, of which friction props using in mining heading are made of and samples of different materials in order to determine dynamic parameters describing this phenomenon, which are dynamic and restitution coefficients.

2. The results of stand tests

Aim of the first stage of stand tests was to determine dynamic parameters for samples made of different materials during impact with freely falling mass. Samples made of steel, wood and rubber, were tested on a special test stand, which description with the testing methodology was presented in a paper [2].

As a result of performed tests, time courses of displacement of impact mass and force with which this mass acts on tested samples were determined. In Figure 1, there is presented overall schematic of the system for tests (Fig. 1a) and time courses of registered forces (Fig. 1b) for samples made of different materials for impact energy equal to 62,3 J.

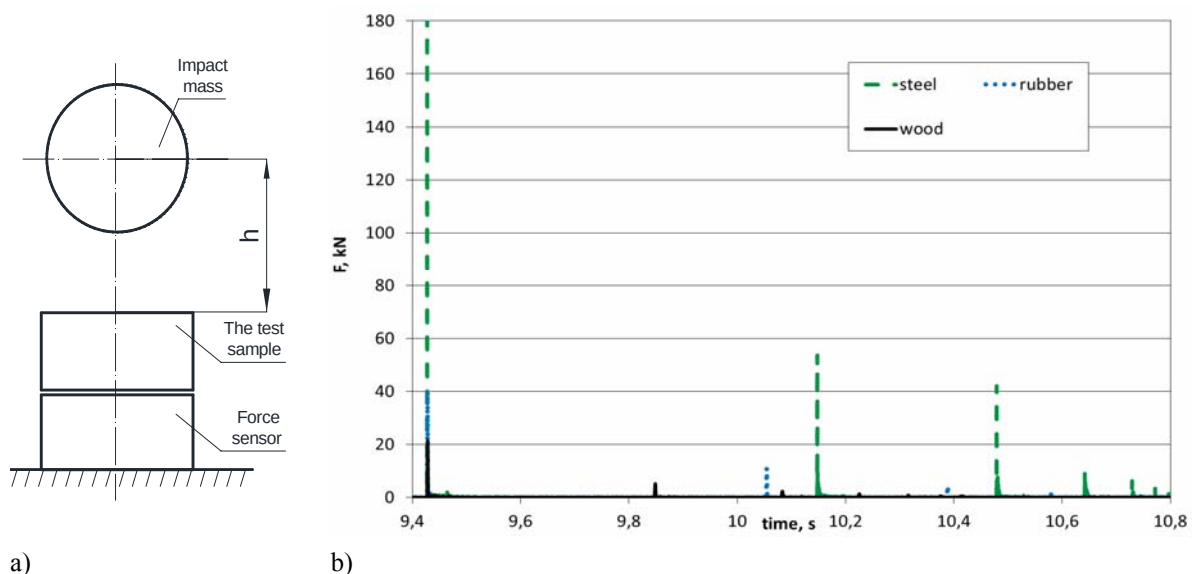


Fig. 1. Scheme of the system for tests (a) and time course of registered forces (b)

Based on obtained results, values of restitution and dynamic coefficients for tested samples were determined (Fig. 2). During determination of these dependences also data from paper were used [2].

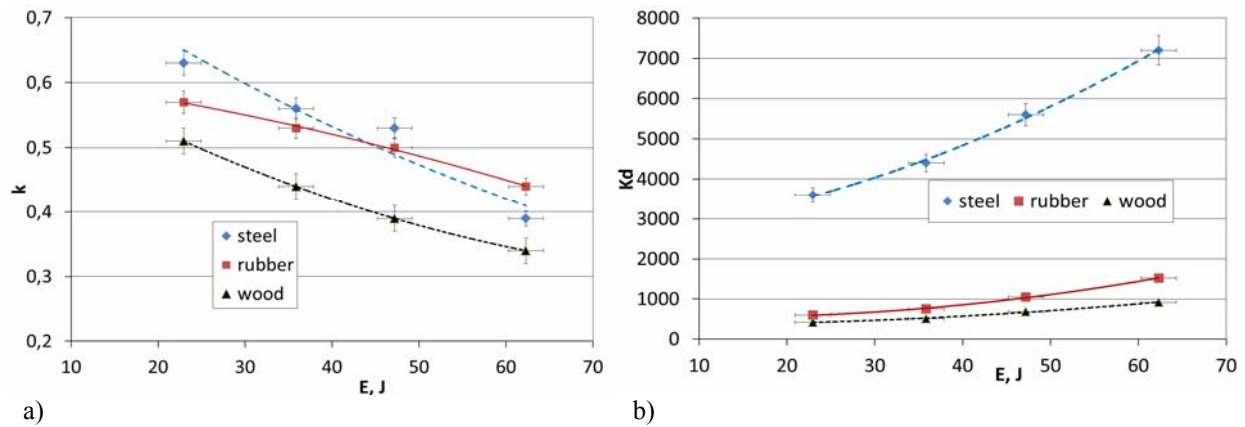


Fig. 2. Values of restitution (a) and dynamic (b) coefficients for tested materials

Obtained results clearly indicate, that together with increasing impact energy, values of determined coefficients increase.

In next stage, stand tests included straight segment of section V29 from which friction props used in yielding support of dog heading axial loading with impact of freely falling mass.

Test consisted on loading section of length 2m with mass (4000kg) falling from specified height supported on cross-bar (1600kg) of section [3]. As a result of performed tests, time courses of forces acting at the ends of sections were determined.

In a Figure 3, there is presented loading scheme of tested section (Fig. 3a) and determined during the test, time courses registered forces (Fig. 3b) for impact energy equals to 7,85 kJ. Sensor (1) registered force (T) on section, and sensor (2) force (R) under the section.

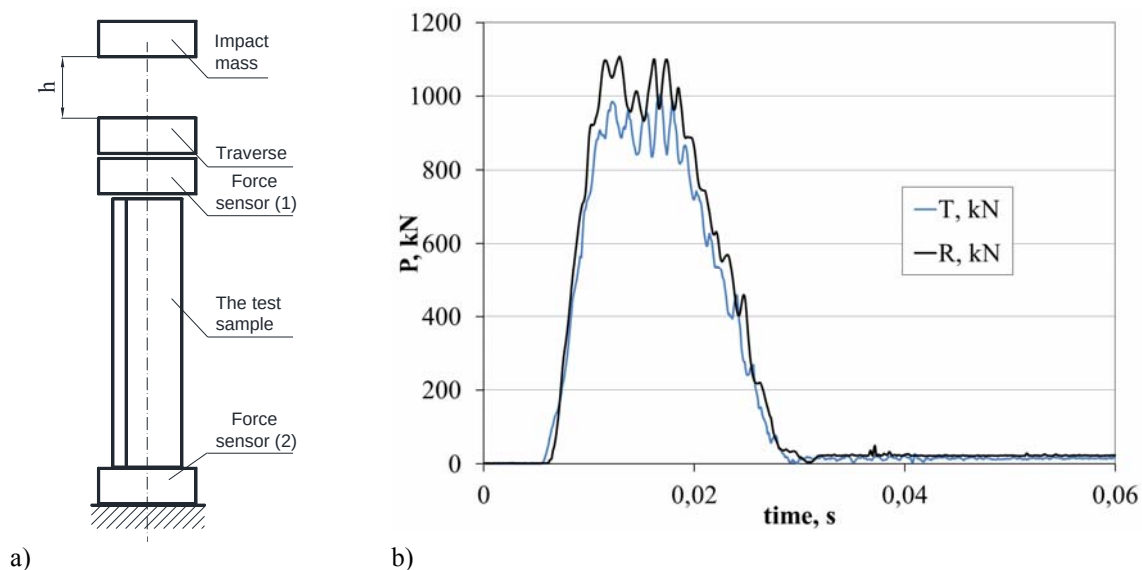


Fig. 3. Scheme of loading tested section (a) and time course of registered forces (b)

Based on obtained courses values of restitution and dynamic coefficients for different values of impact energy were determined (Fig. 4).

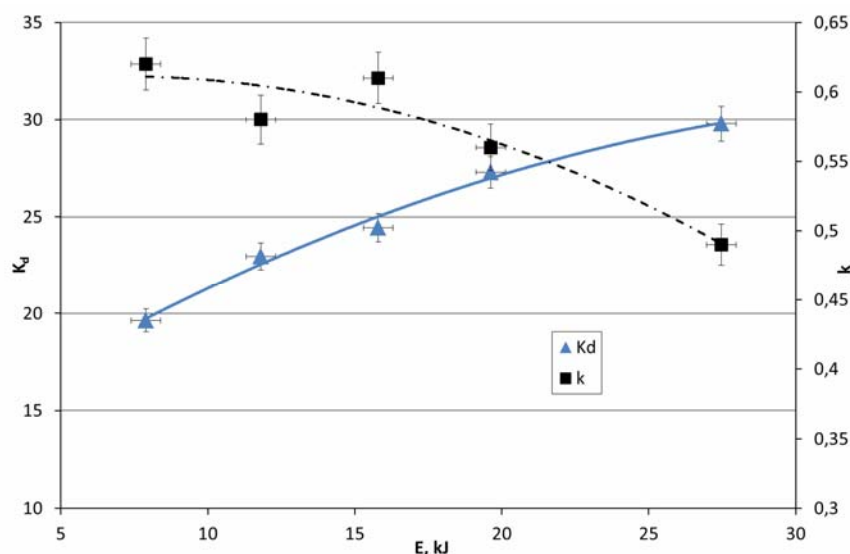


Fig. 4. Values of restitution (k) and dynamic (K_d) coefficients for different values of impact energy (E)

3. Conclusions

Based on performed tests and analyses, one can conclude, that dynamic loading is more inconvenient way of loading of mechanical elements than static load.

This is based on determined values of dynamic coefficient for tested samples and sections. Values of forces acting on tested elements in a case of their loading with the impact mass are few times greater than in a case of static load with the same mass.

Analysis of the values of dynamic coefficient for both tested cases, indicated that very significant impact on its values has way of implementation of load and rigidity of tested system. Greater height from which the impact mass falls (despite its small value), results in significant increase of value of this coefficient (first stage of tests).

Whereas values of restitution coefficient enable to determine the type of impact, and its decreasing value together with increase of the impact energy results from increasing deformation of contacting bodies.

Also material from which tested elements are made of and their geometry has very significant impact on the values of both determined coefficients.

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MATEMATIKA V PROJEKTU AMBASADOŘI PŘÍRODOVĚDNÝCH A TECHNICKÝCH OBORŮ

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Abstrakt: Zaměřením projektu Ambasadoři přírodovědných a technických oborů je změna postojů studentů středních škol v Moravskoslezském kraji k přírodovědným a technickým oborům formou popularizace těchto oborů netradičními formami a zvýšení motivace žáků ke vzdělávání se v přírodovědných a technických oborech. V rámci tohoto projektu jsme se zapojili nejenom jako ambasadoři, ale snažili jsme se přesvědčit studenty, že není důvod bát se matematiky.

Abstract: The one of the case of dynamic load often occurring in practice is the impulse load caused by the freely falling impact mass. Quantities characterizing this phenomenon are time of action of dynamic impulse and maximum value of force acting on the struck body. On this basis one can determine values of dynamic and restitution coefficients. In the article there are presented results of tests based on which, values of these coefficients for elements of support of dog headings used in mining excavation were determined. Also there was presented analysis of dynamic impulse and ways of determination of its characterizing parameters were discussed.

1. Projekt Ambasadoři přírodovědných a technických oborů

Cílem projektu je motivovat žáky středních škol na území celého Moravskoslezského kraje ke studiu přírodovědných a technických oborů při volbě dalšího studia (zaměřeno primárně na žáky gymnázií, tam je zaznamenán vysoký odliv talentů směrem k humanitním a sociálním oborům).

Aktivita 1 - Ambasadoři přírodovědných oborů

- popularizace přírodovědných oborů s cílem motivovat žáky středních škol ke studiu matematiky, fyziky a chemie
- prohloubení znalostí studentů v přírodovědných oborech - M, F, CH

Aktivita 2 – Ambasadoři technických oborů

- popularizace technických oborů s cílem motivovat žáky středních škol ke studiu technických oborů
- prohloubení znalostí studentů v technických oborech
- pomoc pedagogům při výuce technických oborů

Aktivita 3 – Studentská síť ambasadorů

- vybudování sítě vrstevnické motivace

2. Naše aktivity v projektu

Do projektu jsme se zapojili jako ambasadoři přírodovědných oborů.

Požadavkem pro naši účast na projektu bylo, aby práce byla systematická, dlouhodobá, pravidelná a inovativní s konkrétní, předem známou cílovou skupinou studentů SŠ z Moravskoslezského kraje. Měla by mít konkrétní výstupy. Aktivita by měla být zaměřena především na studenty se zájmem o výzkumnou a vývojovou činnost a její pokračování v budoucnu. Mělo by se jednat o určitou konkrétní skupinu zájemců, ne celou třídu či všechny žáky školy. Aktivita se bude konat na dobrovolném základě a ve volném čase cílové skupiny.

Výsledkem našich úvah, jak splnit zadání projektu, bylo, že ambasadoři přírodovědných oborů budou působit ve dvou krocích.

V prvním, čistě motivačním vstupu, představí studentům matematiku jako zajímavý předmět studia: zdůrazní, kde všude se s ní setkáváme v praxi; co všechno nám pomáhá řešit; atd. Součástí představení bude lidský aspekt, osobní příběh prezentujícího. Úkolem hodinové diskuze se studenty tedy nebude odborný výklad daného tématu, ale představení matematiky jako zajímavé vědy, která je součástí konkrétního života jedné osoby (typicky otevřeného vědce, který žáky dokáže nadchnout nebo doktorandského studenta, který je studentům blízko i věkově). Tyto diskuze budou realizovány odbornými pracovníky VaV a doktorandy či studenty magisterských programů, vždy místo 1 vyučovací hodiny na vybraných středních školách. Jejich cílem je zapůsobit na co nejširší masu studentů SŠ, vytvořit v nich povědomí o možnostech matematiky, eliminovat její vnímání jako „příliš těžké disciplíny“ a vzbudit zájem.

Druhý krok na motivaci navazuje. Je časově náročnější, vyžaduje aktivní účast studentů a má za cíl prohloubení poznatků žáků SŠ. Jedná se o vzdělávací moduly matematiky a deskriptivní geometrie „jinak“. Moduly budou, oproti motivačním diskuzím, výběrovou aktivitou. Účastní se jich ti žáci, kteří projeví zájem o matematiku a to ve svém volném čase.

Moduly jsme nazvali „Není nutné bát se matematiky“ a „Není nutné bát se deskriptivní geometrie“.

3. Není nutné bát se matematiky

První částí tohoto modulu bylo víkendové setkání se studenty SŠ v prostotách VŠB-TU Ostrava. Studenti tak měli možnost se seznámit s životem na naší škole.

Setkání jsme zahájili motivační přednáškou, ve které se studenti seznámili s tématy, která se probírají na jednotlivých fakultách během studia, ukázali jsme jim, jaké studijní materiály jsou pro jejich studium připraveny.

Fakulty VŠB – TU Ostrava a rozsah výuky

HGF – **Základy matematiky, Bakalářská matematika I** – funkce jedné proměnné a její diferenciální počet, základy algebry a analytické geometrie; **Bakalářská matematika II** – integrální počet funkce jedné proměnné, diferenciální počet funkce dvou proměnných, obyčejné diferenciální rovnice, **Matematika na počítači a základy programování, Inženýrská matematika** – pravděpodobnost, diferenciální a integrální počet funkcí více proměnných, křivkový integrál.

FMMI – **Matematika I** – funkce jedné proměnné a její diferenciální počet, základy algebry a analytické geometrie; **Matematika II** – diferenciální počet funkce dvou proměnných, integrální počet funkce jedné proměnné, obyčejné diferenciální rovnice.

FS – **Základy matematiky, Matematika I** – funkce jedné proměnné a její diferenciální počet, základy algebry a analytické geometrie; **Matematika II** – diferenciální počet funkce dvou proměnných, integrální počet funkce jedné proměnné, obyčejné diferenciální rovnice, **Matematika III** – pravděpodobnostní počet a statistika.

FAST – **Matematika I** – funkce jedné proměnné a její diferenciální počet, základy algebry a analytické geometrie; **Matematika II** – diferenciální počet funkce dvou proměnných, integrální počet funkce jedné proměnné, obyčejné diferenciální rovnice, **Matematika III** - pravděpodobnostní počet a statistika.

FBI – **Matematika I** – funkce jedné proměnné a její diferenciální počet, základy algebry a analytické geometrie; **Matematika II** – diferenciální počet funkce dvou proměnných, integrální počet funkce jedné proměnné, obyčejné diferenciální rovnice, **Inženýrská matematika** – integrální počet funkcí více proměnných, křivkový integrál, řady.

Po úvodní motivační přednášce práce probíhala ve skupinách, kde na jednoho lektora připadalo cca 15 studentů.

Pro studenty byly vytvořeny pracovní listy, ve kterých si studenti ve spolupráci s lektorem „snad“ našli odpověď na jejich věčnou otázku „Proč se to máme učit?“

Kvadratická rovnice	
Zadání : Nalezněte stacionární body funkce $f(x) = \frac{x-1}{e^{3+17x}}$.	
Řešení : Nalézt stacionární body funkce znamená nalézt taková $x \in D(f)$, že platí $f'(x) = 0$. Jsou to body, v nichž může funkce nabývat svého lokálního minima nebo maxima. $f'(x) = e^{\frac{-x-1}{3+17x}} \cdot \left(\frac{3+2-x-x^2}{(3+x)^2} \right) \stackrel{?}{=} 0$.	
Exponenciální člen $e^{\frac{-x-1}{3+17x}}$ je vždy kladný a tedy nemulový. Stejně tak jmenovatel $3+x^2$. Jedinou možností, kdy může být derivace nulová, jsou nulové body čitatele. Jednoduchým vytknutím převedeme čitatele do normovaného tvaru, tj. $3+2x-x^2 = -(x^2-2x-3)$. Jeho kořeny tak můžeme kořeny určit pomocí Vietových vzorců. Budeme hledat x_1 a x_2 , takové že pro ně platí $x_1 + x_2 = 2, \quad x_1 x_2 = -3.$	
Takovými čísly jsou $x_1 = -1, x_2 = 3$ a čitatele člen lze zapsat jako $3+2x-x^2 = -(x+1)(x-3).$	
Nulové body čitatele, jsou hledanými stacionárními body funkce $f(x)$. Řešením tedy jsou $x_1 = -1$ a $x_2 = 3$.	



JA...

Kvadratická rovnice : Vyřešte rovnice, použijte vzorce s diskriminantem	
i) $x^2 - 2x + 3 = 0$ ii) $x^2 - 2x - 3 = 0$ iii) $x^2 + 2x - 3 = 0$ iv) $x^2 + 2x + 3\frac{2}{3} - \frac{1}{6} = 0$	Řešení : Řešení: $a = 1, b = -2, c = 3$. Diskriminant D rovn $D = b^2 - 4ac$. Ukážeme si, že $D < 0$, tedy rovnice nemá řešení. $D = (-2)^2 - 4 \cdot 1 \cdot 3 = 4 - 12 = -8$. Protože $D < 0$, rovnice nemá řešení. ii) $a = 1, b = -2, c = -3$. $D = (-2)^2 - 4 \cdot 1 \cdot (-3) = 4 + 12 = 16$. $\sqrt{D} = \sqrt{16} = 4$. $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$. $x_1 = \frac{-(-2) + 4}{2 \cdot 1} = \frac{2 + 4}{2} = \frac{6}{2} = 3$. $x_2 = \frac{-(-2) - 4}{2 \cdot 1} = \frac{2 - 4}{2} = \frac{-2}{2} = -1$. Řešení: $x_1 = 3, x_2 = -1$. iii) $a = 1, b = 2, c = -3$. $D = 2^2 - 4 \cdot 1 \cdot (-3) = 4 + 12 = 16$. $\sqrt{D} = \sqrt{16} = 4$. $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$. $x_1 = \frac{-2 + 4}{2 \cdot 1} = \frac{2}{2} = 1$. $x_2 = \frac{-2 - 4}{2 \cdot 1} = \frac{-6}{2} = -3$. Řešení: $x_1 = 1, x_2 = -3$. iv) $a = 1, b = 2, c = 3\frac{2}{3} - \frac{1}{6} = \frac{20}{6} - \frac{1}{6} = \frac{19}{6}$. $D = 2^2 - 4 \cdot 1 \cdot \frac{19}{6} = 4 - \frac{76}{6} = 4 - \frac{38}{3} = \frac{12}{3} - \frac{38}{3} = -\frac{26}{3}$. Protože $D < 0$, rovnice nemá řešení.



JA...

Obr. 2 - ukázka materiálů – látka 1. ročníku SŠ




Kvadratická nerovnice : Vyřešte	
i) $x^2 - 3x - 2 < 2$	Rozhodnutí : Provedeme vše na pravou (nebo levou) stranu, kde abychom posunovali kvadratickou nerovnici vzhledem k znaménku. Pokud je znaménko kladné, budeme dělit, pokud záporné, budeme násobit. Pokud je znaménko nulové, budeme dělit. i) $x \in (-1, 4)$ ii) $x \in (-\infty, -3) \cup (2, \infty)$ iii) $x \in (-2, 0)$
ii) $1 - x - x^2 < -5$	
iii) $x - 3 \leq 3 - 3x - 2x^2$	






JA...

Obr. 3 - ukázka materiálů – látka 1. ročníku SŠ

Druhé části modulu, která se konala vždy v pátek odpoledne po 12 týdnu, se zúčastnili ti studenti, kteří měli zájem o téměř individuální lekce. Ve skupinkách o 2-4 studentech jsme se jim věnovali pouze podle jejich potřeby. Sami přišli s tématy, která pro ně byla obtížná, často se jednalo o maturitní otázky. Naší snahou bylo nejen osvětlit problém, ale také poukázat na souvislosti s jinou látkou, které jim zatím unikaly.

4. Závěr

Projektu se zúčastnili studenti SŠ, kteří měli opravdový zájem o matematiku. Velmi kladně hodnotili propojení jednotlivých oblastí matematiky. Potvrdili, že možnost seznámit se s tím, jak probíhá výuka matematiky a deskriptivní geometrie na VŠB-TU Ostrava, je zbavila obavy ze studia na vysoké škole technického zaměření.

Chceme tímto také poděkovat organizátorům projektu Ambasadoři přírodovědných a technických oborů, že nám umožnili se ho zúčastnit. Byla to pro nás velmi příjemná zkušenost.

PEWNE NIETYPOWE KSZTAŁTY NIECKI POWYROBISKOWEJ W MODELU KNOTHEGO-BUDRYKA TYPU CAUCHY'EGO.

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Abstrakt: W artykule omówiono zmiany kształtu powyrobowiskowej niecki obniżeniowej przy różnych kształtach wyrobiska.

Abstract: In the article discusses some land change in the model S. Knothe – W. Budryk with Cauchy type influence function and unusual excavation.

Jednym z najważniejszych problemów w górnictwie podziemnych jest określenie jaki wpływ na powierzchnię ma prowadzona eksploatacja. Zmiany terenu powstałe w wyniku podziemnych prac górniczych przyjmują postać tak zwanej powyrobowiskowej niecki obniżeniowej. Istnieje wiele metod opisujących kształt powyrobowiskowej niecki obniżeniowej. Jedną z najpopularniejszych jest metoda S. Knothe – W. Budryka. Do opisu powyrobowiskowej niecki obniżeniowej wykorzystuje ona tak zwaną funkcję wpływów. W modelu S. Knothe - W. Budryka stan ustalony (końcowy nie zmieniający się już w czasie) powyrobowiskowej niecki obniżeniowej opisany jest wzorem

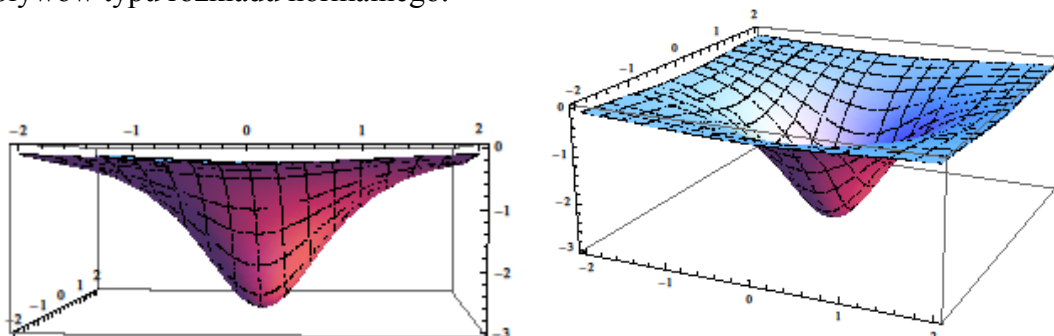
$$O(x,y) = \iint_K W(x-s, y-k) ds dk$$

gdzie: O – funkcja obniżenia terenu, W – funkcja wpływów, K – obszar wyrobiska. Ogromny wpływ na kształt powyrobowiskowej niecki obniżeniowej ma kształt samego wyrobiska.

W artykule zostanie omówione kilka przykładów zmian powyrobowiskowej niecki obniżeniowej w zależności od kształtu wyrobiska przy wykorzystaniu funkcji wpływów typu Cauchy'ego. Wykorzystywana funkcja wpływów typu Cauchy'ego ma postać

$$W(x, y) = \frac{1}{2\pi} \frac{hkr}{(1 + rx^2 + ry^2)^{3/2}}$$

gdzie: h – grubość pokładu, k – współczynnik kierowania stropem, r – parametr rozproszenia wpływów. Wykres tej funkcji jest zbliżony do klasycznej funkcji wpływów typu rozkładu normalnego.



Funkcja wpływów typu Cauchy'ego.

W wersji typowej wyrobisko znajduje się na stałej głębokości, posiada stałą wysokość oraz rzutem jego jest prostokąt. W efekcie posiada kształt prostopadłościanu.



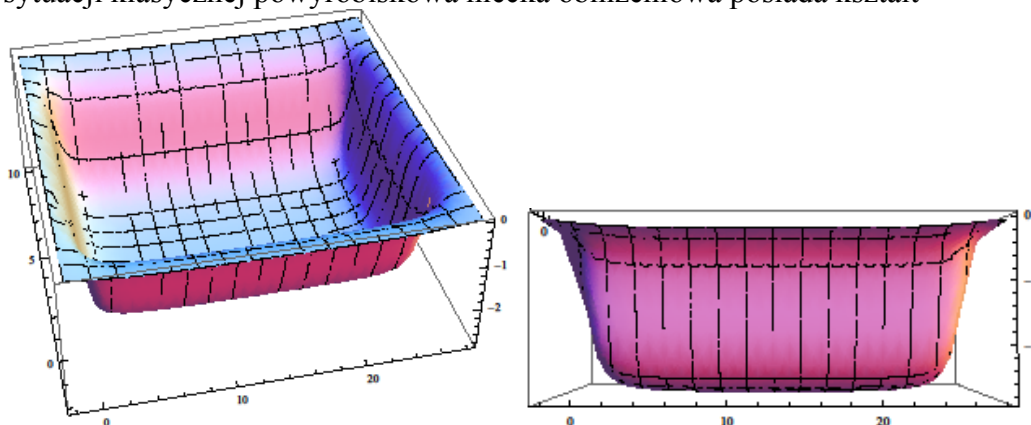
Wyrobisko klasyczne

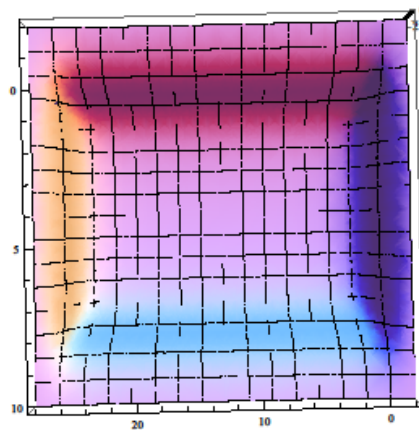
Dla takiego wyrobiska kształt powyrobiskowej niecki obniżeniowej można obliczyć z następującego wzoru

$$O(x, y) = \iint_K \frac{1}{2\pi} \frac{hkr}{[1 + r[(x-s)^2 + (y-k)^2]]^{3/2}} dsdk$$

gdzie $K = \{(x, y) | x \in [a, b], y \in [c, d]\}$.

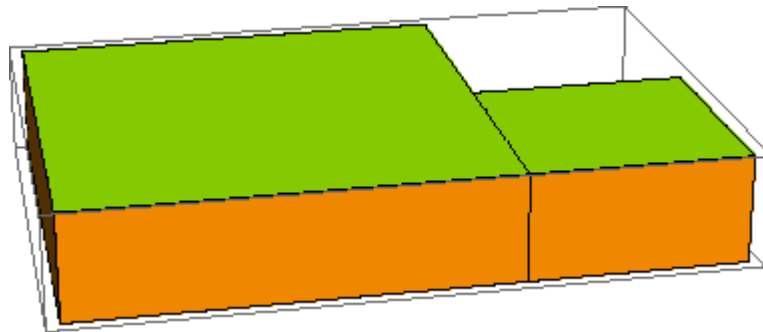
W sytuacji klasycznej powyrobiskowa niecka obniżeniowa posiada kształt





Powrobiskowa niecka obniżeniowa wersja klasyczna.

W praktyce nie zawsze udaje się zachować front robót o tej samej długości. W takiej sytuacji wyrobisko nadal zachowuje tę samą głębokość i wysokość, natomiast zmienia się kształt rzutu. Wtedy w obliczeniach kształtu powrobiskowej niecki obniżeniowej zmianie ulega figura po której obliczamy całkę podwójną. Omówione zostaną dwa przykłady opisującą tą sytuację. Pierwszy przykład opisuje sytuację jednostronnego skrócenia frontu robót.



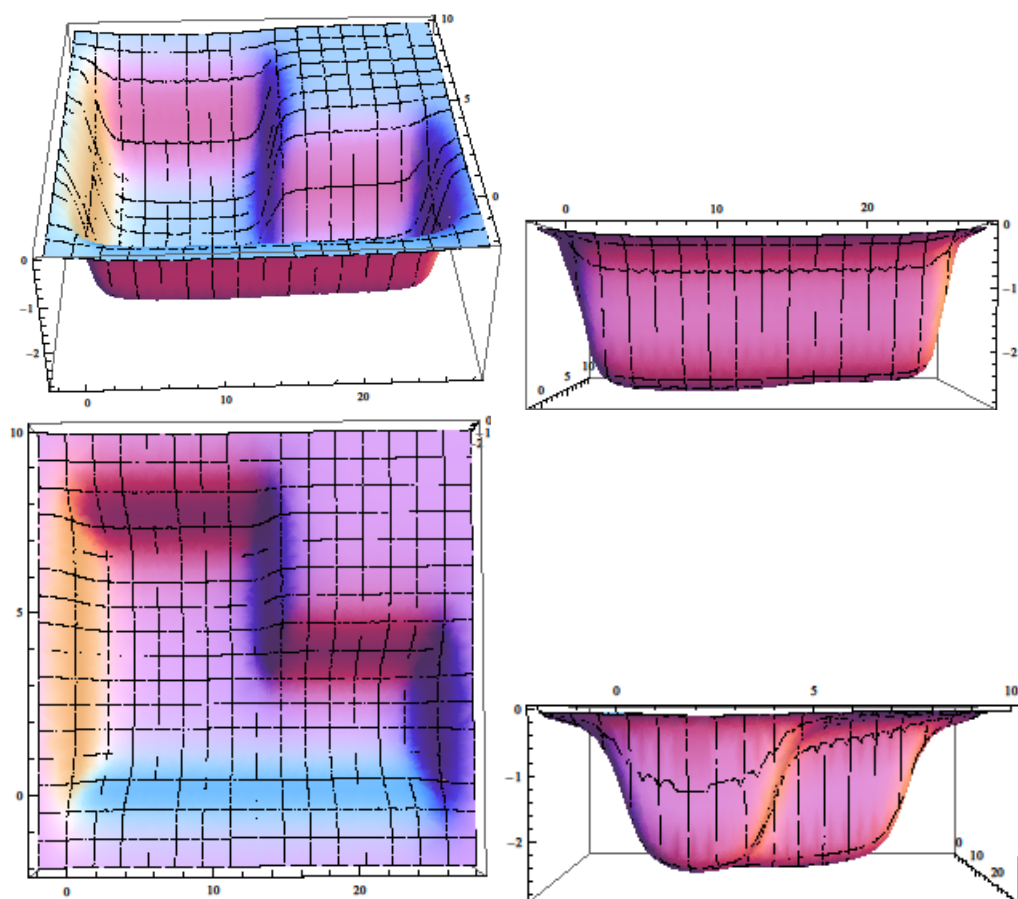
Wyrobisko o jednostronnym skróceniu frontu robót

Powrobiskową nieckę obniżeniową dla powyższego wyrobiska opisuje wzór

$$O(x,y) = \iint_K \frac{1}{2\pi} \frac{hkr}{[1 + r^2((x-s)^2 + (y-k)^2)]^{3/2}} dsdk$$

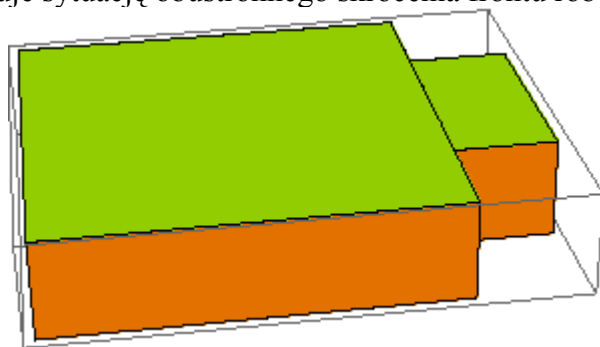
gdzie $K = \{(x,y); x \in [0,a], y \in [0,c]\} \cup \{(x,y); x \in [a,b], y \in [0,d]\}$.

W przypadku jednostronnego skrócenia frontu robót powrobiskowa niecka obniżeniowa ma kształt.



Powyrobiskowa niecka obniżeniowa dla jednostronnie skróconego frontu robót.

Drugi przykład opisuje sytuację obustronnego skrócenia frontu robót.



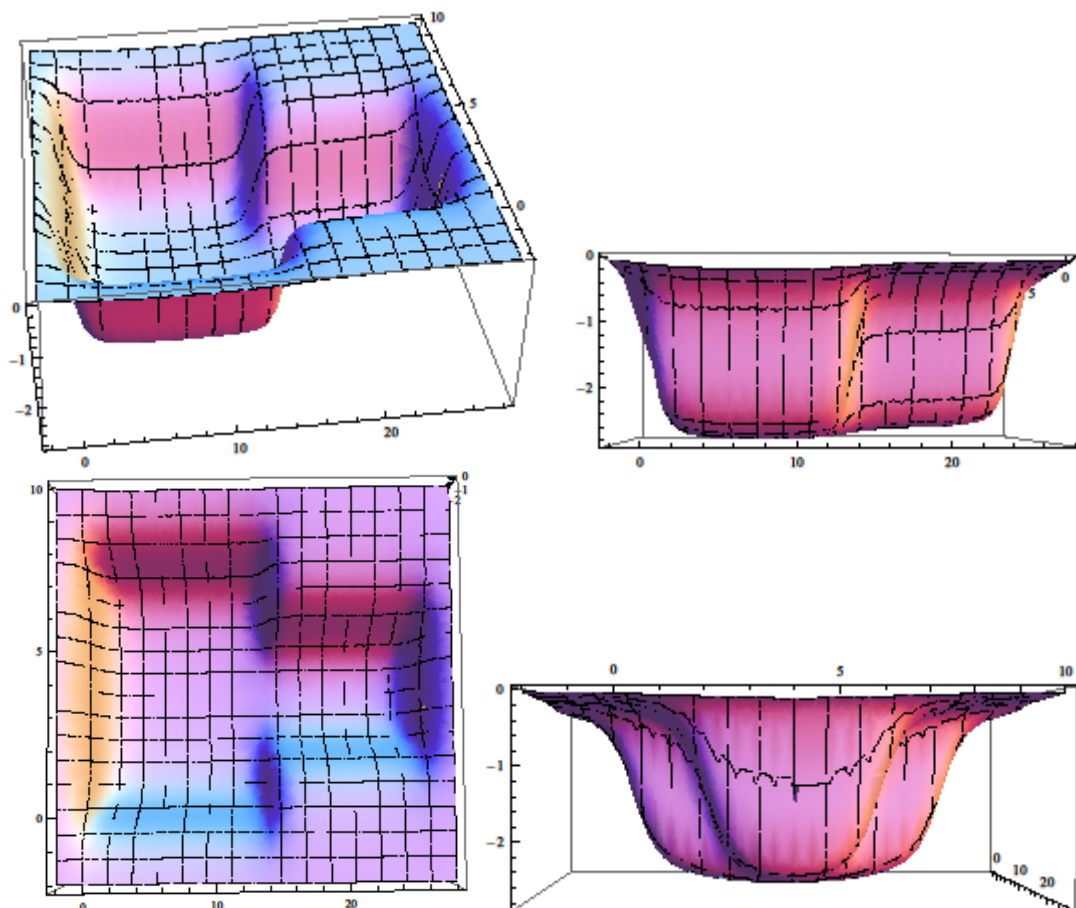
Wyrobyśko o obustronnym skróceniu frontu robót

Powyrobiskową nieckę obniżeniową dla powyższego wyrobiska opisuje wzór

$$O(x,y) = \iint_K \frac{1}{2\pi} \frac{hkr}{[1+r^2([x-s]^2 + [y-k]^2)]^{3/2}} dsdk$$

gdzie $K = \{(x,y), x \in [0,a], y \in [0,c]\} \cup \{(x,y), x \in [a,b], y \in [c,d]\}$

W przypadku obustronnego skrócenia frontu robót powyrobiskowa niecka obniżeniowa ma kształt.



Powyrobiskowa niecka obniżeniowa dla obustronnie skróconego frontu robót. W przypadku zmiennej wysokości frontu robót we wzorze ogólnym stała k ulega uzmiennieniu, natomiast w sytuacji zmiennej głębokości uzmiennia się stała r .

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PROBLÉMY STUDIA MATEMATIKY V I. ROČNÍKU FS VŠB-TU OSTRAVA

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Abstrakt: Matematické znalosti studentů technických vysokých škol vykazují dlouhodobě klesající tendenci [1-6]. Názory studentů na výuku předmětů garantovaných katedrou matematiky a deskriptivní geometrie v prvním roce studia na FS, FAST a FBI VŠB-TUO jsme zjišťovali formou anonymního dotazníku. Zajímá nás pohled studentů na úroveň jejich vstupních znalostí z matematiky a geometrie i přístup k přednáškám, cvičením a zkouškám. Ve svém příspěvku budu prezentovat výsledky týkající se strojí fakulty.

Abstract: Mathematical skills of students of technical universities have long-term downward trend. With the form of an anonymous questionnaire, we investigated the students' opinions on the teaching of subjects that are guaranteed by the Department of Mathematics and Descriptive Geometry in the first year of study at the Faculty of Mechanical Engineering, Faculty of Civil Engineering and Faculty of Safety Engineering at VSB - Technical University of Ostrava. We have been also interested in the opinion of students on their level of entry knowledge of mathematics and geometry as well as their attitude to lectures, exercises and exams. In my paper I am going to present results related to the Faculty of Mechanical Engineering.

1 Úvod

Na VŠB, nyní VŠB-TU, učím matematiku od roku 1974. Za tu dobu se v souvislosti s výukou matematiky mnohé změnilo, podle mne k horšímu. Znalosti, které se kdysi považovaly za naprostou samozřejmost (např. počítání se zlomky, mocninami, odmocninami a odtud se odvíjející úpravy algebraických výrazů, ...), teď už jsou často výjimečné. Máme počítače, proč bychom se to učili – takový je názor mnoha studentů. Tito studenti si ale neuvědomují, že k tomu, aby uměli číst, se museli dokonale naučit abecedu. Naučit se dokonale matematickou abecedu však považují za zbytečné.

Stále se snižující úroveň znalostí studentů se spolu s klesající hodinovou dotací odráží v postupné úpravě osnov. Jen pro srovnání – před 40 lety se matematika na FS učila první čtyři semestry vždy v rozsahu 4 hodiny přednášek a 4 hodiny cvičení týdně. Dnes mají první tři semestry rozsah 2+2, a v navazujícím studiu 1 semestr 2+3. Je jasné, že některé partie z osnov zcela zmizely (např. kvadratické plochy, implicitní diferenciální rovnice I. řádu, Fourierovy řady, ...), další se ilustrují na jednodušších příkladech (průběh funkce, parciální

derivace složené funkce více proměnných, ...). Největší problém ale podle mne spočívá v tom, že po probrání základních matematických prostředků nezůstane obvykle čas na aplikace. Studenti proto nevidí, k čemu matematika slouží a považují za zbytečné se ji učit.

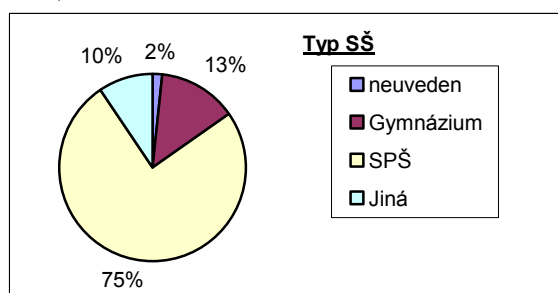
Podobné je to s konstruktivní geometrií. Z původního rozsahu 2+3 (první semestr) a 2+2 (druhý semestr) zbylo 2+2 ve druhém semestru. Vůbec se už neučí např. kinematika, projektivní geometrie, řezy na kuželi, ...

Otázkou je, proč se úroveň znalostí stále snižuje. Příčina může být ve složení studentů (stále větší procento populace přichází na vysoké školy) a z toho vyplývajících (ne)zájmu o studium, v absenci maturitní zkoušky z matematiky, v připravenosti z matematiky a geometrie ze SŠ, případně se mohou projevit další vlivy.

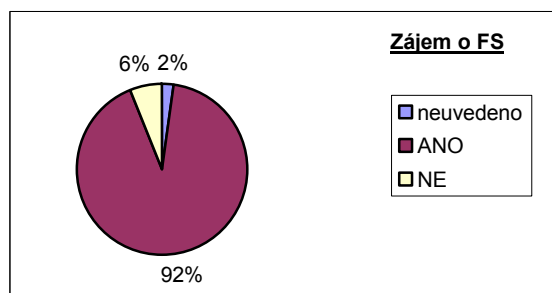
2 Složení studentů

Na některé z těchto otázek jsme se pokusili získat odpověď formou dotazníku. Anonymní dotazník vyplnilo celkem 231 studentů FS ve třetím semestru, šlo tedy o studenty, kteří získali dostatečný počet kreditů k zápisu do druhého ročníku. Podotýkám, že data v grafech jsou uváděna v procentech).

Plné tři čtvrtiny studentů absolvovaly střední průmyslovou školu (většinou strojní). Je to největší podíl absolventů průmyslovek ze tří sledovaných fakult (FAST 73%, FBI jen 15%), viz obr. 1.



Obr. 1: Složení studentů



Obr. 2: Zájem o studium na FS

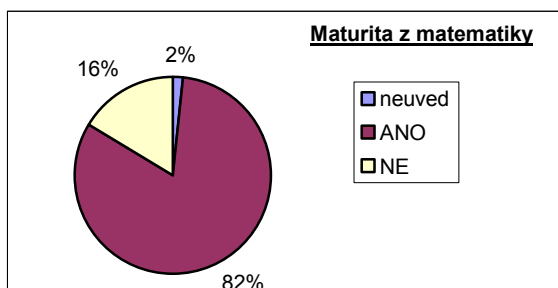
Z druhé strany absolventů gymnázií je na FS jen 13 % oproti 44% na FBI. V kategorii Jiná (10%) překvapily na FS absolventky Střední zdravotnické školy nebo umělecky zaměřených škol.

Vzhledem k převažujícímu počtu absolventů SPŠ nepřekvapí skutečnost, že 92% studentů si vybralo FS ze zájmu, jen pro 6% FS byla náhradním řešením, viz obr. 2. Studijní výsledky v prvním ročníku (nejen v matematice) jsou s tímto údajem bohužel většinou v rozporu.

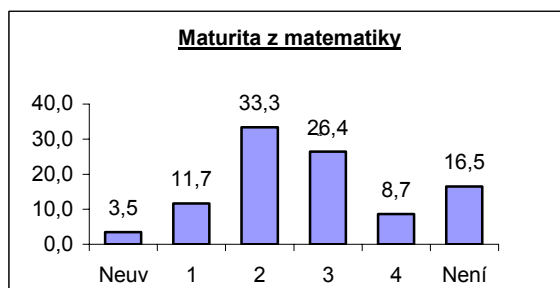
Pokud se týče maturitního roku, 72,5% respondentů maturovalo v roce 2012, 19% v roce 2011 a zbytek ještě dříve. S největší pravděpodobností tedy více než čtvrtina opakovala ročník, přerušila nebo studovala jinde.

3 Maturita z matematiky

Velmi překvapivý byl pro mne údaj, že 82% studentů složilo maturitu z matematiky, viz obr. 3, z toho 12% s výsledkem výborně a 33% s výsledkem velmi dobře, viz obr. 4. Ve srovnání s rokem 2003/04, kdy z matematiky maturovalo 46% studentů prvního ročníku, je to podstatný nárůst [3]. Aktuální znalosti studentů jej však příliš nepotvrzují.



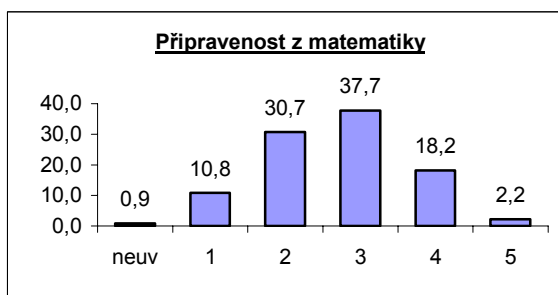
Obr. 3: Maturita z matematiky



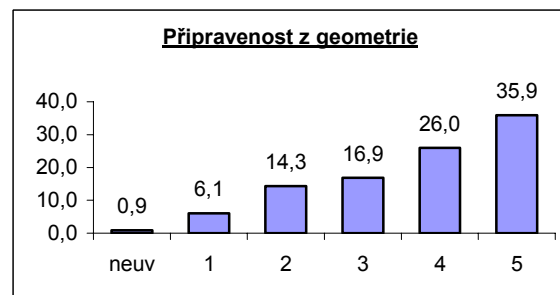
Obr. 4: Výsledky maturity z matematiky

4 Jak studenty připravila střední škola z matematiky a geometrie?

Data v následujících dvou grafech vykazují výrazný posun doprava, směrem k horším známám. V matematice skoro 60% studentů odpovídá na výše položenou otázku známkou 3 a hůře, viz obr. 5, z geometrie je to plných 79% studentů, viz obr. 6. Přitom 36% studentů hodnotí své znalosti z geometrie ze střední školy nedostatečnou známkou! Tato skutečnost velmi nepříznivě ovlivňuje výuku předmětu konstruktivní geometrie v letním semestru 1. ročníku. Základy geometrie, v nichž se studenti seznámili s tím, co měli znát ze střední a základní školy, byly zrušeny před dvěma lety. Vybudovat prostorovou představivost na čisté louce za 1 semestr je velmi obtížné a podaří se to jen u schopných, dobře motivovaných studentů. Studenti, kteří získali dost kreditů a postoupili do 2. ročníku, mají možnost nedostatky odstranit v repetitoriu z konstruktivní geometrie (zimní semestr 2. ročníku) a ve značné míře ji využívají.



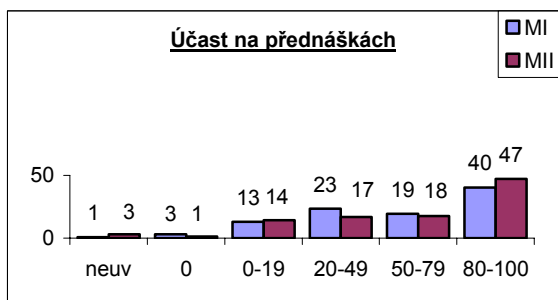
Obr. 5: Připravenost z matematiky ze SŠ



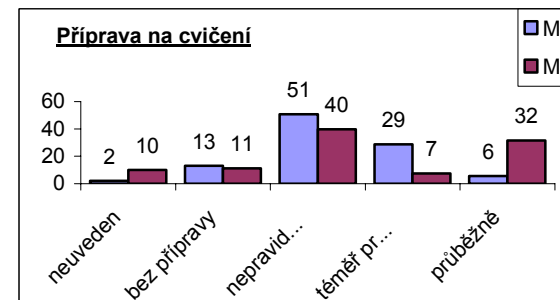
Obr. 6: Připravenost z geometrie ze SŠ

5 Účast a příprava na výuku

Účast na přednáškách je nepovinná a obr. 7 ukazuje, že studenti této skutečnosti využívají. Někteří z euforie, že 13 let povinné docházky do školy je minulostí, někteří si na studium vydělávají a jsou v zaměstnání, jiní spoléhají na studijní materiály na internetu, další nemají zájem. Byli i takoví, kteří se nezúčastnili ani jediné přednášky. Přesto lze konstatovat, že se zkušenosti ze ZS projeví v jistém zlepšení docházky v LS v kategorii (80-100)%.



Obr. 7: Účast na přednáškách z MI a MII

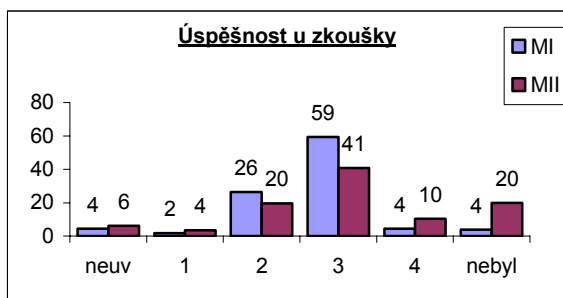


Obr. 8: Příprava na cvičení z MI a MII

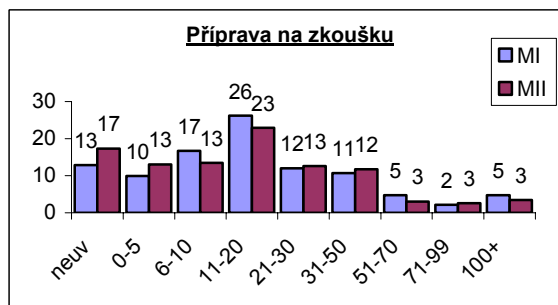
Docházka na cvičení je naštěstí stále ještě povinná. Na katedře tolerujeme 20% omluvené neúčasti. Zajímavý je výrazný rozdíl v přípravě na cvičení, viz obr. 8. Zatímco v ZS se na cvičení průběžně připravuje pouze necelých 6% studentů, v LS jejich počet vzrostl skoro na 32%. Asi se už pozitivně projevila zkušenost ze ZS (čím více bodů za zápočet, tím snazší průběh zkoušky). Přesto ani v jednu semestru se průběžně, případně téměř průběžně nepřipravuje ani polovina respondentů.

6 Problematika zkoušek

Nízká účast na přednáškách a minimální příprava na cvičení se pochopitelně projevuje i ve výsledcích zkoušek z MI a MII, viz obr. 9. U obou předmětů převládá známka dobře. Dvacetiprocentní neúčast na zkoušce z MII je způsobena do značné míry tím, že MI je pro MII prerekvizitou. Slabé výsledky u zkoušek objasňuje obr. 10, na kterém je znázorněn počet hodin, které studenti věnovali přípravě na zkoušky. Většina dat je rozmístěna v levé polovině grafu, převážná většina studentů věnovala přípravě na zkoušku méně než 20 hodin času. Uvědomíme-li si předcházející skutečnosti (špatná připravenost ze střední školy, nízká docházka na přednášky a nevalná příprava na cvičení), není se výsledkům u zkoušek co divit.

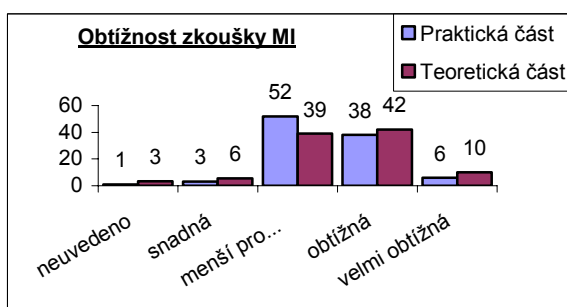


Obr. 9: Úspěšnost u zkoušky z MI a MII

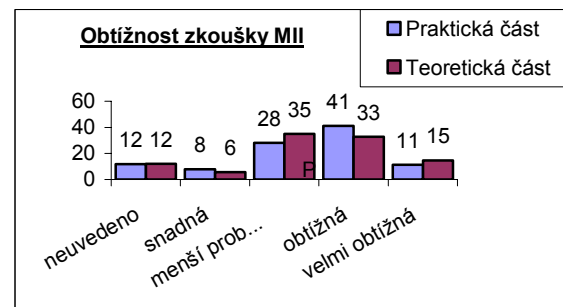


Obr. 10: Příprava na zkoušku z MI a MII

Počet hodin přípravy na zkoušku je v rozporu s tím, jaký mají studenti názor na obtížnost zkoušek, obr. 11 a 12. Polovina respondentů považuje praktickou i teoretickou část zkoušek z MI a MII za obtížné, případně velmi obtížné, nicméně počet hodin přípravy na zkoušky s těmito údaji nekoresponduje.



Obr. 11: Obtížnost zkoušky MI

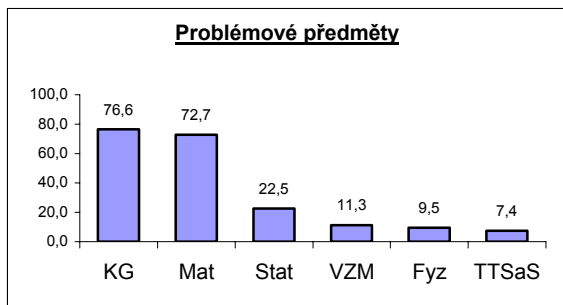


Obr. 12: Obtížnost zkoušky MII

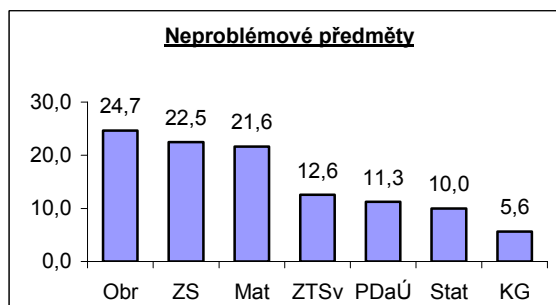
8 Oblíbené a neoblíbené předměty

Oblíbenost či neoblíbenost předmětu zřejmě úzce souvisí s úspěšností u zkoušky v daném předmětu. Mezi problémovými předměty jasně vévodí konstruktivní geometrie, viz obr. 13. Tento předmět je pro studenty vzhledem k minimálním znalostem ze střední školy, viz výše obr. 6, a v důsledku toho minimálně rozvinuté prostorové představivosti mimořádně

náročný. Pokud nezačnou hned od začátku studovat průběžně, brzy se pro ně stane změtí čar a pak se ke zkoušce připravují způsobem: „Bod A spojm s bodem B a vedu kolmici na bod C, ...“ se naučí jako báseň nazpaměť, aniž chápou princip řešení. V závěsu za konstruktivní geometrií je matematika (respondenti většinou nerozlišovali mezi MI a MII) a s velkým odstupem následuje statika.



Obr. 13: Problémové předměty

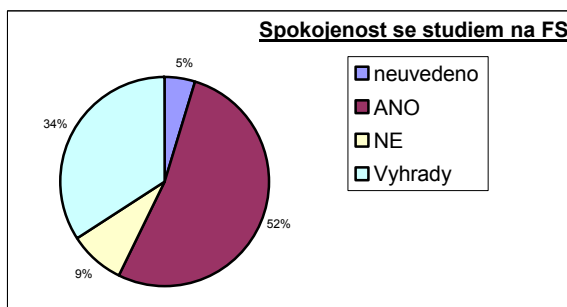


Obr. 14: Neproblémové předměty

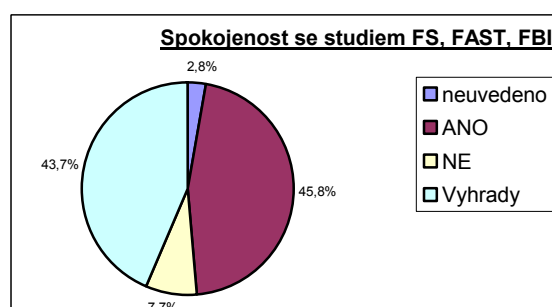
Zajímavé je, že názor na matematiku je výrazně polarizován (pro 73% studentů je to předmět problémový, pro 22% naopak neproblémový. Dalším zajímavým výsledkem je skutečnost, že u bezproblémových předmětů žádný neuvedla více než čtvrtina respondentů (na rozdíl od předmětů, které činí potíže), obr. 14.

9 Závěr

Výše uvedené skutečnosti nejsou příliš povzbudivé. Potvrzují známou skutečnost, že výsledky studentů v matematice jsou slabé. Na druhé straně je potěšitelné, že studenti si dobře sami uvědomují nedostatečné základy ze střední (i základní) školy, jsou si vědomi i z toho plynoucí náročnosti zkoušky z matematiky. Nechává je to však bohužel většinou v klidu, nic je nenutí k vyššímu úsilí. Studenti často neumí pracovat samostatně, studovat z literatury. Někteří předpokládají, že všechny potřebné vědomosti získají ve škole (nejlépe pomocí nějaké moderní verze norimberského trychtýře). Jiní často spoléhají na internet, kde skutečně existuje nepřehledné množství studijních materiálů různé úrovně (někdy amatérské s chybami), ve kterých se však neumí dostatečně orientovat. Teorie studenty nebaví a nechápou, že pro praktické úlohy nemají zase potřebný teoretický základ.



Obr. 15: Spokojenost se studiem na FS



Obr. 16: Spokojenost se studiem celkem

Jejich spokojenost s daným stavem věcí potvrzuje obr. 15, který zobrazuje skutečnost, zda studium na strojní fakultě odpovídá jejich představám. 52% respondentů je zcela spokojeno, 34% je spokojeno s výhradami. Pro srovnání uvádím odpovídající graf, na kterém je zobrazen souhrn z FAST, FBI a FS, viz obr. 16.

Dalším důvodem současného stavu je snížení povinného počtu hodin matematiky na středních školách. Pokud se současně neredukovaly učební plány, pak se učivo nestačí v dostatečné míře procvičit a povrchní vědomosti studenti velmi rychle zapomínají.

Jinou příčinou je způsob financování škol (od základních přes střední po vysoké) podle počtu studentů. Vezmeme-li v úvahu nárůst počtu středních a vysokých škol po roce 1990 a současně nižší populační ročníky, vede to velmi často ke snižování náročnosti.

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ZAPOJENÍ STUDENTŮ OBORU MATEMATICKÉ INŽENÝRSTVÍ DO AKTIVIT ÚSTAVU MATEMATIKY

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Abstrakt: Studenti oboru Matematické inženýrství – oboru, který patří mezi Aplikované vědy v inženýrství - se během studia mají možnost zapojit do řady aktivit garantujícího Ústavu matematiky. Jednou z aktivit, která mimo jiné slouží k propagaci studijního oboru Matematické inženýrství, je soutěž pro studenty středních škol nazvaná Internetová matematická olympiáda. Při přípravě a realizaci této soutěže je invence a zápal studentů obrovským přínosem.

Abstract: During their studies, students of Mathematical Engineering - a study branch that belongs to the Applied Sciences in Engineering - have the opportunity to participate in numerous activities by the Institute of Mathematics. A competition for high school students called the Internet Mathematical Olympiad is one of such activities that among other things serves to promote the Mathematical Engineering branch of study. The inventiveness and enthusiasm of students are a huge benefit to the preparation and implementation of this competition.

1. Studijní obor Matematické inženýrství

Na Fakultě strojního inženýrství Vysokého učení technického v Brně je možné studovat jak obory z programu Aplikované vědy v inženýrství (viz Obr. 1 vlevo), tak klasické obory z oblasti Strojírenství (viz Obr. 2 vpravo). Ústav matematiky garantuje studijní obor Matematické inženýrství a se svými studenty je v úzkém kontaktu od počátku jejich studia. Studenti mají zájem zapojit se do činnosti ústavu a mají k tomu několik možností. Jednou z nich je pomoc při organizaci Internetové matematické olympiády pro studenty středních škol.

Oblast aplikovaných věd	Obory profesního typu
	B-S1R Strojírenství
	B-STG Strojírenská technologie
	B-EPP Energetika, procesy a životní prostředí
	B-EPE Energetika, procesy a ekologie
	B-SSZ Stavba strojů a zařízení
B-PDS Průmyslový design ve strojírenství	B-PRP Profesionální pilot
	B-AIŘ Aplikovaná informatika a řízení
	Obor obecného typu
B-MAI Matematické inženýrství	B-STI Strojní inženýrství
B-MET Mechatronika	
B-FIN Fyzikální inženýrství a nanotechnologie	Ostatní obory
B-MTI Materiálové inženýrství	B-KSB Kvalita, spolehlivost a bezpečnost

Obrázek 1: Obory Aplikovaných věd v inženýrství, obory Strojírenství (bakalářské studium)

2. Internetová matematická olympiáda

Internetová matematická olympiáda je soutěž, která byla založena Ústavem matematiky na podzim roku 2008. V tu dobu šlo o naprosto originální formát týmové soutěže.

Soutěžní tým je tvořen 7 žáky jedné střední školy (ne nutně z jediné třídy a ročníku). Každý tým si vybere svého kapitána, který je zodpovědný za veškerou komunikaci s organizátory (od registrace, přes zadání jmen členů týmu, korektního názvu školy až po odeslání elektronické verze řešení). Soutěž probíhá virtuálně přes webové stránky matholymp.fme.vutbr.cz (viz Obr. 3).

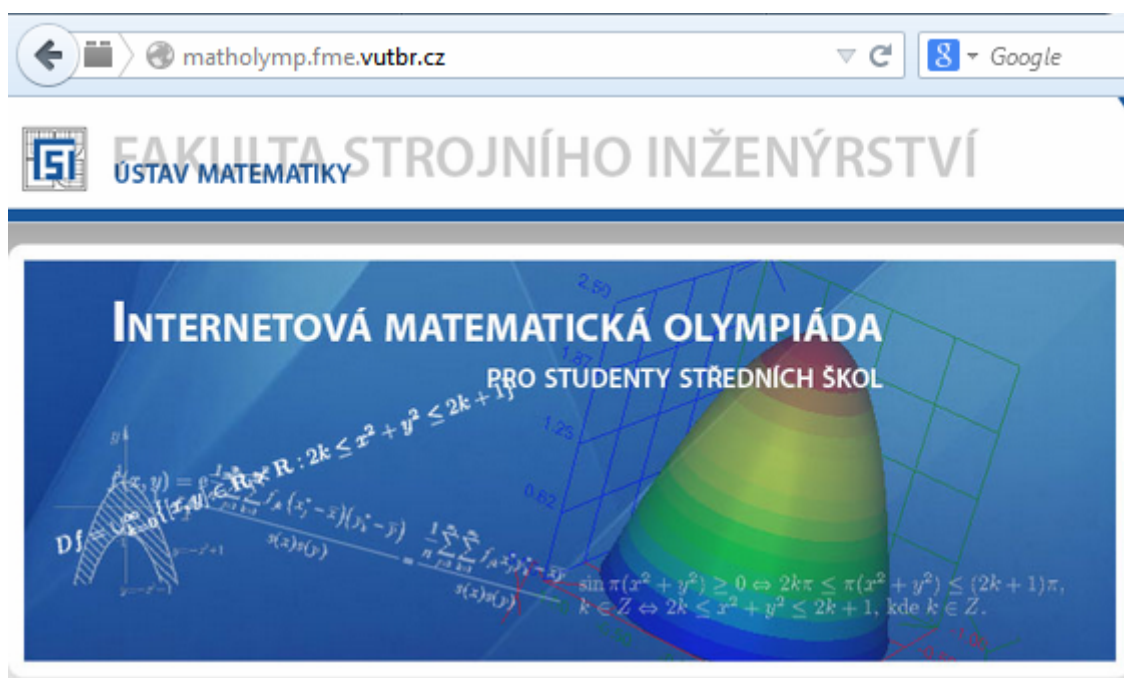
Poslední listopadové úterý v 9:00 je organizátory na webové stránce olympiády vyvěšeno zadání 10 příkladů. Náročnost úloh se liší, jsou zařazeny velmi jednoduché úlohy, ale i velmi obtížné. Snahou je pozitivně motivovat účastníky a docílit mimo jiné i dobrého pocitu z úspěšně vyřešené úlohy, což považujeme za důležité zvláště v případě mladších řešitelů. Originální je především systém hodnocení jednotlivých příkladů, kde záleží nejen na náročnosti úlohy, ale také na tom, kolik týmů a do jaké míry zvládne úlohu vyřešit.

3. Zapojení studentů Matematického inženýrství

Sestavení zadání 10 příkladů je nejnáročnější fází příprav každého ročníku soutěže. Obrovskou výhodou zapojení studentů VŠ do přípravy soutěže pro SŠ je to, že mají dokonalý přehled o tom, na jaké úrovni jsou aktuální znalosti žáků SŠ v oblasti matematiky a mají přehled o náročnosti tradičních matematických olympiád. Mnozí z našich současných studentů na VŠ, kteří se na organizaci soutěže podílejí, také soutěžní úlohy v době svého studia na SŠ řešili. Díky nim máme výbornou zpětnou vazbu.

Při přípravě zadání 10 úloh je kladen důraz na pestrost zadání. Studenti disponují obrovskou invencí, jsou schopni pojmout zadání i velmi netradičně a s jistou dávkou humoru. Vznikají tak originální příklady, které středoškoláky baví řešit.

Motivací k zapojení studentů VŠ do vymýšlení originálních příkladů bývá především nadšení pro matematiku a zájem o tvořivou činnost s hmatatelnými výsledky. Naši snahou je však v rámci možností také ocenit jejich práci finančně. Zapojení studentů je pro úspěch celé akce klíčové.



Obrázek 3: Webová stránka Internetové matematické olympiády

4. Přínos pro studenty a pro ústav

Soutěž Internetová matematická olympiáda je příkladem šikovně zvolené metody propagace oboru Matematické inženýrství mezi středoškoláky. Díky soutěži se informace o existenci oboru Matematické inženýrství dostane do širokého povědomí na středních školách a samotní organizátoři z řad studentů VŠ díky, osobnímu podílu na zadání úkolů, udržují úzký kontakt se svou střední školou a předávají osobní zkušenosti ze studia na VŠ svým mladším vrstevníkům.

Studenti mají oprávněně dobrý pocit z práce, kterou při přípravě soutěže odvedli. Získají možnost podílet se na běžném chodu Ústavu matematiky a zapojit se do činnosti, která je nad rámec jejich povinností.

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THE USE OF BLUEPRINTING TO ASSESS THE QUALITY OF SERVICES IN AUTOMOTIVE INDUSTRY

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Abstract: The purpose of this study is to present the blueprint scheme for the process of customer service. The assessment and analyses questionnaire of customer service standards in car showrooms will be based on this scheme. Next there will be performed the comparative research in Poland and Czech Republic. The process of customer service in car showrooms was described according to the L. Shostack blueprint methodology.

Keywords analysis of customer service standards, car showrooms, Polish-Czech comparative study

Streszczenie: Celem niniejszej pracy jest prezentacja schematu blueprinting dla procesu obsługi klienta. Na podstawie schematu zostanie opracowany kwestionariusz oceny i analizy standardów obsługi klienta w salonach sprzedaży samochodów osobowych w Polsce i w Czechach. A następnie, w dalszej kolejności przeprowadzone zostaną porównawcze badania naukowe. Proces obsługi klienta w salonach sprzedaży samochodów osobowych został opisany i opracowany według metodologii L. Shostack dla metody blueprinting.

Keywords analiza standardów obsługi klientów, salony sprzedaży samochodów osobowych, polsko-czeskie badania komparatywne

Scientific background

Service-blueprinting (SB - Service Blueprinting) is suggested to be used in modelling the components of the service process [8; 9; 18, 49-63; 19, 133-139; 20, 27-43; 21, 34-43]. In the other words, SB is the optical process recording, that presents the image of the communication between the exchange participants. Shostack's arguments were accepted by the scientists and a lot of analysts showed the advantages of using this method. In the literature about this subject, the matter of positive contribution of SB is being raised for: internal marketing [10, 325-351], service development process [18], monitoring of the customer preferences [16, 7-12], services innovation [3, 66-94], services efficiency [6, 732-747], process effectiveness [8, 580-591], business processes [4, 475-496; 13, 410-420] and first of all for management and control of already existing customer service processes [19, 20, 22]. The use of blueprinting known from the literature regards to specific services such as for example: non-profit [15, 1-20],

pro-health education [1, 508-535], industry [2, 932-957], or the hotel industry [12, 606-621].

The service- blueprinting profile

The visualization of each phase and action, which are realised or by the customers themselves or with the assistance of the customer service representative, is the base for modelling the SB. The main objective of this modelling is the communication, analysis and control of the realised actions [5, 359-361; 11, 312-319]. All these elements cause the continuous improvement. Due to the services nature, every meeting with the customer is different. However, it is possible to plan the central component of the service process—each subsequent service element is to be realized by the employees, who are expected to behave professionally [7]. Blueprinting map shows visually the sequence of events in a two-dimensional system. What is important, the actions taken by the customer are fundamental for mapping—they determine the whole mapping system. The horizontal axis presents the chronological schedule of actions taken by the customer and employees. The vertical axis creates the second dimension of analysis. It means that it puts the activities in order based on their type and their role in a service process. There are two groups visible here: customers—in a role of potential buyers and the workers—in a role of first contact people and the support of actions implemented in the whole process. Separating particular groups of stakeholders is additionally marked by the lines isolating each zone of influence. There are mentioned the lines of: interaction, visibility and internal connections [19]. They respectively mean the relations between:

- customers with frontline employees,
- frontline employees with second-line employees and
- second-line employees with the other workers.

There is one more element visible in the blueprinting map: the material properties of service [14, 41-50; 23, 35-48]. In the whole service process, the material service elements are still being assessed by the client. Although the customer is not always aware of this assessment, it has the influence on his service perception.

All these components together create the blueprinting matrix, which is composed of lines and sequence of the process participants' actions, between whom there are interactions.

Blueprinting map

Blueprinting matrix was prepared for the car showroom according to the following stages:

1. Identification of main events and typical sequences (flowchart).
2. Modelling the process of customer service from the client's point of view.
3. Modelling the actions of: frontline employees, second-line employees and material aspects.
4. Creating the direct and indirect communication connections.

Blueprinting map was divided into five thematic parts: material aspects, customer actions, frontline employees' actions, direct activities of second-line employees and the supporting process. These areas were additionally separated with the dividers named after the lines of:

- material aspects,
- interactions between the customer and frontline workers,
- visibility—this is the area of interaction between frontline and second-line personnel,
- internal interaction, it shows the relation at the joint of second-line workers and those who accomplish the processes supporting the whole organization working.

The lines of interaction and visibility are the most important for the customer. Those lines set the zones of service process, where the client takes part directly and assesses the results of any management and operational actions taken at all company levels-including material aspects, which are assessed by the results of the procedures supporting the organization's activities. When the client visits the car showroom, he assesses all available elements, including the material aspects of surroundings. By the material aspects are meant all real attributes, which are available for the company and which are used in a day-to-day work. These elements are part of organizational culture of the company [17]. Material aspects make it also possible to create the organization image, that is important from the client's point of view, especially in comparison to the competition. The components of surroundings in case of car showrooms include material and immaterial aspects. They create the quality standards of customer service on the one hand, and on the other hand, they are judged by the customer and have an influence on his shopping decisions.

If the delivery is to be effective and efficient, it is essential to prepare the professional offer to sell. In practice, it means using all possible attributes of the showroom in order to delay and/or eliminate the competition.

Material aspects creating the customer service standards in car showroom include:

- a) the elements outside the showroom:
 - the website,
 - the outdoor advertising,
 - information boards indicating the way to the showroom,
- b) the elements in the showroom area:
 - car park for the customers,
 - parking with the cars for sale,
 - localization,
 - aesthetics of the square by the showroom,
 - showroom entrance,
 - additional customer services,
- c) the interior of the showroom:
 - hall,
 - interior design,
 - additional customer services inside,
 - advertising materials,
 - appearance and uniforms of the frontline personnel,
 - showroom equipment.

Material aspects of the service design only the external image of the showroom. It is evaluated by the customer, but the main point of car selling is the process of customer service, including the direct contact with the showroom employee. The actions taken by the showroom frontline employees are the mirror image of the customer's activity. It is important to be aware that preparation of sales consists of many stages. In the showroom the customer see the finished product in the ready "package", which means the car in the arrangement. It is supposed to influence on the client's decision about buying a car. All material and immaterial properties, which are the part of an image, are previously arranged and organized by the managerial and support staff. The employees are appropriately motivated, and the effect of their work is monitored and assessed As a result of interactions, there are taken amending and preventive actions, which are to led to working out of the best customer service standards.

Included activities are not directly visible for the client. That is why they are situated behind the line of visibility- that really means, that they are not directly available for a

customer. The clients use the work effects of many people without even knowing them. However, meeting the standards of customer service would be impossible without these actions. These required actions are realized by the second-line employees at the strategic, tactical and operational level.

Conclusion

In the blueprinting method, the modelling of the dynamic service process, including the perspective of customer and all staff involved in this process, is being accomplished. Modelling of reality in this context means making an effort to map, analyse and then make up the process of service implementation taking into account the customer's perception. In this paper there was presented the original map of the service process in a car showroom. This map contains elements that form sequence of actions. Comparison of actions and targets, which determine behaviour of exchange participants let us understand the mechanisms that occur in particular transactions.

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DESIGN OF RESEARCH TOOL FOR PROBLEMS ANALYSIS IN QUALITY MANAGEMENT SYSTEMS

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Introduction

When talking about the issues connected with quality management we usually use descriptive data. We talk about nonconformance or development possibilities, strengths and weaknesses. However, comparing them is not easy due to a large number of descriptive data, what may cause a lack of information clarity. The problem occurs not only with the way of description, if research is conducted on a larger sample, one is searching for generalizations allowing comparison of a maximal amount of information and maintaining the simplicity of recording. In the Department of Quality and Safety Management there was a various type research conducted [1-8]. The existing studies of problems in certified quality management system has shown difficulties for the collection and processing of data [9-13]. Data gathering did not constitute a problem itself but very often their proper gathering and possible processing in the clearest form.

In order to conduct research on large groups of varied objects it is important to find the adequate grounds enabling comparison of the aforementioned largest possible group. The simplest variant is to gather such data that allow comparing results on more than one level. Such solution was created for the needs of information gathering on a number and type of nonconformance in quality management systems for ISO 9001 series of standards. It was crucial to collect data in such a way so that it would be possible not only to process or select them later depending on the need, but also to compare or verify them.

Development of research tool for data collection

In preparation tool for collecting and processing data were used the possibilities offered by the MS Excel. It was necessary to create a simple tool for quick enter data. Also important was the fact that the tool was universal and minimizing the possibility of mistakes during data entry, and later stage of their analysis. When building a spreadsheet it is worth paying attention to such data that may be grouped in a range. For instance, this could be: time of certificate possession, enterprises size and all types of industry generalization. It helped not only in building a spreadsheet focused on an interesting area but also improved the data analysis later.

In this case the subsequent enterprises had to be marked and the most important data shortly characterized. For this purpose a general division into industries was created as well as key to table with the generalized data in the first part of the table (Figure 1).

	A	B	C
2	Nr Organizacji		
3	Branża		
4	przemysł wydobywczy		
5	przemysł energetyczny		
6	przemysł chemiczny		
7	przemysł elektromaszynowy		
8	przemysł mineralny		
9	przemysł drewno - papierniczy		
10	przemysł lekki		
11	przemysł spożywczy		
12	przemysł inny		
13	nauka i rozwój		
14	oświata i wychowanie		
15	kultura i sztuka		
16	ochrona zdrowia i opieka publiczna		
17	turystyka, wypoczynek, kultura fizyczna		
18	administracja		
19	sądowictwo		
20	obrona narodowa i bezpieczeństwo publiczne		
21	finanse i ubezpieczenia		
22	gospodarka mieszkaniowa		
23	pozostałe usługi		
24	wielkość		
25	mikro		
26	małe		
27	średnie		
28	duże		
29	bardzo duże		
30	Czas posiadania systemu		
31	poniżej 1 roku		
32	od 1 rok do poniżej 3 lat		
33	od 3 lat do poniżej 6 lat		
34	od 6 lat do poniżej 9 lat		
35	9 lat i więcej		
36	Rozaj audytu		
37	Certyfikujący		
38	Recertyfikujący		
39	Punkty normy - rok 2009		

Figure 2 Division characterizing the selected enterprise
Source: own work

In the part directly linked to data gathering there was a division included not only according to the ISO standards. Dividing the subsequent sections along with indicating the corresponding years made the search easier in case of data analysis with maintaining the

simplicity of recording mentioned before as well as leaving room for a possible modification, depending on the needs (Figure 3).

39	Punkty normy - rok 2009		
40	4 System zarządzania jakością	4	
41	4.1 Wymagania ogólne	4.1/ 2009 N	
42	4.1 Wymagania ogólne	4.1/ 2009 S	
43	4.2 Wymagania w zakresie dokumentacji	4.2/ 2009 N	
44	4.2 Wymagania w zakresie dokumentacji	4.2/ 2009 S	
45	5 Odpowiedzialność kierownictwa	5	
46	5.1 Zaangażowanie kierownictwa	5.1/2009 N	
47	5.1 Zaangażowanie kierownictwa	5.1/2009 S	
48	5.2 Orientacja na klienta	5.2/2009 N	
49	5.2 Orientacja na klienta	5.2/2009 S	
50	5.3 Polityka jakości	5.3/2009 N	
51	5.3 Polityka jakości	5.3/2009 S	
52	5.4 Planowanie	5.4/2009 N	
53	5.4 Planowanie	5.4/2009 S	
54	5.5 Odpowiedzialność, uprawnienia i komunikacja	5.5/2009 N	
55	5.5 Odpowiedzialność, uprawnienia i komunikacja	5.5/2009 S	

Figure 4 Division according to ISO standard sections along with assigning the year of research conducted
Source: own work

After ascribing nonconformances or weaknesses to a letter mark when data are displayed, there is information presented for the particular enterprise in terms of number of nonconformances or weaknesses along with indication of both the ISO standard section and the year. .

Figure 3 illustrates the practical application of developed tool. It was about the development of a tool to clearly and readable both collect, collect and process large amounts of data with various values, and which can be studied the differences in terms of value.

In the research conducted there was also a division used into weaknesses and nonconformances. The example below presents the weaknesses marked in the section/year cell as S and nonconformances – section/years as N, The fragment placed concerns the enterprises from the auto-moto industry or a related one, that employs 3 people. It has possessed the certificate since 2011 and the results presented refer to the audit of supervision that took place in the year 2014.

AX13		
	A	B
1	Nr organizacji do badań	AX 48
2	Informacje	Pkt normy
3	Nr Organizacji w systemie	X
13	przemysł motoryzacyjny i pokrewny	x
29	wielkość	x
35	Czas posiadania systemu	x
45	AN 1	x
52	4.2 Wymagania w zakresie dokumentacji	4.2/ 2014 S
59	5.3 Polityka jakości	5.3/2014 S
65	5.6 Przegląd wykonywany przez kierownictwo	5.6/2014 S
92	8.2 Monitorowanie i pomiary	8.2/2014 S
97	8.5 Doskonalenie	8.5/2014 N
98	8.5 Doskonalenie	8.5/2014 S

Figure 5 Fragment of research based on the particular enterprise – presentation of possibilities of using the tool proposed
Source: own work

However, it is not the only possibility of using the tool presented. It is a handy solution for all types of gathering a large number of varied data in a way enabling their further processing or detailed analysis.

Summary

With clear notation it is possible to verify information in the course of their introduction, and very easy applied possibly changes. But at the same time there is possible in the same time both collecting large number of data and analyze them on many levels. An important advantage of such solution is a possibility to analyze both, entirety of results as well as selected elements. It allows a thorough selection of comparison range, not affecting clarity at the same time and reducing a possibility of mistake in the data grouped.

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Results of statistical analysis of clayey rocks

geochemical studies

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Summary. The paper shows statistical analysis of results of trace elements concentration in clayey rocks coexisting with 610, 620 and 630 coal seams (Poruba beds). The samples were taken from Mineral Mining Facility “Jadwiga” and mines: „Sośnica”, „Rydułtowy”, „Marcel–ruch” „1 Maj” and „Anna”.

The statistical analysis included: calculation of average concentrations, testing frequency distribution and correlation analysis.

Introduction

Trace elements in the barren rocks being accompanying with coal seams in the Upper Silesia Coal Basin (USCB), until now was subject of only few studies, dedicated mainly to the rocks from the bands, and less occasionally from the roofs and floors of coal seams (Adamczyk 1997, 1998; Parzentny 1992, 1999; Hanak and Kokowska 2002; Hanak and Kokowska-Pawłowska 2003).

The barren rocks consist trace elements, inter alia: B, Ba, Cd, Co, Cr, Cu, Ni, Pb. Set of analyzed elements included: elements, which in case of their concentration in wastes, may have negative impact on the environment and elements, whose content may indicate the geochemical character of rocks sedimentation environment.

Sampling and Methods

The samples of clayey rocks were taken from Mineral Mining Facility “Jadwiga” and mines: „Sośnica”, „Rydułtowy”, „Marcel–branch 1 Maj” and „Anna” from 610, 620 and 630 coal seams.

Content of trace elements: B, Ba, Cd, Co, Cr, Cu, Ni, Pb was determined using atomic emission spectrometer with inductively coupled plasma (ICP-AES).

The results of laboratory studies were the basis of determination of trace elements frequency distribution and correlation coefficients between content of trace elements in samples of clayey rocks. Statistical analysis were made using „Statistica” software (Rabiej 2012).

Results of statistical analysis

The results of statistical analysis included: calculation of average concentrations of trace elements, frequency distribution and correlation coefficient were shown in figures below and table 1.

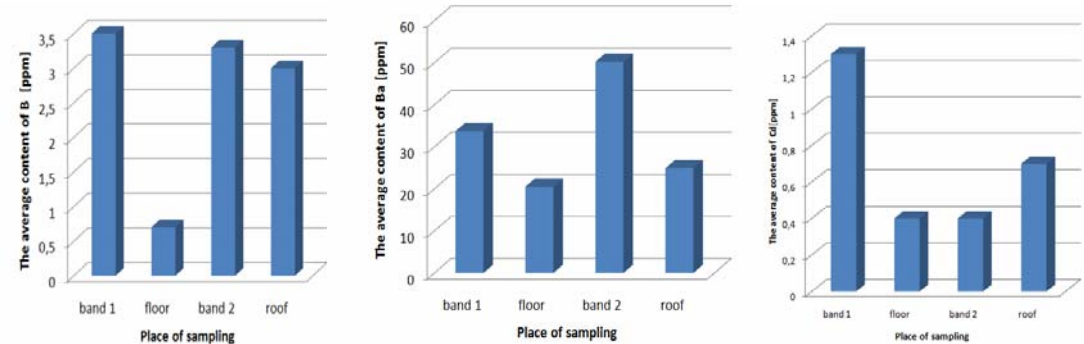


Fig. 1. The average content of B, Ba and Cd in clayey rocks samples

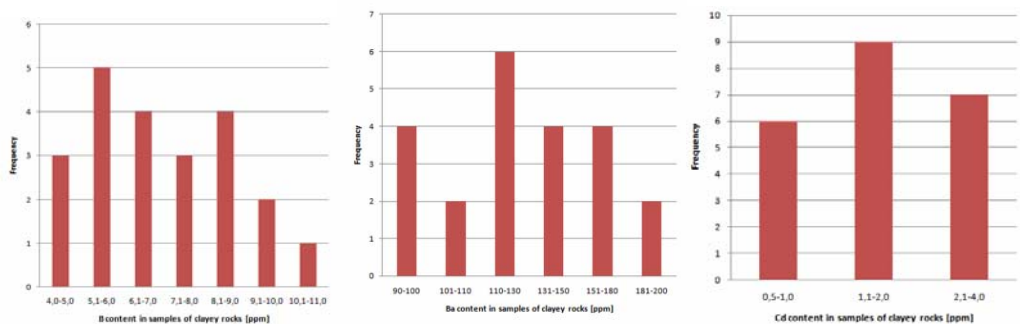


Fig. 1a. A frequency histogram of B, Ba and Cd content in clayey rocks samples

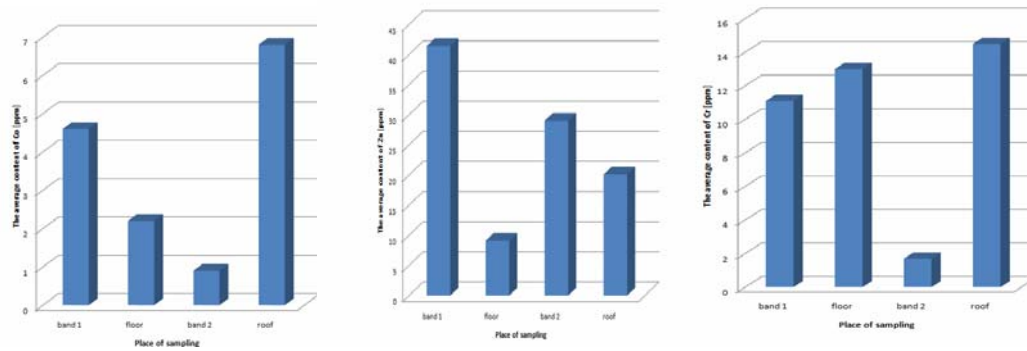


Fig. 2. The average content of Co, Zn and Cr in clayey rocks samples

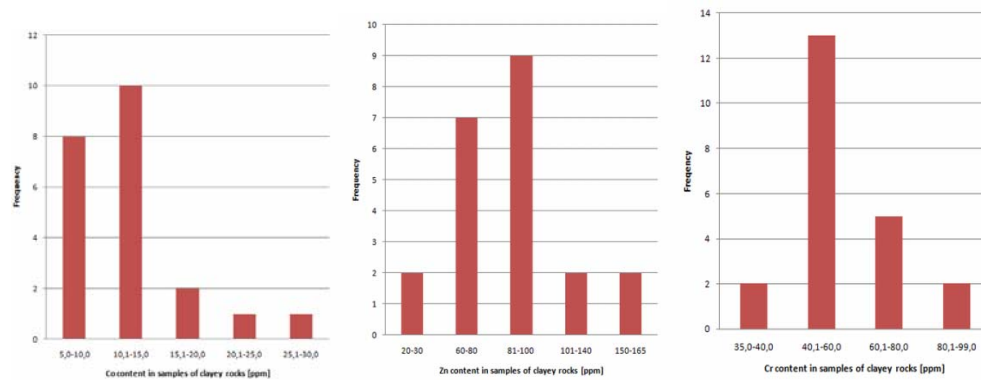


Fig. 2a. A frequency histogram of Co, Zn and Cr content in clayey rocks samples

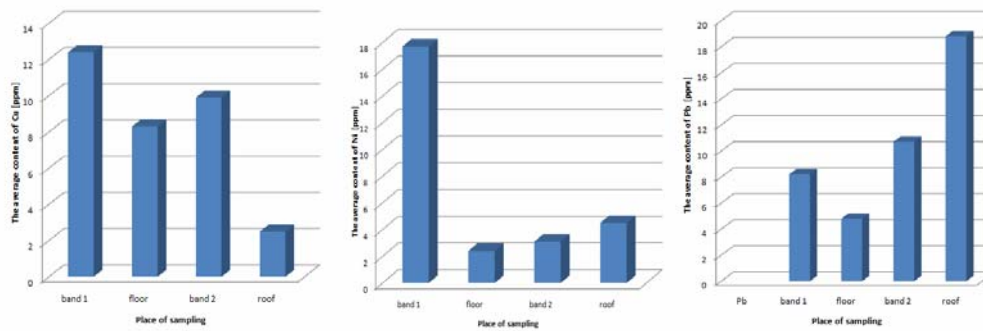


Fig. 3. The average content of Cu, Ni and Pb in clayey rocks samples

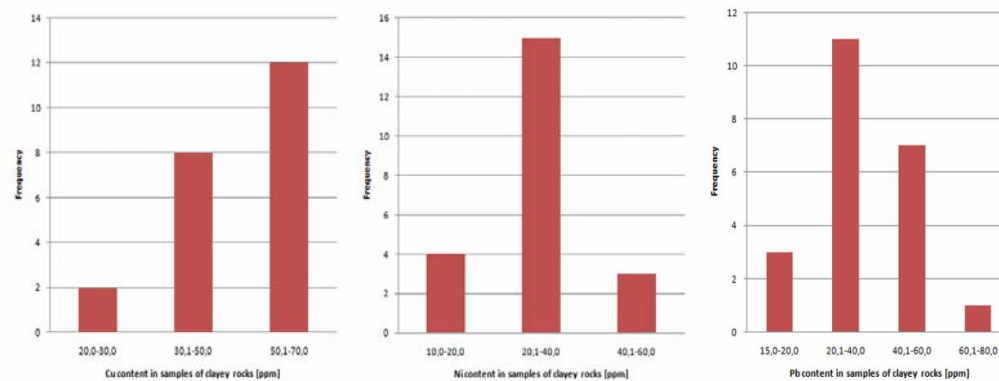


Fig. 3a. A frequency histogram of Co, Zn and Cr content in clayey rocks samples

Table1.

The correlation coefficient between content of trace elements in samples of clayey rocks

	Ba	Cd	Co	Cr	Cu	Ni	Pb	Zn
B	0,63	0,04	-0,26	-0,20	0,27	0,01	0,74	0,30
	0,85	0,89	-0,53	-0,34	0,21	-0,94	-0,50	-0,81
	0,84	0,02	-0,59	0,26	0,43	-0,60	0,33	0,17
	Ba	0,63	-0,30	-0,26	-0,44	0,05	0,57	0,18
		0,89	-0,14	-0,74	-0,25	-0,80	-0,11	-0,81
		0,34	-0,74	-0,24	0,05	-0,58	0,74	0,36
		Cd	-0,06	-0,31	-0,60	0,19	0,07	0,13
			-0,48	-0,38	-0,19	-0,92	-0,46	-0,92
			0,25	-0,35	-0,24	0,53	0,23	0,94
			Co	0,76	0,38	0,37	-0,02	0,15
				-0,54	-0,46	0,62	0,60	0,37
				0,31	0,16	0,92	-0,79	0,27
				Cr	0,34	0,36	0,07	0,30
					0,47	0,23	-0,43	0,38
					0,38	0,16	-0,79	-0,04
					Cu	0,35	0,12	0,32
						-0,05	-0,30	0,25
						-0,18	-0,26	-0,21
						Ni	0,20	0,84
							0,55	0,88
							-0,62	0,55
							Pb	0,53
								0,55
								0,04
								Zn

very low correlation 0,3 – 0,5

low correlation 0,5 – 0,7

mean correlation 0,7 – 0,9

High correlation > 0,9

Correlation value ranges are given for Krawczyk and Słomka (1986).

Conclusions

Results of statistical analysis of trace elements participation in clayey rocks showed some variation.

The increased content of trace elements was found in rock samples coming from roofs and bands (main bands located closer to the floors). Exceptions were Co and Cd whose participation was relatively high in the floor.

Increased participation of trace elements in floors, evidence of their partially infiltration origin from solutions, which had probably easier access to the floor rocks.

Correlation analysis showed significant relationships both positive and negative. Most of the correlation is geochemical confirmed, for example significant positive correlation between B and Ba, Cd; Ni and Zn, Co, Pb.

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THE ESTIMATION OF PROBABILITIES IN DECISIONS MADE UNDER RISK – PSYCHOLOGICAL ASPECTS

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Abstract: Most decisions of individuals as well as organizations, are taken under conditions of risk or uncertainty, in which the consequences of the actions are uncertain (the condition of risk) or unpredictable (conditions of uncertainty). An important element of the decision containing risk is the estimation of the probability of positive or negative results. Psychological studies have shown that in situations where the probability of an event cannot be set, the probability is identified as the degree of belief of a person regarding the veracity of the statement or the chance of an uncertain event occurrence. Article discusses the problem of subjective probabilities.

Introduction

Decision making by individuals, groups or organizations in many cases is associated with a choice of one of the available options. In the literature the term decision making often occurs in the context of human economic choices. In this light, for a relatively long time, it was emphasized that the decisions made by humans are rational, and thus leading to obtain the best results compared to the funds or energy involved. One of the most frequently cited theory here is the expected utility theory of von Neumann and Morgenstern [1]. The theory of utility allows to deal with the risks and risk minimalization measures. However, research conducted by psychologists a few years after the publication of the work of von Neumann and Morgenstern, provided evidences that human decisions are not always rational, which initiated the development of behavioral (descriptive) theory of decision [2]; [3]; [4]; [5].

The classification of decision-making situations

According to the literature on the theory of decision-making, there are three types of decision-making situations: decisions made in the conditions of certainty, decisions under risk and under uncertainty. These situations differ in terms of the level of ambiguity and – as a consequence – the chance of making a bad decision. Decisions taken in conditions of certainty (also called deterministic decisions) are characterized by the fact that their effects are well defined, ie. the outcome of every action is known in advance and certain. In such cases, the possible difficulty can be associated with the issue of the optimal (best) solution to the

problem. For this purpose mathematical methods are used, which on the basis of calculations indicate the optimal choice to make.

The second type of decisions are called probabilistic and relate to situation in which the outcomes of possible alternatives are uncertain (eg. often cited example is the relationship between the economic situations of many industries and the weather conditions). However, the level of uncertainty in this situation is limited and it can be estimated on the basis of previous experiences or data (eg. the frequency of negative climatic phenomena which may affect the business plans or upward and downward trend in the stock exchange, which allows to predict the direction of further adjustments to the price of the securities held). In this situation under risk, the decision maker can assume the future outcomes of particular alternative with some probability, but cannot be sure about it.

The third type of distinguished decision making situations is connected with the conditions of uncertainty, which mean that the outcomes of the alternatives under consideration are completely unpredictable. In such a situation, the decision maker is aware that particular action leads to one among a set of consequences but he or she cannot estimate the probability of its occurrence. The literature distinguishes also decisions made under conditions of complete uncertainty in which the outcomes of considered alternatives are totally unpredictable. The above characteristic indicates, therefore, that the choice under uncertainty is much more difficult than the choice under risk, due to the lower amount of available information [6].

The above distinction shows that the process of decision making varies depending on the information held by the decision maker. As it was shown, the best described situation is a situation of certainty, in which the decision will consist on the choice of one of the possible alternatives, based on the analysis of available options. Whereas the situation of risk and uncertainty, which does not provide the decision maker with the necessary information, leads him or her to rely on a random, often subjective assumptions, which may result in a suboptimal decision. The problem of decisions made in the uncertain conditions is the issue of many studies conducted by psychologists who distinguished several cognitive biases which people use when making such decisions and choices [3]; [4]; [5]; [6]; [7].

The common known definition of risk states that it can be described by multiplying the probability of incurring losses and its size. In majority of risky decisions the estimation of probability is the result of insights and people do not use all available information in this process or there is no relevant information because the action relates to the separate events. In the following paragraphs the examples of probability estimations in the situation of risk and uncertainty will be described.

Heuristics as the subjective method of probability estimations

Tversky and Kahneman presented the behavioral economic theory (Prospect theory) which described people's way to choose between probabilistic alternatives that contains risk and the probability of outcomes are known. The author claimed that people evaluate the possible gains and losses using several heuristics [3]. Heuristics are defined as a mental shortcuts which are not optimal but in many cases save time and effort in the decision-making process. Moreover, Tversky and Kahneman stated that people use heuristics instinctively but do not understand them which may result in wrong decisions.

The prospect theory [7] was presented almost forty years ago and has inspired numerous studies that have contributed to the creation of a list of heuristics used by people. The four following are examples of rules particularly often used in the process of decision-making and related to the estimation of the probability of the alternatives: the representativeness heuristic, availability, anchoring and unrealistic optimism.

According to the representativeness heuristic, people judge the probability of the event on the basis of its similarity to a typical event of this category. In a classic experiment, Tversky and Kahneman [3] presented students examples of the results of six coin rolls:

- a) H (heads) – T (tails) – T – H – H – T
b) H – H – H – T – T – T
c) T – T – T – T – T – T

and then asked students which on the above result seemed to them as the most probable. Majority of respondents chose the answer a, while each of these results is as probable because the results of individual roll are independent of each other, so in each next roll there may be heads or tails. In this particular case, people have a tendency to believe that the distribution of rolls in the small sample is the same as in the population, and therefore select the first answer. The representativeness heuristic is linked to other cognitive error that affects the estimation of the probability and erroneous assessment – conjunction fallacy. In another experiment, Kahneman and Tversky presented a description of the woman, and then asked to sort sentences about her according to their probability [8]:

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Linda is a teacher in elementary school.

Linda works in a bookstore and takes Yoga classes.

Linda is active in the feminist movement. (1)

Linda is a psychiatric social worker.

Linda is a member of the League of Women Voters.

Linda is a bank teller. (2)

Linda is an insurance salesperson.

Linda is a bank teller and is active in the feminist movement. (1&2)

Result of this research showed that respondents more often claimed that Linda is a bank teller and is active in the feminist movement (1&2) than just a bank teller (1), which is illogical.

Availability heuristic in turn explains how people assess the likelihood of events based on previously observed event rate. In one of the experiments respondents were asked whether there is more likely that the English word starts with a *R*, or that *R* is its third letter.

Consider the letter R

Is R more likely to appear in - the first position?

- the third position?

My estimate for the ratio of these two rates is

Results showed that people answering this question compared the availability of two recalled categories: words starting with *R* and words with *R* as a third letter. It is much more easier to recall words beginning with *R*, therefore this option was judged as more frequent [9].

In another study respondents were presented with a list of names of known and unknown personalities of both sexes. After listening to the list, subject were asked whether there were more male or female name. According to the obtained results, respondents claimed that there were more female names when those names were well-known (in comparison with male names) and analogously – respondents judged male names as more frequent when they were well-known (and female were unknown). The actual frequency of male and female names in each list was the same [4].

Another heuristic which affects the process of assessing the probability is anchoring. Anchoring refers to a situation when judgment is based on the first assumption about a particular event. Consequently, different starting point leads to different estimates. Moreover, Tversky and Kahneman proved that anchoring occurs also when people are not given the

starting point, as in one of the Tversky and Kahneman experiment conducted among two groups of students. The instruction was as following:

Group 1

Estimate, within 5 seconds, the product:

$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Group 2

Estimate, within 5 seconds, the product:

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$

As the instruction was to answer rapidly, students made a few steps of computation and then tried to estimate the product by extrapolation or adjustment, which led them to different results [10]. The median estimate according to the group 1 was 2,250 while for the second group: 512. The correct answer is 40,320.

The experiment showed that first assumptions (starting point) made while dealing with the task affected the given answer and led people to incorrect estimations. Regarding to the probability judgment, Bar-Hillel experiments are worth mentioning [6]. In this experiment subjects were presented with three urns containing different proportions of white and black balls (Urn A: 9 white balls and 1 black ball; Urn B: 5 white balls and 5 black balls; Urn C: 1 white ball and 9 black balls). In a series of experiments Bar-Hillel asked respondents which option they would choose to win 100\$:

- a) draw seven times from urn A (each time returning the ball to the urn) and win when white ball is drawn seven times
- b) draw once from urn B and win when white ball is drawn

or:

- b) draw once from urn B and win when white ball is drawn
- c) draw seven times from urn C (each time returning the ball to the urn) and win when white ball is drawn at least once.

Results showed that people more often chose the option a than b but at the same time more respondents chose option b than c, while the probability of winning is 48% in option a, 50% in option b and 52% in option c. This phenomenon is explained by the anchoring heuristic, according to which the preponderance of white balls on the black ones in the urn A was the starting point in subjects' estimations and resulted in more frequent choosing the option a – even though the actual probability of winning was lower. Analogously, the urn C anchors the chances of drawing the white balls as very small and even the conditions of option c did not revise the estimation.

The fourth mentioned above phenomenon which affects people's probability estimations is connected with unrealistic optimism. This term is defined as a human's tendency to be unrealistically optimistic about future life events. Weinstein conducted an experiment in which students estimated their own chances of 42 different (positive and negative) events (e.g. like postgraduation job; living past 80; in 10 years, earning > 40,000\$ a year; marrying someone wealthy or having a drinking problem; not finding a job for 6 months; divorces a few years after married; attempting suicide) in comparison with their classmates' chances [11]. Results of this study showed that generally people have a tendency to overestimate their own chances of positive events and lower the probability of negative events which may affect them – in comparison with other members of a particular group or whole population. Of course, the observed optimism may be justified in relation to a particular person (and linked to his or her individual capabilities and resources), but at group level it is a cognitive distortion which significantly affects the estimation of the probabilities by the individual.

The above examples of heuristics describe methods used by people in the decision making process associated with probabilistic information. As defined, heuristic is a way to select one of the available alternatives while reducing the cognitive effort associated with the decision.

However, according to the above examples, heuristic thinking is associated with focusing on only one aspect of the situation and ignoring others which – as it was shown – results in incorrect estimations. On the other hand, heuristics in many cases allow for fast decisions, especially with regard to non-essential issues, and save mental energy that can be used for making decisions on a larger significance. Awareness of the existence of these heuristics can also be the first step to resist them when the consequence of using particular heuristic could be making the wrong choice.

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APPLICATION OF THE RANKING METHOD IN THE PROCESS OF RELIABILITY ESTIMATION

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Abstract: The functioning of a system man-technology-surroundings (M-T-S) is always inseparably connected with disruptions which can generate loss (human or material) in the system. The scope of this loss and its occurrence probability instigates researchers to investigate the problem of reliability in various areas of engineering activity, including also the management of work safety. The objective of the paper is to discuss the estimation possibility of events whereof occurrence is probable when working as a driver of an electric trolley mine locomotive when there is no available information on statistical data involving the disruptions taking place in the past. In this case, the modeling of reliability was carried out using the opinions of experts, who, by determining the positions of events (disruptions) in the ranking, carry out the calibration of their array, whereby the numerical values involving the probability of event occurrence can be determined.

Introduction

Reliability is one of the key problems of the occupational health and safety, and the relation between such notions as risk, hazard and unreliability constitutes a background of all methods of risk analysis [3], [4]

$$(risk\ measure) = (unreliability\ measure) \times (hazard\ measure).$$

Therefore, when describing the functioning of the system “man-technology-surroundings” (M-T-S), we must also take into consideration factors which can contribute to the occurrence of adverse events (malfunctions). The malfunction can involve the M-T-S system as a whole, technical equipment¹ (whole subassemblies or their components) and the personnel (groups or individuals). We can distinguish physical malfunctions i.e. the events which bring about an immediate dysfunction of an object, and symbolic malfunctions whereof occurrence does not lead to any breaks in the functioning of the object, but it may generate a damage which can bring about big loss and/or functionality deterioration of an object. Therefore, when the risk measure is

¹ - in the case of technical equipment also the a term “damage” is applied as an alternative.

accepted as the occurrence probability of loss $C(t)$ not lower than c in the operating time t of the M-T-S system [3], [4]

$$\Lambda(c, t) = P\{C(t) \geq c\}$$

then the above relation can be also presented in the following form $\Lambda(c, t) = Q(t) Z(c)$,

where: $Q(t)$ – occurrence probability of adverse events,

$Z(c)$ - hazard measure.

For the quantitative reliability analysis, so called reliability measures are commonly applied, such as:

- a reliability function $R(t) = P\{T > t\}$ $R(t) \in \langle 0; 1 \rangle$

where: T – operating time of the object until the first malfunction or between the malfunctions,

$P\{T > t\}$ - probability that the malfunction does not occur until time t
(random events $T \leq t$ and $T > t$ are events opposite to $P\{T > t\} + P\{T \leq t\} = 1$),

- an unreliability measure $Q(t) = 1 - R(t)$.

When we assume that there exists a time derivative of the distribution function of a random variable T , i.e. the probability density $f(t)$ is equal to

$$f(t) = \frac{dQ(t)}{dt}$$

$$\text{then: } Q(t) = \int_0^t f(\tau) d\tau; \quad R(t) = 1 - \int_0^t f(\tau) d\tau.$$

When the operation time without malfunction T has the exponential distribution, then:

$$f(t) = \lambda e^{-\lambda t}$$

$$Q(t) = \int_0^t f(\tau) d\tau = 1 - e^{-\lambda t}; \quad R(t) = e^{-\lambda t}$$

where: λ - a constant parameter interpreted as the number of malfunctions of the object in time unit, τ - operating time between malfunctions.

In the estimation process of object reliability, apart from modeling the phenomena with the use of relations facilitating the calculation of the above parameters, also statistical methods (access to the information and data involving the events which have already occurred is required) and expert methods are applied. In the case of the latter ones, the access to so called data banks is not required – the estimation of uncertainty is based on

the opinions of people (experts) who are acquainted with the analyzed problem. The opinions expressed independently by each expert are based on their knowledge and/or experience.

Application of experts' opinions in the estimation process of reliability – a calculation example

With respect to expert methods we can distinguish the method of direct estimation, in which the experts express their opinions on e.g. the occurrence frequency of definite events, providing the number of these events in an arbitrary time interval, and the ranking method [1], [2]. The latter method is applied in the cases when it is necessary to estimate the occurrence probability of a greater number of events within a definite time period (week, month, year etc.). In the discussed method, each of the appointed experts is individually arraying the analyzed events, e.g. from the least to the most probable, and then a joint position of each of the events is determined within the scope of final ranking. In order to determine the probabilities of events, a so called arraying calibration of events must be carried out, using for that purpose the following relation:

$$\log Q = a (\text{poz}) + b$$

where: a, b – unknown calibration coefficients,

poz – joint position of the event in the ranking.

In order to determine the calibration coefficients a and b , the information involving the probability value of at least two events in the ranking is required, and the more distant the events are in the ranking, the more accurate the calibration is².

Let us assume that for the position of a driver of a train of mine carriages³, 5 events have been defined:

$X^{(1)}$ – a fall when entering/leaving the locomotive;

$X^{(2)}$ – a fall of the employee when moving along the transport route (e.g. to change the settings of the crossover, to walk to a telephone stand etc);

$X^{(3)}$ – uncontrolled shift of the transported cargo;

$X^{(4)}$ – collision of the train with some obstacle;

$X^{(5)}$ – derailling of the train

for which occurrence probability is to be estimated.

Since there is a necessity to determine calibration coefficients, the above set has been complemented with two events:

$Y^{(1)}$ – electric shock;

$Y^{(2)}$ – hand injury when connecting/disconnecting particular carriages of the train,

² - the events whereof the occurrence probability is known, can but do not have to belong to the set of the analyzed events.

³ - the transport is realized with the use of locomotives Ld31/2 and carriages of Granby type.

for which occurrence probability within the time of one year is respectively:

$$Q_Y^1 = 0,250 \text{ and } Q_Y^2 = 0,8571.$$

For the needs of the studies, a panel of experts was appointed which comprised seven members of supervision staff of the GPT department (low and medium supervision staff) and six laborers (drivers of electric mine locomotives). The mentioned employees, individually and independently from one another, allocated the positions $X^{(1)} \div X^{(5)}$ and $Y^{(1)} \div Y^{(2)}$ to particular places in the ranking. The list of the opinions of experts taking part in the investigation studies is presented in Table 1.

Tab.1. List of events in the ranking

Number of expert	Location of event in the ranking						
	$X^{(1)}$	$X^{(2)}$	$X^{(3)}$	$X^{(4)}$	$X^{(5)}$	$Y^{(1)}$	$Y^{(2)}$
1	1	4	6	2	5	7	3
2	2	3	5	4	7	6	1
3	2	1	4	6	5	7	3
4	1	3	6	4	5	7	2
5	3	4	5	1	7	6	2
6	3	2	7	6	5	4	1
7	1	3	7	6	5	4	2
8	3	2	5	6	7	4	1
9	1	3	5	4	7	6	2
10	1	4	6	2	5	7	3
11	2	3	6	7	4	5	1
12	1	4	5	3	6	7	2
13	2	4	7	3	6	5	1

The summary of average ranks for individual items in the ranking is presented in table 2, while the relative measures of variation (coefficients of variability) for individual events – in table 3.

Tab.2. Average positions of particular positions in the ranking

$X^{(1)}$	$X^{(2)}$	$X^{(3)}$	$X^{(4)}$	$X^{(5)}$	$Y^{(1)}$	$Y^{(2)}$
1,7692	3,0769	5,6923	4,1538	5,6923	5,7692	1,8462

Tab. 3. Relative measures of variation for individual events $V(X^i)$ [%]

$V(X^{(1)})$	$V(X^{(2)})$	$V(X^{(3)})$	$V(X^{(4)})$	$V(X^{(5)})$	$V(Y^{(1)})$	$V(Y^{(2)})$
47,0	31,0	16,6	45,9	18,1	21,4	43,4

Using the position averages and the occurrence probability of events $Y^{(1)}$ and $Y^{(2)}$, calibration coefficients a and b were determined:

$$\begin{cases} \log(0,250) = 5,7692a + b \\ \log(0,8571) = 1,8462a + b \end{cases}$$

$$a \approx -0,1364$$

$$b \approx 0,1849$$

Hence, the calibration expression in the investigated case has the following form:

$$\log Q^{(k)} = -0,1364 (\text{poz}^{(k)}) + 0,1849$$

where: $k = 1, 2, \dots, 5$.

The values involving the occurrence probability of events $X^{(1)} \div X^{(5)}$ are presented in Table 4.

Tab. 4. List of occurrence probability of events $X^{(1)}, X^{(2)}, \dots, X^{(5)}$

$Q^{(1)}$	$Q^{(2)}$	$Q^{(3)}$	$Q^{(4)}$	$Q^{(5)}$
0,8782	0,5824	0,2562	0,4152	0,2562

Conclusions

Due to the need to satisfy legal regulations and also to improve work efficiency, the problems of occupational risk management are becoming more and more attractive. Due to the ability to identify the most dangerous elements or the most dangerous stages (phases) which occur within the system M-T-S the problems of reliability are becoming an inseparable element of work safety management. The focus on determining hazard potentials brings about a situation where modeling reliability is becoming a necessity. Yet, the commonly applied statistical methods for the estimation of reliability require the access to so called data banks (information about the events), which is not always possible. By the application of the ranking method presented in the article, which belongs to a group of expert methods for reliability estimation, it is possible to determine the probability values of events which disrupt the functioning of the M-T-S system, basing on the opinions of experts. An appointed group of people who know the specificity of the analyzed problem determine the position of events (disruptions) in the ranking and then calibrate their array.

With respect to the example presented in the paper, the highest occurrence probability out of the five analyzed events involves the fall when entering/leaving the mine locomotive $Q^{(1)} \approx 0,8782$ and the fall of the employee while moving along transport routes ($Q^{(2)} \approx 0,5824$). These events in most cases are not followed by serious consequences as compared, for example, to the consequences caused by an uncontrolled shift of transported cargo, collision of the train with some obstacle or by derailling.

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FRANTIŠEK ČUŘÍK (†JUNE 7, 1944) THE FIRST PROFESSOR OF MATHEMATICS AND DESCRIPTIVE GEOMETRY AT MINING UNIVERSITY (VŠB)

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Abstract: In this paper we commemorate the life story of František Čuřík (born on June 23, 1876), from 1919 to 1939 professor of mathematics and descriptive geometry at the Mining University (VŠB) in Příbram, the first one holding the chair. He was originally a mechanical engineer and held a doctorate in probability theory. He was an ardent teacher, skilful organizer, philanthropist and active textbook writer. He took his own life during the time of Nazi rule over the destroyed Czechoslovakia, exactly 70 years ago.

Introduction

The Mining University had its seat in Příbram, a mid-sized county centre lying 50km south-west from Prague, until 1945, and it was then moved to Ostrava, which is, even if a rather large city, 350km from Prague by train. Hence, it has always had a seat in a periphery. Moreover, mathematics was just one of the subjects forming general background, not a subject of study in itself. Thus, there was just one professor with two assistants at the department. These are the main reasons why its history has not been in the focus of historians of mathematics. In any case, there is no literature on the subject written in English.

As this paper is based on our previous papers published in Czech and German, see [2]–[4], we have chosen to reduce citations to the minimum and refer the interested reader to [4], where all sources are properly cited.

Studies and beginning of career

František Čuřík was born in Smíchov, today a part of Prague. His father Josef Čuřík was an official at district authorities (*C. k. okresní hejtmanskství*). Sadly, his parents died untimely and František grew up supported by his uncle Karel Havránek, a pastor in the nearby Slivenec, who gave him the opportunity to study at *Realgymnasium* in Příbram. Having graduated from it in 1895, František studied mechanical engineering at Czech Technical University in Prague (*C. k. Česká vysoká škola technická v Praze*). His interest for mathematics and descriptive

geometry had risen noticeably there, mainly under the influence of Augustin Pánek (1843–1908) and Eduard Weyr (1852–1903).

During his studies, František fulfilled his obligatory military service (as a machine operator by the navy in the Austria's main naval base in Pula), then he worked as a volunteer and later technical officer in a company owned by one of the most successful Czech industrialists Emil Kolben. Finally, he chose a scientific occupation, though, and became assistant of Professor Augustin Pánek at his alma mater. From 1907/8 he substituted Augustin Pánek in his lectures in calculus.

Having graduated in mechanical engineering in 1904, František studied further, namely mathematics and descriptive geometry at the Charles-Ferdinand University in Prague and passed examinations allowing him to teach at secondary schools. At Czech Technical University, he defended doctoral thesis on Bernoulli theorem in probability theory and got the degree *doctor rerum technicarum* (a Ph.D. equivalent) in 1910. At the same time, he got a tenured professorship in mathematics and geometry at a public secondary technical school in Prague (*C. k. Státní průmyslová škola v Praze*).

However, František held on the lectures in mathematics at Czech Technical University up to 1919, he wrote in the meantime his first two papers for the local mathematical journal *Časopis pro pěstování matematiky a fyziky* and published first volume of textbook on calculus under the title *Základy vyšší matematiky*. However, he did not try to get *venia legendi*.

Professor at Mining University (VŠB)

After the birth of the first Czechoslovak Republic, VŠB quickly adapted to the new conditions. In 1919/1920, a thorough reorganization of VŠB took place. Six new chairs were established, the free chair in mathematics a descriptive geometry was created by splitting the former chair for mathematics and physics, while its professor Josef Theurer, being a physicist himself, held on the chair for physics.

On September 24, 1919, order of the candidates was decided at the faculty meeting of VŠB. Applications to be considered came from dr. František Čuřík (*1876), dr. Václav Hruška (*1888) and dr. František Nachtikal (*1874). From the today's point of view, Čuřík's chances did not look very promising, both Hruška, later Professor of applied mathematics at Czech Technical University in Prague and Nachtikal, later Professor of physics at technical universities in Brno and Prague, are more eminent names. However, Nachtikal's serious disadvantage was his lack of experience in descriptive geometry, and Hruška, even if scientifically better, was in opinion of the committee, too young for the chair.

Finally, apart from his scientific qualification, Čuřík's great teaching experience was stated as the main reason for choosing him. We note that Čuřík, as a graduate from Příbram Realgymnasium, like some members of the faculty, also knew the local situation well, which could have played some role in the decision process.

On January 25, 1920, František Čuřík was appointed Extraordinary Professor for Mathematics and Descriptive Geometry. On July 18, 1921, he was promoted to Ordinary Professor. He quickly joined the academic life of the University, he worked for many academic associations, he served as the (vice-)chairman of the committee for the first state exams, etc. In 1924/25 he was first elected rector of the University. In this office, he successfully organised celebrations of the 75th anniversary of VŠB. At the event, honorary doctorate was awarded to Tomáš Garrigue Masaryk, first President of Czechoslovakia.

Čuřík tried hard, for the whole four terms of his rector office, to enforce the long-lasting claims of the professorial board of VŠB, namely moving the school to Prague and building a

modern campus in Prague-Dejvice. This ambitious project did not come to being, first mainly due to political obstacles, later also due to economic crisis.



Picture 1. František Čuřík (1876–1944). Příbram, 1920s.

Textbooks on “technical mathematics”

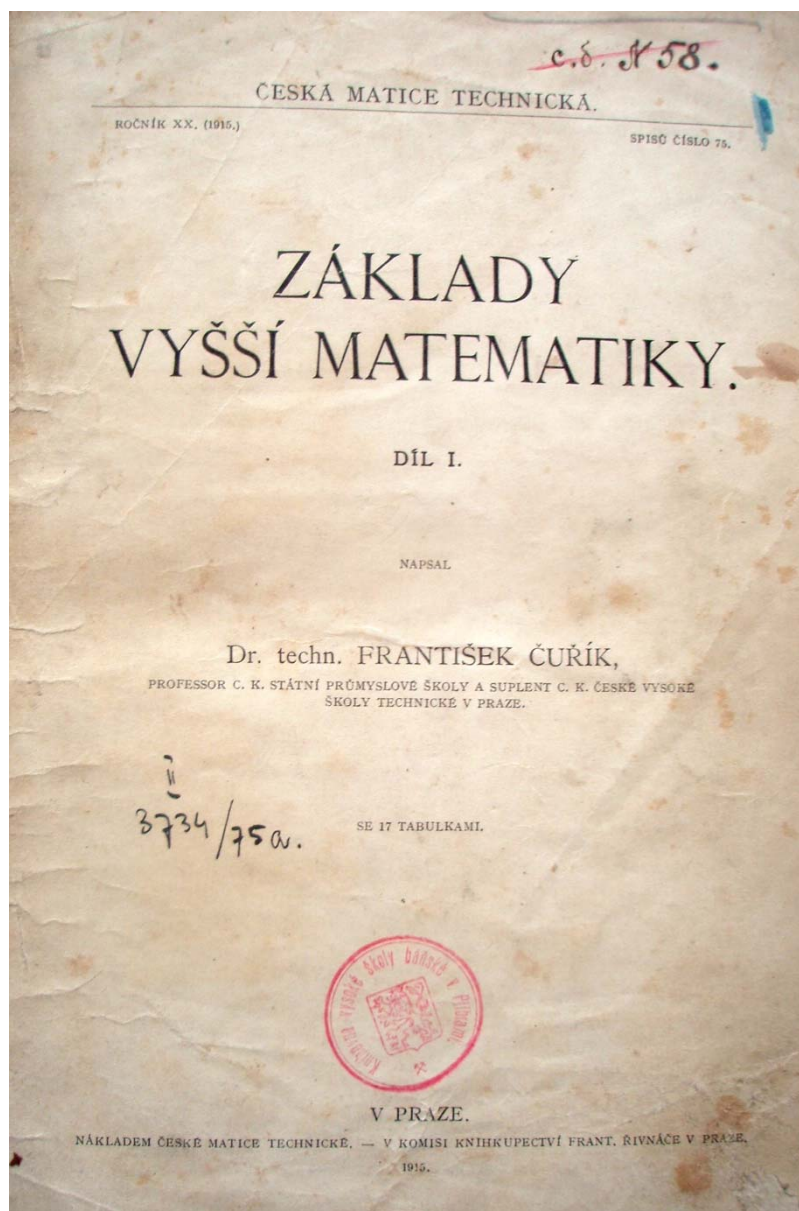
By the notion of technical mathematics I understand calculus for engineers, as well as Čuřík and contemporary engineers as Vladimír List did, see [5]. Thus, it denoted textbooks on calculus written with respect to the needs of engineering practice. The book *Základy vyšší matematiky* by Čuřík [1] was the first textbook on higher mathematics (= calculus) written from this point of view. Nevertheless, it was heavily criticized by some pure mathematicians, mainly Matyáš Lerch, professor of mathematics at (Czech) Technical University in Brno.

Lerch criticized mainly, apart from a couple of evident mistakes, “loose style” of the book that led to imprecise formulations. The book indeed relaxed the over formalized style typical for the contemporary pure mathematics, definitely the positive for an engineering student, but on the other hand sometimes his unfortunate phrasings could have been easily misinterpreted. He gave, e.g., function

$$f: y = \begin{cases} x \cdot \sin \frac{1}{x} & \text{for } x \neq 0, \\ 0 & \text{for } x = 0, \end{cases}$$

as an example of continuous function without derivative at $x = 0$, cf. [4], p. 190. Comparison of the treatment in the first and the second edition (see page 55 of the first edition,

respectively on page 37 of the second edition of [1]) shows that Čuřík took the objections seriously, revised the textbook thoroughly and corrected most of the inaccuracies. Still, I rate the book to be very demanding. Nevertheless, it was profusely used by the whole generation of engineers.



Picture 2. Well-worn title page of the first edition of the textbook on *Foundations of higher mathematics* by František Čuřík (Courtesy of the library of VŠB).

In 1918 Čuřík finished second volume of *Základy vyšší matematiky*, devoted mainly to integral calculus, in 1921 contributed into engineer's guidebook series *Technický průvodce* with the mathematical formula tables (appeared as volume 1), in 1922 published his lectures on the method of least squares, *Počet vyrovnávací* (as a regular textbook, it only appeared in 1936). Later in the 1920s and 1930s, he revised thoroughly all the textbooks. At the end of the 1930s he published mathematical and statical tables (volume 19 of *Technický průvodce*) and he did not break up the work even during the German occupation, cf. below.

Life of a mathematician under German occupation

After the occupation of Czech lands by Nazi troops, an illusion of a normal situation was maintained initially. However, after heavy demonstrations in Prague on the occasion of 21st anniversary of the birth of Czechoslovak Republic on October 28, 1939, the route of oppression was taken. All Czech universities were closed on November 17, 1939 and about a thousand of students were arrested and sent to the Nazi concentration camp for political prisoners located in Sachsenhausen-Oranienburg. Full professors were sent on forced leave, and also their retirement age was gradually decreased. The other teaching staff (extraordinary professors, associate and honorary professors, assistants) had to take jobs at secondary schools or in the industry.

František Čuřík was sent on the forced leave at the end of 1939. He devoted himself, with the help of the professors František Klokner (1872–1960) and Zdeněk Bažant (1879–1954) from Czech Technical University in Prague, to revising his last books from the series *Technický průvodce*. However, he worked in isolation, which meant that he could not even use the library of the institute, because it was locked in the building of the closed Mining University. In this situation, as a member of *Sokol*, with friends and colleagues being arrested, it was definitely not easy to find peace of mind for scientific work.

In October 1940 František Čuřík was sent to permanent retirement. He died tragically on June 7, 1944. Reasons for his suicide remain unclear. From the last period of his life, we do not have any primary sources. Hence, we have to rely on indirect sources and their combination.

Some of his colleagues from VŠB promoted the story that František Čuřík was forced by the Nazis to collaborate in their company Waffen-Union research institute in Příbram (contemporary German name of the town was *Pibrans*) on the ballistic computations of missiles V-2. He resolutely refused it as treason and, because he could not see any other way out of this situation, he took his own life, cf. [7].

However, the story is probably only partially based on the truth. The holding company Waffen-Union Skoda-Brünn worked in weapon industry within the concern Reichswerke Hermann Göring. The top management was exclusively German, but research divisions were under Czech governance, mathematical research division was led by Miloslav Hampl (1897–1974) and physics division by Professor Václav Dolejšek (1895–1945), who handed his responsibilities to dr. Miloslav Tayerle during 1940. Tayerle should have set up the research institute of Waffen-Union in Příbram. He brought his collaborators from Skoda, hired some researchers from *Zbrojovka Brno* (Brünn) and counted also with some professors from the closed VŠB in Příbram. Their names are listed in the report about the visit of the Mining University from October 9 1943: Jirkovský, Šebesta, Čechura, Glazunov and Mitinský. František Čuřík, then already pensioned for three years, was not considered. However, in this period, there was also no rocket research.

The situation changed rapidly in August 1944, when SS-Hauptsturmführer Rolf Engel, head of the *Versuchsanstalt für Strahltriebwerke* in Grossendorf, evacuated his institute from West Prussia to the Protectorate of Bohemia and Moravia and overtook the governance of the research facilities in Příbram. He had already worked intensively in the rocket research for SS and he strictly opposed the German army project of V-2, cf. [6]. When Engel came to Příbram, Čuřík was already dead. But even if Engel had looked for collaborators in the spring 1944, he would not have chosen Čuřík. Ballistics was a key discipline, controlled entirely by the Germans, and Engel had his own mathematicians with expertise in ballistics, namely dr. Uwe Tim Bödewadt (1911–2003), dr. Franz Kalscheuer (1913–2002), Niels W. Larsson or H. Teichmann, see [3].

The role of some professors of VŠB in constructing the story on Čuřík's suicide is not clear either. Did they want to pay tribute to their former colleague? Or did they need a story excusing their own collaboration? Probably both motivations together formed the story. Čuřík was a respected long-time member of the faculty, appreciated for his pedagogical and moral qualities. On the other hand, they knew that certain people did not endorse their decision about collaboration in Waffen-Union. Moreover, some of them stood trial after the war, namely in front of retribution court in Příbram. Hence, we do not know what happened; we know only what did not happen.

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METRIC'S INFLUENCE FOR CLUSTERING

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Abstract: This paper presents the grouping of elements in a relatively homogeneous class. Matlab's procedure CLUSTER was used for computing. It interacts with the other two procedures PDIST and LINKAGE. The procedure PDIST defined metric by which to calculate the distance between the elements of the set. The procedure LINKAGE determined the method to clustering. The paper shows examples of the division a one hundred points set of into subsets according to various metrics.

1. Introduction

Data clustering is a method of grouping components making relatively homogeneous class. The basis for the grouping in most algorithms is the similarity between the elements - expressed using the function (metric) similarities.

The purpose of cluster analysis is to organize observed data into meaningful structures or groups by analyzing the similarities in the elements of tested according to established criteria. In other words, the essence here search elements in the test population (experimental results, variables, objects) in such a way as to form a group (concentration, clusters), where in terms of specific features of these components to each other are as similar as possible and at the same time other than the maximum in other groups. Grouping is based on the isolation of groups (classes, subsets).

2. Procedures CLUSTER, PDIST and LINKAGE Matlab

2.1. CLUSTER procedure

In Matlab is prepared CLUSTER procedure allows grouping of elements in the set. Before use, you must specify the metric by which to calculate the distance between the elements of the set. For this serves PDIST procedure. Also, specify the method by which it is to be clustering. For this procedure is LINKAGE. The set of these three instructions in Matlab is as follows: `y=pdist(x,'metric');` `z=linkage(s,'method of grouping');` `t=cluster(z,'number of groups');` As a result of the procedure is obtained CLUSTER vector length equal to the number of points to be grouped. The elements of this vector are numbers from 1 to 'number of groups'.

These numbers indicate the number of the group (cluster) to which the selected point will be assigned.

2.2. Procedure PDIST

Coordinates of points to be grouped should be given in a set of x second input parameter is 'metric'. You can use the following metrics: 'euclid', 'seuclid', 'cityblock', 'mahal', 'minkowski'.

a) 'euclid' – Euclidean metric

It is the Euclidean distance defined by the formula

$$d(X, Y) = \left(\sum_{i=1}^n (x_i - y_i)^2 \right)^{1/2} \quad (1)$$

where X, Y are a set of points in n -dimensional space, and x_i and y_i coordinates of those points.

For the points $X(1,3), Y(2,5), Z(8,7)$ $d(X,Y)=2.2361, d(X,Z)=8.0623, d(Y,Z)=6.3246$.

b) 'seuclid' – Standardized Euclid metric

It is a standardized Euclidean distance defined by the formula

$$d(X, Y) = \left(\sum_{i=1}^n \frac{1}{\hat{s}_i^2} (x_i - y_i)^2 \right)^{1/2} \quad (2)$$

where $\hat{s}_i^2$ is the variance of the i -th coordinate points.

For the points $X(1,3), Y(2,5), Z(8,7)$ $d(X,Y)=1.0343, d(X,Z)=2.7237, d(Y,Z)=1.8739$.

c) 'cityblock' – City Block metric or Manhattan distance

This is the so-called "distance city" defined by the formula

$$d(X, Y) = \sum_{i=1}^n |x_i - y_i| \quad (3)$$

For the points $X(1,3), Y(2,5), Z(8,7)$ $d(X,Y)=3, d(X,Z)=11, d(Y,Z)=8$.

d) 'mahal' – Mahalanobis metric

It is the Mahalanobis distance defined by the formula

$$d(X, Y) = \left((X - Y) \cdot C^{-1} \cdot (X - Y)^T \right)^{1/2} \quad (4)$$

where C is the covariance matrix

$$C = \text{cov}(X, Y) \quad (5)$$

For the points $X(1,3), Y(2,5), Z(8,7)$ $d(X,Y)=2, d(X,Z)=2, d(Y,Z)=2$.

e) 'minkowski' – Minkowski metric, distance in an absolute Minkowski power metric

It is the Minkowski distance defined by the formula

$$d(X, Y) = \left(\sum_{i=1}^n |x_i - y_i|^p \right)^{1/p} \quad (6)$$

The size p is given as the third parameter `pdist` procedure: `y=pdist(x,'minkowski', p)`. If you use only the first two parameters without a "p", this parameter is assumed to be $p=2$ (in the example is $p=4$).

For the points $X(1,3), Y(2,5), Z(8,7)$ $d(X,Y)=2.0305, d(X,Z)=7.1796, d(Y,Z)=6.0184$.

2.3. Procedure LINKAGE

LINKAGE procedure is used to determine the method by which it is to be clustering. The input parameters LINKAGE procedures are vector y obtained before the procedure PDIST and 'method of grouping'. You can use the following names for the grouping: 'single', 'complete', 'average', 'centroid', 'ward'.

a) 'single' – nearest distance

This method is called by nearest neighbor or by a single bond. The distance between two clusters is measured as the shortest distance from the distance between the units of one and the other focusing [1].

If we assume that we have in the n-dimensional set $X=\{x_i\}$, where x_i are the elements of this set ($i = 1, \dots, k$), the set $Y=\{y_j\}$, where y_j are elements of the set Y ($j=1, \dots, m$) and the point P does not belong to the set X , the distance of the point P from the set X is given by

$$d(P, X) = \min_i d(P, x_i) \quad (7)$$

The distance of X from the set Y is given by

$$d(X, Y) = \min_{i,j} d(x_i, y_j) \quad (8)$$

In this method combine the groups based on the spacing between adjacent elements belonging to the combined clusters. The group with the smallest distance between their closest elements are combined as the first [2].

b) 'complete' – furthest distance

This method is called by the farthest neighborhood method or full binding. The distance between two clusters is determined as the greatest distance from the distance between the units of one of the other focus [1].

For the point P does not belong to the set X distance of the point P from the set X is given by

$$d(P, X) = \max_i d(P, x_i) \quad (9)$$

For two sets X and Y distance between them in the formula

$$d(X, Y) = \max_{i,j} d(x_i, y_j) \quad (10)$$

In this method, we use the distance between the most distant elements belonging to the groups, in order to decide which of the two clusters to combine the first [2].

c) 'average' – average distance

Medium is a method call. The distance between two clusters is calculated as the arithmetic average of the distances between individuals from one of the other focus [1]. For the point P and the set X defined as above, the distance between them is

$$d(P, X) = \frac{1}{k} \sum_{i=1}^k d(P, x_i) \quad (11)$$

For two sets of X and Y are defined as above, the distance between them is

$$d(X, Y) = \frac{1}{k \cdot m} \sum_{i=1}^k \sum_{j=1}^m d(x_i, y_j) \quad (12)$$

In this case, the distance between clusters is defined as the average distance between all pairs of elements belonging to both groups. This method is abbreviated as UPGMA (unweighted pair group method with arithmetic mean) [2].

d) 'centroid' – center of mass distance

For each group we calculate the centroid - as the average of all objects (vectors) belonging to the group. The distance between clusters is defined as the distance between the centroids of these clusters. This method is abbreviated - UPGMC (unweighted pair group method centroid) [2].

The main idea of the algorithm is to determine the k centroids - one for each group. Then, each data object is assigned to the nearest centroid. So you create a k -clusters that we receive. We continue the algorithm for each of the clusters obtained pre-calculate a new centroids and objects are distributed in the manner already discussed anew.

Repeat this operation as long as none of the objects does not change the cluster.

e) 'ward' – inner squared distance

In this method, two groups of objects are combined into one group so as to minimize the sum of squared deviations of all the objects of the two groups of the center of gravity of the new group, which will result from combinations of these two groups [1].

At each step of merging groups of objects, of all possible to combine groups of objects are combined into one group, these groups, which in effect form a group of objects with the smallest variation due to variables that describe them.

3. Examples of clustering

For purposes of illustration we use the coordinates of the points distributed over the one hundred plane. As a method of clustering method was chosen for the farthest neighborhood (full binding method). Points on a plane is illustrated in Figure 1.

The first version of the program used in the calculation of the following commands `y=pdist(x, 'euclid');` `z=linkage(y, 'complete');` `t=cluster(z,3);`

This means that it was decided to Euclidean distance.

Clusters are presented in the following three figures.

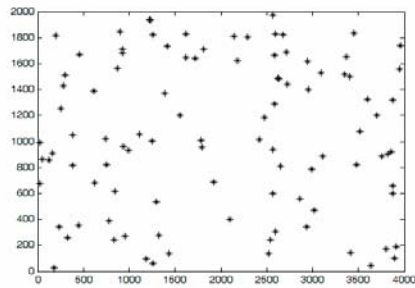


Fig. 1. Location hundred points on a plane

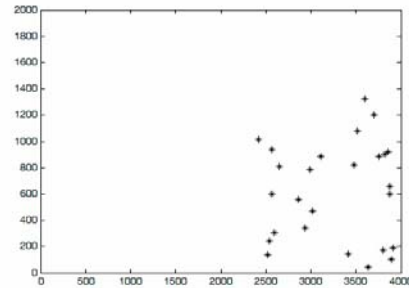


Fig. 2. Cluster 1 ('euclid')

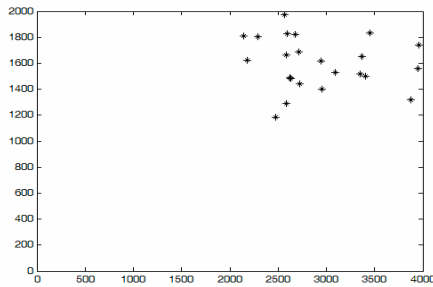


Fig. 3. Cluster 2 ('euclid')

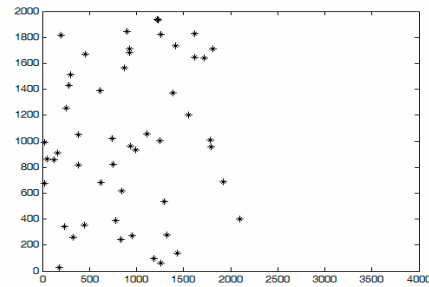


Fig. 4. Cluster 3 ('euclid')

The number of clusters was as follows: 26, 23, 51.

In the second version is used in the calculation of the following commands

`y=pdist(x, 'cityblock');` `z=linkage(y, 'complete');` `t=cluster(z,3);`

This means that the decision was made to city distance.

The clusters in this case are shown in the next three figures 5, 6, and 7.

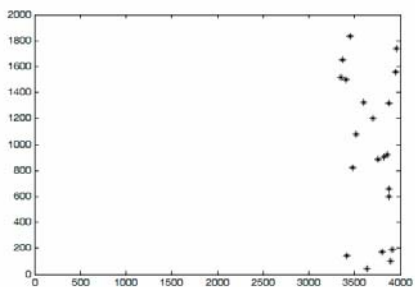


Fig. 5. Cluster 1 ('cityblock')

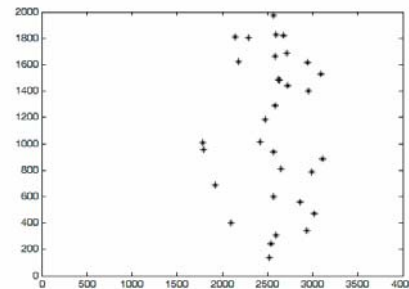


Fig. 6. Cluster 2 ('cityblock')

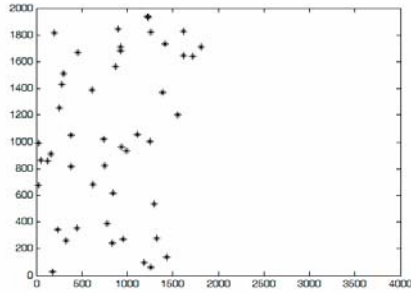


Fig. 7. Cluster 3 ('cityblock')

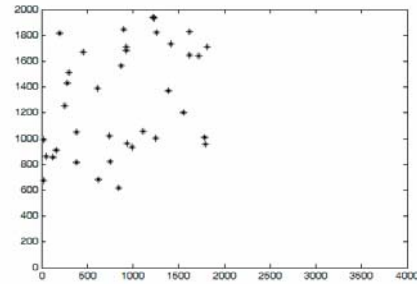


Fig. 8. Cluster 1 ('mahal')

The number of clusters was as follows: 21, 32, 47.

The third version of the calculations used in the following commands
`y=pdist(x, 'mahal'); z=linkage(y, 'complete'); t=cluster(z, 3);`

This means that it was decided to Mahalanobis distance.

The clusters in this case are shown in the next three figures 8, 9 and 10.

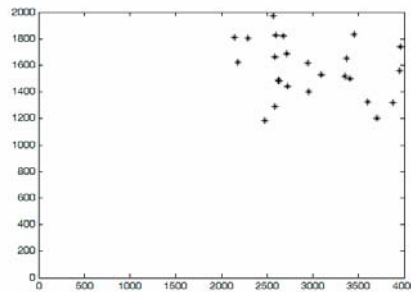


Fig. 9. Cluster 2 ('mahal')

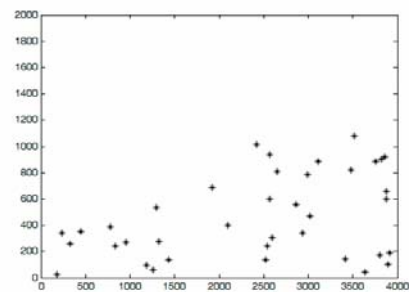


Fig. 10. Cluster 3 ('mahal')

The number of clusters was as follows: 37, 25, 38.

4. Summary

The paper shows examples of the distribution of a set of one hundred points into subsets according to various metrics. Used three metrics: 'euclid', 'cityblock', 'mahal'. Received different clusters. One can also use different methods of clustering. Thus, the use of different metrics and different clustering methods has significant impact on the final result of calculation. Been calculated using MATLAB.

References

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RADIATION HEAT TRANSFER IN THE CRUCIBLE FURNACE

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Abstract: Radiation heat transfer between two bodies is mathematically a surface integral, whose integrand is again a surface integral. The article concerns the transport of heat in the crucible furnace and suggests a solution of expressing the "inner" of these integrals.

Keywords: crucible furnace, view factor.

Introduction

This article deals with the mathematical problem of calculating radiation heat transfer in the crucible furnace. Crucible furnaces are mainly designed to melt metals and to maintain the molten metals in liquid state. Crucible furnace is a furnace of a circular cross section which is inserted into a refractory crucible. The shape of the crucible can also influence energy consumption. However, to calculate the appropriate shape of the crucible, there must be a sufficiently reliable mathematical model introduced at first.

Mathematical formulation of the problem

The calculation of the radiation heat transfer between the crucible and the furnace, i.e. between the crucible (body A bounded by surface S_A) and the furnace (body B with the inner surface S_B) can be mathematically expressed by the surface integral over the surface S_A whose integrand is a surface integral over the surface S_B . This article will only deal with the case in which the crucible (solid of revolution) is centrally located in a cylindrical furnace, i.e. surfaces S_A and S_B are rotational and coaxial.

The basis for calculating over the surface S_B is the so-called local view factor. Local view factor is a dimensionless number that expresses what proportional part of the radiation from the appropriate dS_A element of the surface S_A falls on a certain part of surface S_B (the rest falls on another part of the surface S_B or falls on the surface S_A itself, if it is concave).

The local view factor (hereinafter referred to as the view factor) for radiation from the dS_A element to a certain part of surface S_B (Fig. 1) is given by

$$\Phi_{A,B} = \frac{1}{\pi} \int_{S_{B^*}} \frac{\cos \alpha_A \cdot \cos \alpha_B}{d^2} \cdot dS_B ,$$

where dS_B is the element of surface S_B . Angles α_A , α_B are the angles between the normals of elements dS_A , dS_B and the line connecting both elementary surfaces; d is the distance of dS_A and dS_B . Furnace interior is the surface of the cylinder that, for purposes of this article, will be divided only into three sub-areas: bottom, wall and cover. For the calculation of the integral is generally not the whole surface S_B , but only the "visible" part of the S_B marked as S_{B^*} . The dS_A element radiates only into the outer half-space. Part of the area S_B which is located in this half-space may be overshadowed by the crucible. Area S_{B^*} is thus that part of the bottom, the wall and the cover surface, which is located in the half-space specified by the dS_A element and can be seen directly from the dS_A element.

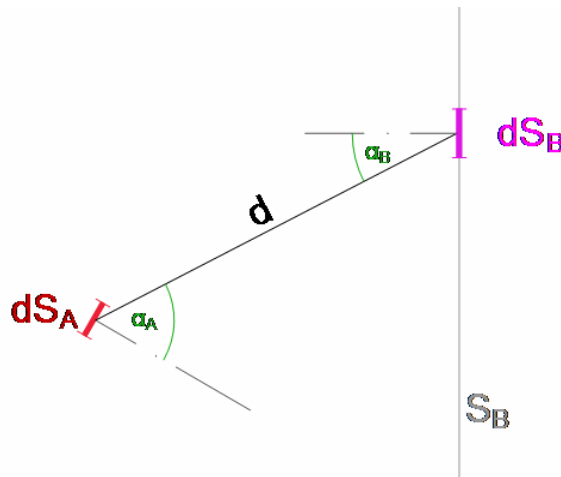


Fig 1: Local view factor for radiation from the dS_A element to the dS_B element of the surface S_B .

For instance, in Fig. 2 the cover cannot be seen from the red dS_A element and only a part of the bottom and a part of the wall of the furnace can be seen.

Proposed solution for the local view factor

In the available resources [1], an analytical solution of formulated problem has been found; however, it concerns only the case in which the area is cylindrical. The solution of [1] is therefore in our case (as seen in Fig. 2) directly applicable only for the vertical dS_A elements which are located on the "widest" cylindrical part of the crucible.

This article introduces a different design method of calculating these three view factors for the vertical dS_A element. The proposed calculation is only approximate but its principle can be used for the horizontal element as well. In the calculations for the

general shape of the crucible, it is then mathematically possible to replace any "oblique" dS_A element with a pair of appropriate vertical and horizontal elements.

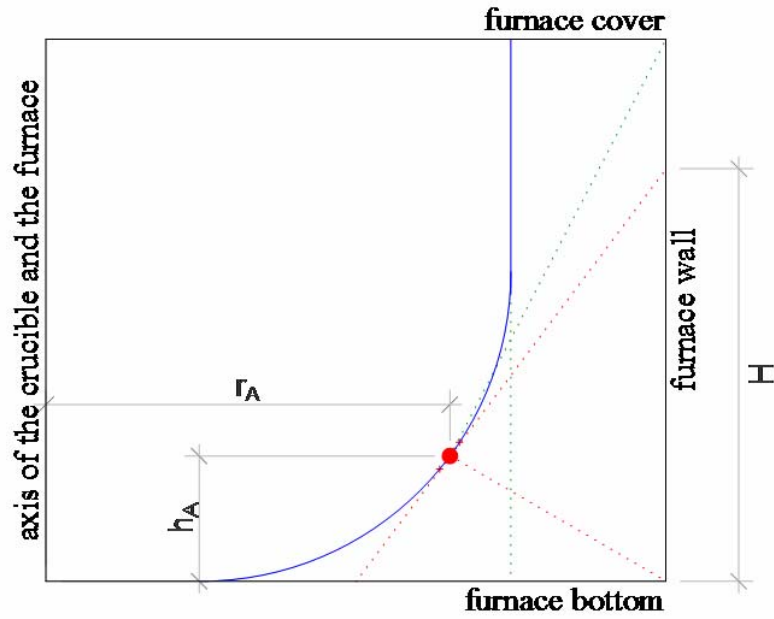


Fig. 2: Schematic axial section through the crucible and the furnace

The proposed calculation is based on the fact that the surfaces S_A and S_B are both rotational and coaxial. In fig. 3 (horizontal section) the solid green line represents the two dS_A elements, which are the same size and are at the same height.

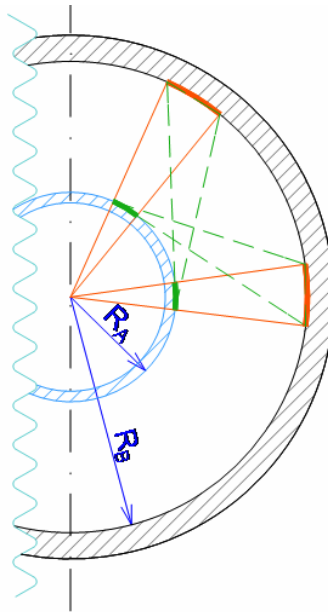


Fig. 3: Horizontal section of crucible furnace

Each of the elements determines an elementary cylindrical sector which corresponds to an identically large area on the furnace wall (in Fig. 3 appearing as an orange arch). The same amount of radiation that one dS_A element provides to another into his "arch" on the wall of the furnace, it also gets back. The amount of radiation that a certain element radiates onto the entire visible part of the furnace wall is the same as if it was all emitted just to its sector. The same applies to the visible part of the bottom and the cover.

If the dS_A element is not in a close proximity to the bottom or the cover, it can be expected that the amount of radiation calculated only for the relevant parts of the elemental sector will be in the same proportion as the corresponding view factors. The amount of radiation in the elemental sector specified by an elemental angle, of course, is once again elementary. For practical use, it is appropriate to formally convert the corresponding amount of radiation to the sectors with angle size 1.

For a simplified form of the crucible (see Fig. 2 – the upper part of the contour is a vertical line, the bottom is convex circular arc), six formulas have been derived, given the assumed conditions. Three of these formulas were designed for a vertical element and three for a horizontal element for radiation on the cover, the wall and the bottom. The formula for the radiation of the horizontal element on the cover for the shape of the crucible as mentioned above (even for normal crucibles) is not necessary.

For instance, the integral to calculate the quantity of radiation on a part of the furnace wall has the form

$$\frac{1}{\pi} \int_0^H \frac{(R_B - r_A)^2}{((R_B - r_A)^2 + (h - h_A)^2)^2} \cdot dh ,$$

where R_B is the radius of the furnace,

r_A, h_A are the coordinates of the element (radius, height),

H is the height of the visible part of the furnace wall (see Fig. 2).

All these six integrals have analytical solutions. To calculate the view factor of the element of the considered crucible on the cover, the wall and the bottom, it is necessary to calculate the appropriate amount of radiation and divide it by their sum. This type of crucible does not radiate on itself, therefore the sum of the calculated three view factors must be 1.

For the dS_A elements of the upper vertical part of the crucible, the obtained results should correspond according to the results in [1]. During control calculations in Matlab, we have achieved a very good agreement with the elements that were not in close proximity to the cover. When the dS_A element "approached" the cover, its view factor on the cover began to increase disproportionately and in an extreme position of the element, it even significantly exceeded the theoretical limit value, which is $\frac{1}{2}$. At the same time, the problems associated with dividing by zero appeared.

These problems could be resolved by another calculation of the amount of radiation on the cover sector and the bottom sector.

This method is based on the mathematical assumption that the sector of the bottom and the cover is "open" and that the rotary cylindrical furnace S_B of the wall surface is not limited by the bottom and the cover. Instead of the amount of radiation, which turns for instance on a certain cover sector, the amount of radiation that passes through a given hole and lands on an extended cylindrical surface is calculated. For the calculation of the amount of radiation on the cover, the wall and the bottom, only the integral for calculation of the radiation on the "wall" is used, namely in the intervals $\langle -\infty, 0 \rangle$, $\langle 0, H \rangle$ and $\langle H, \infty \rangle$.

Control calculations demonstrated a sufficient compliance of the results for the vertical dS_A element with the results obtained according to [1]. The results for the horizontal dS_A element could not be sufficiently verified yet. An analytical solution of this problem could not be found in the available resources. Therefore, there is an opportunity to confront the obtained results with the partial results obtained for some specific cases by much more sophisticated numerical methods.

If the proposed calculations for the horizontal dS_A element could be successfully verified, it would be possible to calculate the view factor of the "oblique" element accurately enough. The replacement of the "oblique" dS_A element by a pair of corresponding vertical and horizontal elements in the test examples led to the results that have the expected properties. It only remains to check their accuracy.

Conclusion

The proposed method of an approximate calculation of the local view factor for the element of the outer surface of the crucible in the crucible furnace is applicable for the general shape of the crucible. It could therefore significantly contribute to a further development of the mathematical modeling of radiation heat transfer between the crucible and the furnace.

Literature

- [1] <http://www.thermalradiation.net/tablecon.html>, section B, B-59 and B-60.

MATEMATIKA V I. ROČNÍKU FAST VŠB-TU OSTRAVA Z POHLEDU STUDENTŮ

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Abstrakt: Názory studentů na výuku předmětů garantovaných katedrou matematiky a deskriptivní geometrie v prvním roce studia na FS, FAST a FBI VŠB-TUO jsme zjišťovali formou anonymního dotazníku. Zajímá nás pohled studentů na úroveň jejich vstupních znalostí z matematiky a geometrie i přístup k přednáškám, cvičením a zkouškám. Ve svém příspěvku zabývám výsledky na stavební fakultě.

Abstract: With the form of an anonymous questionnaire, we investigated the students' opinions on the teaching of subjects that are guaranteed by the Department of Mathematics and Descriptive Geometry in the first year of study at the Faculty of Mechanical Engineering, Faculty of Civil Engineering and Faculty of Safety Engineering at VSB - Technical University of Ostrava. We have been also interested in the opinion of students on their level of entry knowledge of mathematics and geometry as well as their attitude to lectures, exercises and exams. In my paper I am going to present results related to the Faculty of Civil Engineering.

1 Úvod

Dlouhodobě pozorujeme klesající úroveň matematických znalostí studentů technických vysokých škol [1-6]. Rostoucí problémy se zvládnutím látky mají studenti v matematice a geometrii, s nedostatečnými vědomostmi z matematiky se potýkají rovněž vyučující odborných předmětů.

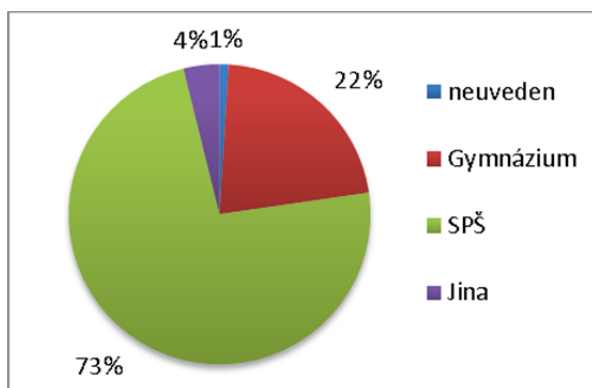
Příčiny tohoto stavu jsme se pokusili zjistit formou anonymního dotazníku, který vyplnili studenti tří fakult VŠB-TU: Fakulty strojní (FS), stavební (FAST) a bezpečnostního inženýrství (FBI). Dotazník celkem vyplnilo 581 studentů [6]. Dotazník měl odrazit pohled očima studenta na připravenost ze střední školy na úroveň výuky v prvním ročníku. Příspěvek je zaměřen na některé odlišnosti na Stavební fakultě VŠB-TU Ostrava (FAST).

2 Složení studentů

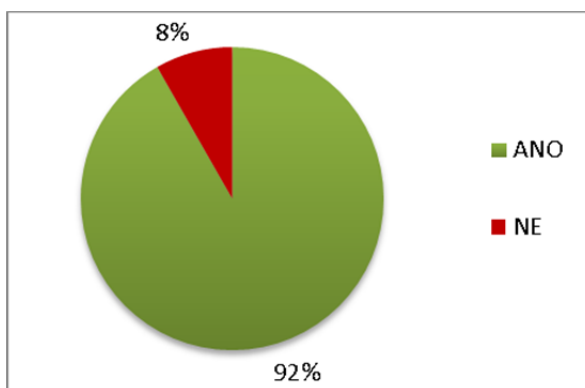
Na začátku třetího semestru celkem 207 studentů FAST vyplnilo anonymní dotazník. Šlo tedy o studenty, kteří získali dostatečný počet kreditů k zápisu do druhého ročníku.

Téměř tři čtvrtiny studentů, viz obr. 1, absolvovaly střední průmyslovou školu (většinou stavební). Je to to srovnatelný podíl jako na FS (na FBI je to jen 15%). Vzhledem

k převažujícímu počtu absolventů SPŠ nepřekvapí skutečnost, že 92% studentů si logicky vybralo FAST ze zájmu, jen pro 8% studentů byla FAST náhradním řešením, viz obr. 2.



Obr. 1: Složení studentů

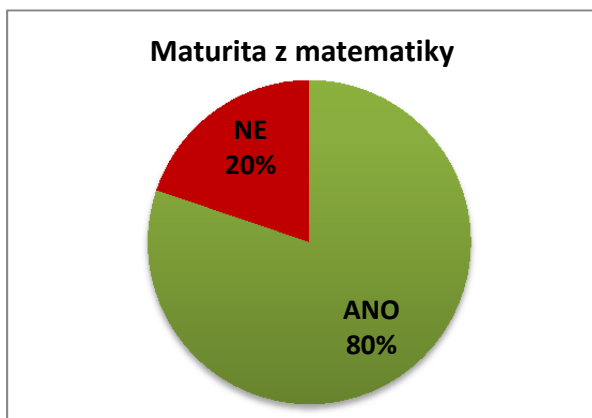


Obr. 2: Zájem o studium na FAST

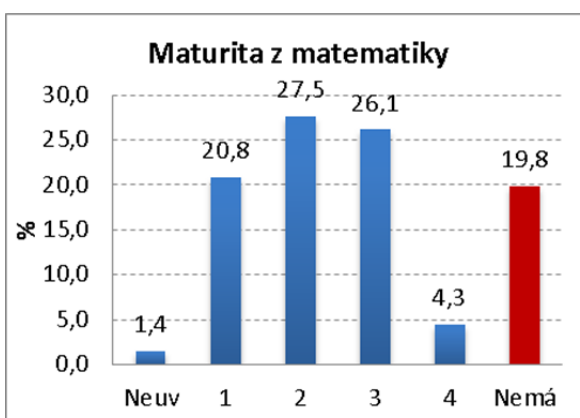
Dá se však předpokládat nižší úroveň znalostí z matematiky. V odborných předmětech by většinou přechod na vysokou školu nemusel činit tak velké problémy.

3 Přípravenost ze střední školy

Překvapením je, jak velký počet studentů (80%) maturovalo z matematiky, viz obr. 3. Z toho téměř 50% s výsledkem výborně nebo velmi dobře, viz obr. 4. Studijní výsledky v prvním ročníku, však příliš nekorrespondují s dobrými výsledky u maturity.

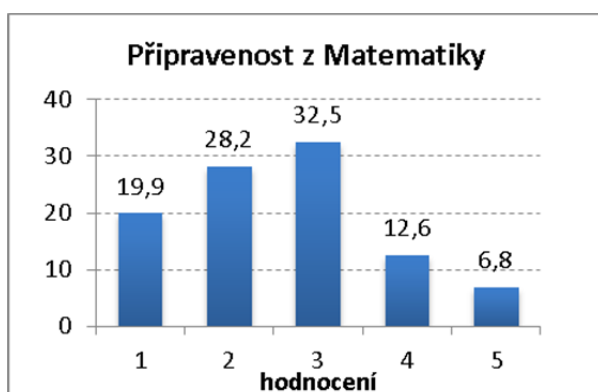


Obr. 3: Maturita z matematiky

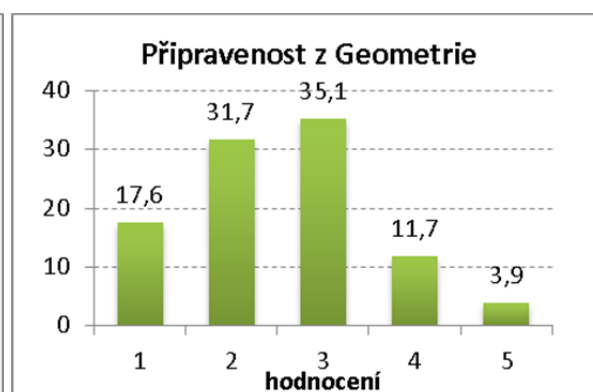


Obr. 4: Výsledky maturity z matematiky

Po absolvování prvního ročníku mohli studenti sami posoudit, jak je střední škola připravila na studium na vysoké škole, viz obr. 3 a 4.



Obr. 5: Přípravenost z matematiky ze SŠ

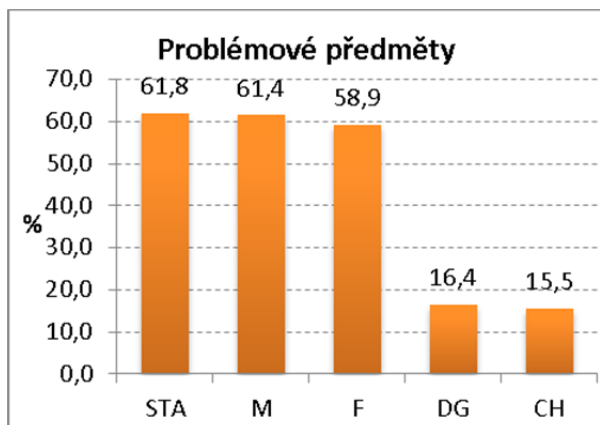


Obr. 6: Přípravenost z geometrie ze SŠ

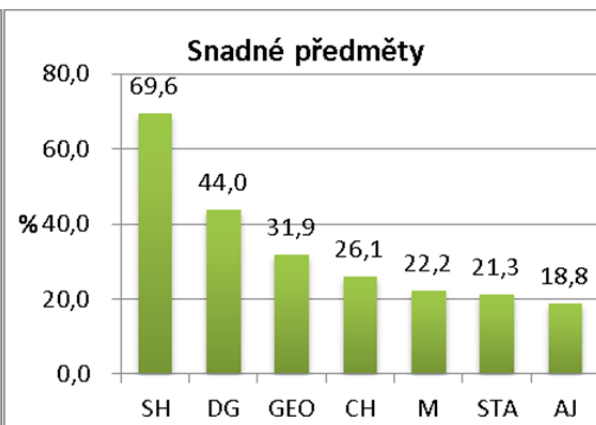
80% studenti hodnotí svoji připravenost z matematiky výborně až dobře a z geometrie je to dokonce přes 84%. Výsledky zkoušek tomu však příliš neodpovídají. Stala se chyba až na vysoké škole?

4 Příprava na výuku a výsledky v 1. ročníku

V dotazníku jsme studentům kladli otázku, které předměty jim v prvním ročníku dělaly největší potíže a které pro ně byly snadné, viz obr. 7 a 8. Oblíbenost či neoblíbenost předmětu zřejmě úzce souvisí s úspěšností u zkoušky v daném předmětu.



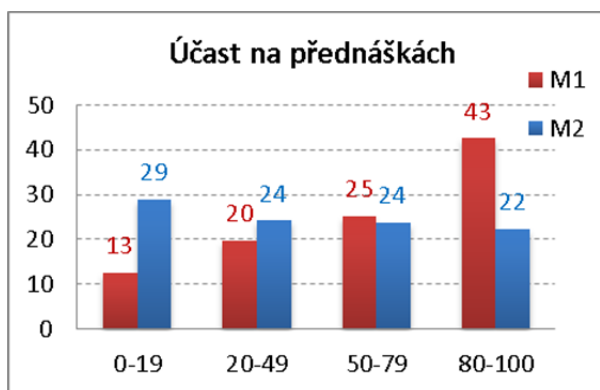
Obr. 7: Problémové předměty



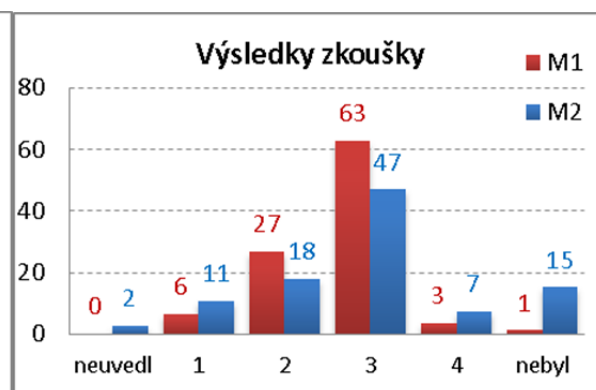
Obr. 8: Neproblémové předměty

Jako nejobtížnější studenti označili předmět stavební statika, následuje matematika a fyzika. Matematika však v sobě zahrnuje M1 a M2, tedy vlastně dva předměty, ostatní jsou jednosemestrální. Obecně jsou za obtížné považovány předměty, které vyžadují matematický aparát. Zajímavé je, že deskriptivní geometrii považuje za obtížnou jen 16% studentů, na rozdíl od FS (77%). Jako nejsnadnější je považován předmět stavební hmoty. Naopak přes 20% studentů považuje matematiku i stavební statiku za snadnou.

Účast na přednáškách je nepovinná a obr. 9 ukazuje, že studenti této skutečnosti využívají. Docházka se zásadně zhoršila zejména ve druhém semestru, kdy téměř 30% studentů přednášky prakticky nenavštěvovalo. Docházka na cvičení je naštěstí stále ještě povinná. Z dotazníku však vyplývá, že více než 50% studentů se na cvičení připravuje jen nepravidelně.



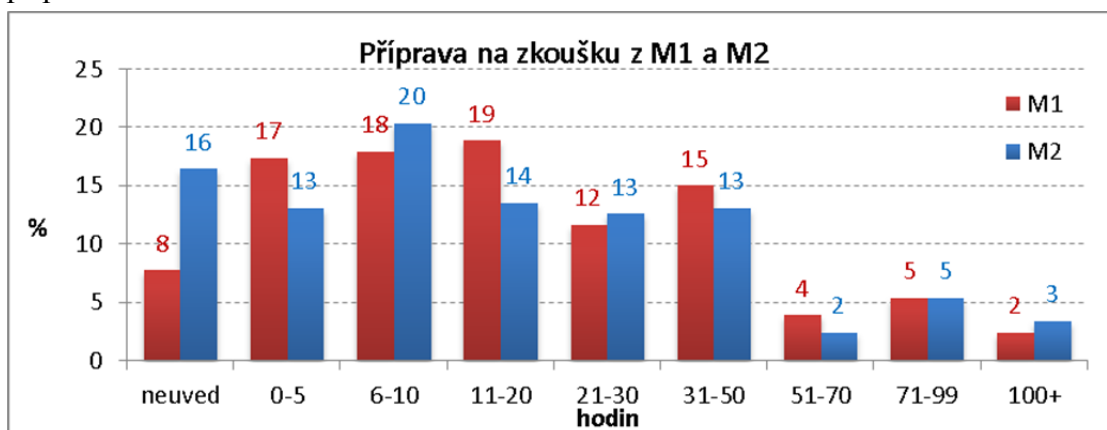
Obr. 9: Účast na přednáškách z MI a MII



Obr. 10: Úspěšnost u zkoušky z MI a MII

Jednou z příčin nízké účasti na přednáškách může být skutečnost, že studenti mají k dispozici elektronické texty, pracovní listy k přednáškám a cvičením. Studenti pak nabývají dojmu, že mají k dispozici veškeré materiály a v případě potřeby je mohou snadno nastudovat. Výsledky zkoušek z matematiky, viz obr. 10, ukazují, že se jim to příliš nedaří.

Značně rozsáhlé studijní materiály by vyžadovaly pravidelné studium a mnohem kvalitnější přípravu na zkoušku.



Obr. 11: Příprava na zkoušku z M1 a M2

Z obr. 11 je však zřejmé, že 35% studentů věnovalo přípravě na zkoušku z matematiky 1 méně než 10 hodin. V přípravě na zkoušku z matematiky 2 to bylo 33%. Pokud se pravidelně nepřipravovali, tak v rozsáhlých studijních materiálech patrně stačili přečíst pouze nadpisy.

5 Závěr

Více než 50% studentů hodnotí úroveň vedení cvičení a přednášek v matematice a deskriptivní geometrii známkou výborně nebo velmi dobře. Hodnocení samozřejmě závisí na přístupu pedagoga. Dosahované výsledky však nejsou dobré. Je to dáno špatnou připraveností ze střech školy a zejména nízkou motivací pro pravidelné studium v průběhu semestru i nedostatečnou přípravou na zkoušky.

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INEXACT SSNM FOR SOLVING FRICTIONAL CONTACT PROBLEMS

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Abstract: We use the semi-smooth Newton method to the solution of 2D contact problems with friction. The primal-dual algorithm for problems with the Tresca friction law is reformulated as the dual one. The conjugate gradient method is used for inexact solving of inner linear systems. Numerical experiments illustrate the performance of the inexact algorithm.

1 Introduction

We start with the algebraic counterpart of the elliptic PDEs describing the contact of two (or more) elastic bodies with Tresca friction. The problem arising from a finite element approximation reads as follows:

$$\text{Find } (u^*, \lambda_\nu^*, \lambda_\tau^*) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^m \text{ such that}$$

$$Ku + N^\top \lambda_\nu + T^\top \lambda_\tau - f = 0, \quad (1)$$

$$Nu - d \leq 0, \lambda_\nu \geq 0, \lambda_\nu^\top (Nu - d) = 0, \quad (2)$$

$$\left. \begin{array}{l} |\lambda_{\tau,i}| \leq g_i \\ |\lambda_{\tau,i}| < g_i \Rightarrow (Tu)_i = 0 \\ |\lambda_{\tau,i}| = g_i \Rightarrow \exists c_i \geq 0 : (Tu)_i = c_i \lambda_{\tau,i} \end{array} \right\} \quad i \in \mathcal{M}, \quad (3)$$

where $\mathcal{M} = \{1, \dots, m\}$ is the index set, $K \in \mathbb{R}^{n \times n}$ is symmetric and positive definite, $N, T \in \mathbb{R}^{m \times n}$ have full row-rank, $f \in \mathbb{R}^n$, $d \in \mathbb{R}_+^m$, and g_i are entries of $g \in \mathbb{R}_+^m$. The formulation (1)-(3) describes the algebraic *primal-dual* contact problem with Tresca friction. The primal unknown u^* approximates displacements, while the dual unknowns λ_ν^* , λ_τ^* approximate the (negative) normal, tangential contact stresses, respectively.

The semi-smooth Newton method (SSNM) uses the primal-dual formulation of contact problems reformulated by non-smooth functions as proposed already in [1]. Later on, it was recognized that the SSNM may be interpreted as a primal-dual active set method [4]. This approach is widely used for solving contact problems in two (2D) as well as three (3D) space dimensions with different friction laws; see e.g. [5]. The standard convergence analysis uses the slant differentiability concept [4] leading to the local superlinear convergence rate. This convergence result assumes exact solutions of inner linear systems that is, however, unrealistic for large-scale problems. The globally convergent variant of the method is analyzed in [3]. Here, we present the inexact implementation of the SSNM leading to the highest computational efficiency.

2 Dual variant of the SSNM

Let $P_{\Lambda_\nu} : \mathbb{R}^m \mapsto \Lambda_\nu$ and $P_{\Lambda_\tau} : \mathbb{R}^m \mapsto \Lambda_\tau$ be the projections onto $\Lambda_\nu = \mathbb{R}_+^m$ and $\Lambda_\tau = \{\lambda_\tau \in \mathbb{R}^m : |\lambda_{\tau,i}| \leq g_i, i \in \mathcal{M}\}$ defined by the max-function as follows:

$$P_{\Lambda_\nu,i}(\lambda_\nu) = \max\{0, \lambda_{\nu,i}\}, \quad (4)$$

$$P_{\Lambda_\tau,i}(\lambda_\tau) = \max\{0, \lambda_{\tau,i} + g_i\} - \max\{0, \lambda_{\tau,i} - g_i\} - g_i, \quad (5)$$

respectively. Let us introduce the function $G : \mathbb{R}^{n+2m} \mapsto \mathbb{R}^{n+2m}$ with $y = (u^\top, \lambda_\nu^\top, \lambda_\tau^\top)^\top$ given by

$$G(y) = \begin{pmatrix} Ku + N^\top \lambda_\nu + T^\top \lambda_\tau - f \\ \lambda_\nu - P_{\Lambda_\nu}(\lambda_\nu + \rho(Nu - d)) \\ \lambda_\tau - P_{\Lambda_\tau}(\lambda_\tau + \rho Tu) \end{pmatrix}, \quad (6)$$

where $\rho > 0$ is an arbitrary but fixed parameter. It is easy to verify that (1)-(3) and the equation

$$G(y) = 0, \quad (7)$$

have the same solution $y^* = (u^{*\top}, \lambda_\nu^{*\top}, \lambda_\tau^{*\top})^\top$. The function G is nonsmooth due to the presence of the max-function. Fortunately, it is semi-smooth in the sense of [?] so that the SSNM can be used. We will present the dual variant of the SSNM.

First of all, we introduce notation. Let $q : \mathbb{R}^{2m} \mapsto \mathbb{R}$ be the quadratic cost function defined by

$$q(\lambda) = \frac{1}{2} \lambda^\top A \lambda - \lambda^\top b, \quad (8)$$

where $\lambda = (\lambda_\nu^\top, \lambda_\tau^\top)^\top$, $A = BK^{-1}B^\top$ with $B = (N^\top, T^\top)^\top$ is symmetric and positive definite, $b = BK^{-1}f - c$, and $c = (d^\top, 0^\top)^\top$. The gradient $r : \mathbb{R}^{2m} \mapsto \mathbb{R}$ to q at $\lambda \in \mathbb{R}^{2m}$ is given by

$$r(\lambda) = A\lambda - b. \quad (9)$$

Denote $\lambda^* = (\lambda_\nu^{*\top}, \lambda_\tau^{*\top})^\top$, $\Lambda = \Lambda_\nu \times \Lambda_\tau$, and introduce the projection onto Λ by $P_\Lambda : \mathbb{R}^{2m} \mapsto \Lambda$ given by $P_\Lambda = (P_{\Lambda_\nu}^\top, P_{\Lambda_\tau}^\top)^\top$. The *reduced gradient* $\tilde{r}_\alpha : \Lambda \mapsto \mathbb{R}$ to q for $\alpha > 0$ is defined by:

$$\tilde{r}_\alpha(\lambda) = \frac{1}{\alpha}(\lambda - P_\Lambda(\lambda - \alpha r(\lambda))). \quad (10)$$

It is well-known [2] that $\tilde{r}_\alpha$ is the optimality criterion to the problem $\min_{\lambda \in \Lambda} q(\lambda)$ in the sense that $\lambda^* \in \Lambda$ solves this problem iff $\tilde{r}_\alpha(\lambda^*) = 0$. Therefore, the reduced

gradient will be used as the stopping criterion.

ALGORITHM SSNM Given $\lambda^0 \in \mathbb{R}^{2m}$, $\varepsilon \geq 0$, and $\rho > 0$. For $k \geq 0$, compute:

(Step 1) If $\|\tilde{r}_\rho(P_\Lambda(\lambda^k))\| \leq \varepsilon$, return $\lambda = P_\Lambda(\lambda^k)$, else go to step Step 2.

(Step 2) Assembly the active/inactive sets at λ^k :

$$\begin{aligned}\mathcal{A}_\nu &= \{i \in \mathcal{M} : \lambda_i^k - \rho r_i^k \geq 0\}, \\ \mathcal{I}_\nu &= \mathcal{M} \setminus \mathcal{A}_\nu, \\ \mathcal{I}_\tau^+ &= \{i \in \mathcal{M} : g_i < \lambda_{i+m}^k - \rho r_{i+m}^k\}, \\ \mathcal{I}_\tau^- &= \{i \in \mathcal{M} : \lambda_{i+m}^k - \rho r_{i+m}^k < -g_i\}, \\ \mathcal{A}_\tau &= \mathcal{M} \setminus (\mathcal{I}_\tau^+ \cup \mathcal{I}_\tau^-).\end{aligned}$$

(Step 3) Find λ^{k+1} so that

$$\lambda^{k+1} = \arg \min q(\lambda) \quad \text{s.t.} \quad \lambda_{\nu, \mathcal{I}_\nu} = 0, \quad \lambda_{\tau, \mathcal{I}_\tau^+} = g_{\mathcal{I}_\tau^+}, \quad \lambda_{\tau, \mathcal{I}_\tau^-} = -g_{\mathcal{I}_\tau^-}. \quad (11)$$

Note that ρ can be discarded from $\mathcal{A}_\nu$ and $\mathcal{I}_\nu$, when the inner subproblems in Step 3 are solved exactly (and $\lambda^0 = 0$, e.g.), since either $\lambda_i^k = 0$ or $r_i^k = 0$. A similar observation is valid also for $\mathcal{A}_\tau$, $\mathcal{I}_\tau^+$, and $\mathcal{I}_\tau^-$ provided that λ^k is sufficiently close to λ^* and g is sufficiently large.

3 Inexact implementation

The computational efficiency of the SSNM depends on a way how the inner subproblems are implemented. We propose to accept inexact solutions to (11), denoted again by λ^{k+1} , that are computed by few CGM iterations. It is referred by

$$\lambda^{k+1} = \text{CGM}(A, b, \mathcal{A}, \lambda^{k+1,0}, \text{tol}^{k+1}),$$

where $\mathcal{A} = \mathcal{A}_\nu \cup \{i + m \mid i \in \mathcal{A}_\tau\}$, $\lambda^{k+1,0}$ is the initial CGM iteration, and tol^{k+1} denotes the stopping tolerance. The implementation ideas are summarized by ALGORITHM ISSNM, where $\text{err}^k = \|\tilde{r}_\rho(P_\Lambda(\lambda^k))\|$ stands for the precision achieved on the outer level. The value tol^{k+1} in Step 3.1 respects err^k but, when the progress is not sufficient, it improves the previous tolerance tol^k . The inner initialization $\lambda^{k+1,0}$ in Step 3.2 is chosen by the previous iteration λ^k and by the constraints in (11).

ALGORITHM ISSNM Given $\lambda^0 \in \mathbb{R}^{2m}$, $\varepsilon \geq 0$, $\rho > 0$, and $r_{\text{tol}}, c_{\text{fact}} \in (0, 1)$.

Set $\text{err}^0 = \|\tilde{r}_\rho(P_\Lambda(\lambda^0))\|$, $\text{tol}^0 = r_{\text{tol}}/c_{\text{fact}}$, and $k = 0$.

(Step 1) If $\text{err}^k \leq \varepsilon$, return $\lambda = P_\Lambda(\lambda^k)$, else go to step Step 2.

(Step 2) Assembly the active/inactive sets at λ^k .

(Step 3.1) $\text{tol}^{k+1} = \min\{r_{\text{tol}} \times \text{err}^k / \text{err}^0, c_{\text{fact}} \times \text{tol}^k\}$

(Step 3.2) $\lambda_{\mathcal{A}}^{k+1,0} = \lambda_{\mathcal{A}}^k$, $\lambda_{\nu, \mathcal{I}_\nu}^{k+1,0} = 0$, $\lambda_{\tau, \mathcal{I}_\tau^+}^{k+1,0} = g_{\mathcal{I}_\tau^+}$, $\lambda_{\tau, \mathcal{I}_\tau^-}^{k+1,0} = -g_{\mathcal{I}_\tau^-}$

(Step 3.3) $\lambda^{k+1} = \text{CGM}(A, b, \mathcal{A}, \lambda^{k+1,0}, \text{tol}^{k+1})$

(Step 3.4) $\text{err}^{k+1} = \|\tilde{r}_\rho(P_\Lambda(\lambda^{k+1}))\|$, $k = k + 1$, and go to Step 1.

4 Numerical experiments

Table 1 shows how ALGORITHM ISSNM behaves with respect to the value $\rho = \beta \times \sigma_{\max}^{-1}$, where $\sigma_{\max}$ denotes the largest eigenvalue of A . We observe that the dependence on ρ is weak.

Table 1: ISSNM with $\rho = \beta \times \sigma_{\max}^{-1}$

β	0.05	1	1.9	20	100
n/m	$iter/n_A$	$iter/n_A$	$iter/n_A$	$iter/n_A$	$iter/n_A$
1320/60	07/35	07/35	07/35	07/36	10/48
11160/180	09/49	09/49	09/49	08/41	10/52
30600/300	09/48	09/48	09/48	08/43	10/55
59640/420	09/49	09/49	09/49	10/59	11/62
98280/540	09/51	09/51	09/51	09/45	12/72
146520/660	09/53	10/57	10/57	10/59	10/54
204360/780	10/59	10/59	10/59	10/61	10/56

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APPLICATION OF STATISTICAL METHODS IN INTERPRETATION OF THE RESULTS OF EMPIRICAL RESEARCH CONCERNING THE QUALITY MANAGEMENT SYSTEM

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Abstract: In the work there are application possibilities of statistical methods presented in interpretation of the results of empirical research. An example of research conducted on the quality management system was used. The attention is paid to the possibility of using the particular statistical tests to interpret the research results obtained. There are the examples of application of statistical methods provided in two groups of research. The examples of application of the selected statistical tests are presented. There is a way of their use emphasized in interpretation of the results obtained.

Introduction

Scientists in their research are seeking innovative methods and tools that enable a better understanding of the analyzed phenomena [1-7]. Empirical research on quality management systems provide a large number of results. In order to interpret the results obtained it is necessary to apply statistical methods. The author of the work conducted a lot of research on quality management systems [8-17]. Most of research was carried out on a large number of research samples. To conduct a proper interpretation of results, verify the research hypotheses stated, it was necessary to use statistical methods. The purpose of the work is to present the examples of statistical methods application in two groups of research. The details concerning research are presented in [12,16]. The first group of research concerned examining the problems in a certified quality management system as well as relating them to organization size and profile. In general, research included 1162 organizations that have been certified by two large certifying bodies. The size of analyzed organizations was determined on the basis of the number of employees and four groups were created: small, medium-sized, large and very large. Organizations were also divided according to the operational profile into two groups: manufacturing and services. Examination of the problems was conducted on the basis of own research method. The reports of third party were examined in which nonconformity and weaknesses were ascribed to the particular sections of the ISO 9001 standard. The research hypotheses were formed for the purpose of research.

The second research group concerned diagnosis of the quality management system based on questionnaire research on quality representatives. Own research tool was elaborated. The questionnaire research was conducted on a total sample of 2500 organizations certified by three certifying bodies. The examined organizations were divided according to size, activity profile and time of system possession. The research hypotheses were then formulated.

The examples of statistical methods application in two groups of research

In the first group of research the author was concentrated on problems determination in the quality management system and their connection with organization size and profile. There were the following types of listings elaborated: identified nonconformities and weaknesses in relation with the particular sections of the ISO 9001 standard, identified nonconformities and weaknesses in accordance with the organization's size, average number of nonconformities and weaknesses per one organization in connection with its size, identified nonconformities and weaknesses related to organization's profile, average number of nonconformities and weaknesses per one organization in connection with organization's profile. There were the appropriate statistical tests conducted in order to check whether there is a relation between the organization's size and types of nonconformities and weaknesses identified as well as between the organization's profile and types of nonconformities and weaknesses identified. The χ^2 independence test was used to determine the relation between the organization's size and types of identified nonconformities or weaknesses in the particular sections of ISO standard and to determine the relations between the organization's profile and types of identified nonconformities or weaknesses in the particular sections of ISO standard. Equality test of multiple means was used for examining the average number of nonconformities or weaknesses related to one organization. In order to conduct such tests it was confirmed if the assumptions concerning its applicability are met: whether the values of observation come from the population of normal distribution or similar to normal one – Hellwig's compatibility test, whether the variations for separate populations are equal - Hartley's test concerning the variations equality in many populations. There was also the equality test of two means for unknown equal standard deviations, equality test of two means for unknown different standard deviations – Welch-Aspin's test as well as two fractions equality test adopted.

In the second group of research the author focused on conducting diagnosis of the quality management system based on the results of questionnaire research. The following types of compilations were made: question number, question contents, possible answers, result obtained for three groups of the examined organizations and total result; answers to the question depending on organization size; answers to the question depending on organization profile; answers to the question depending on the time of system functioning. At the beginning it was verified using Kruskal-Wallis test whether each answer to the question for three groups of organizations belongs to the same general population. Positive verification of such hypothesis enables analysis of answers to the particular questions for all the results, without distinguishing the group of organization. For determination of differences between: organization size and type of answer to the question, organization profile and type of answer to the question, time of system possession and type of answer to the question, χ^2 independence test was used. For examination of average number of problems per one organization equality test of multiple means was used. In order to conduct such tests it was confirmed if the assumptions concerning its applicability are met: whether the values of observation come from the population of normal distribution or similar to normal one – Hellwig's compatibility test, whether the variations for separate populations are equal - Hartley's test concerning the variations equality in many populations.

The detailed information concerning the statistical tests used and the results of the analysis for two groups of research are presented in [12].

Example of application of χ^2 independence test

Let there be two variables given X and Y which may have any form (qualitative, quantitative). Null hypothesis and alternate hypothesis of the test have the following form:

H_0 : Variables X and Y are independent

H_1 : Variables X and Y are dependent

Let there be the empirical sizes of the variables given, which are written in the contingency table below.

Variable X	Variable Y				$n_{i.}$
	y_1	y_2	...	y_k	
x_1	n_{11}	n_{12}	...	n_{1k}	$n_{1.}$
x_2	n_{21}	n_{22}	...	n_{2k}	$n_{2.}$
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$
x_w	n_{w1}	n_{w2}	...	n_{wk}	$n_{w.}$
$n_{.j}$	$n_{.1}$	$n_{.2}$	...	$n_{.k}$	n

where: w – number of rows,

k – number of columns,

x_i – value of the variable X, ($i = 1, 2, \dots, w$),

y_j – value of the variable Y ($j = 1, 2, \dots, k$),

n_{ij} – the size of the empirical group for which the variable X has the value of x_i , and the variable Y the value of y_j ,

n – number of observation.

The following equalities are true:

$$n_{i.} = \sum_{j=1}^k n_{ij}$$

$$n_{.j} = \sum_{i=1}^w n_{ij}$$

$$n = \sum_{i=1}^w n_{i.} = \sum_{j=1}^k n_{.j} = \sum_{i=1}^w \sum_{j=1}^k n_{ij}$$

Hypotheses H_0 and H_1 may be written in the following form:

$$H_0: p_{ij} = p_{i.} \cdot p_{.j}$$

$$H_1: p_{ij} \neq p_{i.} \cdot p_{.j}$$

Where the following terms are the theoretical probability estimators:

$$\hat{p}_{ij} = \frac{n_{ij}}{n}; \quad \hat{p}_{i.} = \frac{n_{i.}}{n}; \quad \hat{p}_{.j} = \frac{n_{.j}}{n}$$

Based on the probability and if the hypothesis H_0 is true, one can calculate the theoretical sizes:

$$\hat{n}_{ij} = n \cdot \hat{p}_{i.} \cdot \hat{p}_{.j} = \frac{n_{i.} \cdot n_{.j}}{n}$$

In order to verify the hypothesis H_0 the χ^2 statistics may be used in the form of:

$$\chi^2 = \sum_{i=1}^w \sum_{j=1}^k \frac{(n_{ij} - \hat{n}_{ij})^2}{\hat{n}_{ij}}$$

The χ^2 statistics have $(w-1) \cdot (k-1)$ degrees of freedom. If the empirical value of χ^2 is greater than the critical value, read from the tables of the critical values for the χ^2 on the significance level α with $(w-1) \cdot (k-1)$ degrees of freedom, then the H_0 should be rejected in favor of the alternate hypothesis H_1 . Otherwise, there is no basis to reject H_0 .

Confirming the dependence using the χ^2 test, does not allow to determine the strength of the relation. In order to do this, the statistical measure of the V – Cramer coefficient was suggested.

The V – Cramer coefficient is calculated using the following formula:

$$V = \sqrt{\frac{\chi^2}{n \cdot (\min(k; w) - 1)}}$$

where $\min(k; w)$ is the lowest of the numbers of rows or columns.

The V - Cramer coefficient is between $<0;1>$. $V = 0$, when the variables are stochastically independent and $V = 1$, if there is a functional relation between them.

Conclusion

The examples presented in the work show a wide range of application possibilities of statistical methods for interpretation of the results of empirical research. There are the examples of two groups of research presented concerning the quality management system. The first group refers to examining problems in the quality management system implemented, basing on audits' results of the third party. The second group concerns a diagnosis of the quality management system grounding on the survey results. The following statistical methods were used for the analysis of the results obtained in two groups of research:

- χ^2 independence test
- equality test of multiple means (test of variance analysis)
- Hartley's test of equality of multiple variances
- Hellwig compatibility test
- the equality test of two means for unknown equal standard deviations
- equality test of two variances
- equality test of two fractions
- Kruskal – Wallis test

- test of equality of two means for unknown unequal standard deviations (Welch – Aspin test).

The results obtained confirm usefulness of the methods adopted. The statistical tests served for testing the research hypotheses, enabled proper interpretation of the results achieved. The presented examples indicate great possibilities for using statistical methods in analysis of research results on quality management. The code of conduct elaborated shows possibilities to use the selected statistical tests a multiple number of times in interpretation of various research results.

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Elementary Proofs of Some Non-Trivial Theorems

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Abstract: Our contribution deals with an intricate problem of teaching mathematical courses at technical universities. Mainly, we expose an issue of a low motivation of students to learn mathematics. We suggest several ways how to encourage students to find the subject exciting and important. Our main interest is to popularize a pure mathematics among students by giving lectures about famous mathematical discoveries enriched by exposing the historical and mathematical background. We propose instructions how to choose an appropriate topic and how to develop it. In the end of the article we provide outlines of two lectures produced by the introduced scheme.

1 Introduction

The mathematical courses at technical universities put accent on the computational techniques at the expense of deeper understanding of the problems. In the environment where results seem to be much more important than reasons that justify them students can very easily lose an ability to differentiate between deep and simple results. Both unexpected and trivial mathematical statements are assigned the same quality by them.

With a given amount of time, the technical approach to the teaching of mathematics is probably unnecessary. The bright side of this approach is that learning (and teaching) of only the mathematical "recipes" is far less difficult than introducing the whole picture. On the other hand, there exists a danger that students will perceive mathematics as something external, distant and meaningless.

Delivering mathematics to the students of technical universities in a way which would decently uncover the framework of the mathematics is certainly a difficult

problem and we do not ambition to solve it. Nevertheless, we can see several possible directions that are worth trying:

1. Expose the unquestionable inner beauty of the mathematics to the students by standard lectures. – *That seems to be idealistic or even impossible.*
2. Introduce mathematics as a tool for applications. "Externalize" mathematics in the real-life examples. – *Probably the most effective but in our opinion, a little non-mathematical solution.*

The main disadvantage of this approach to calculus: Elementary calculus originates in 17. century and its applications are not very spectacular and students usually do not take them seriously. In the best-case scenario, they consider applications of calculus as cute rarities.

3. Pick a relatively recent real and preferably famous mathematical problem to which an elementary proof is known. Such problems are usually accompanied by attractive stories. An extraordinary story overcrowded with acts of mathematical heroism is easily understandable for everyone and it can encourage an interest in mathematics itself and support a healthy respect to the subject.

We devote this article to the development of the last option.

2 Several Possible Sources of Such Problems

The famous Hungarian mathematician Paul Erdős (1913 – 1966) used to believe that there exists The Book containing the perfect proof of every mathematical statement. A lovely attempt to provide such a book is:

- Aigner, M.; Ziegler, G. M. *Proofs from The Book. Including illustrations by Karl H. Hofmann.* Springer, 2001.

Interesting exercises from the examinations for the first-year students of Ph.D. degree in Mathematics on University in Berkeley:

- De Souza, P. N.; Silva, J.-N. *Berkeley problems in mathematics.* Springer, 2004.

As a source of inspiring mathematical problems one can also use a problem book devoted to some reasonable mathematical competition:

High-School Level

- Andreescu, T.; Enescu, B. *Mathematical olympiad treasures.* Springer, 2011.
- Andreescu, T.; Gelca, R. *Mathematical olympiad challenges.* Birkhäuser, 2008.
- Djukic, D., et al. *The IMO compendium.* Springer, 2006.
- Engel, A. *Problem-solving strategies.* Springer, 1998.

University Level

- Alexanderson, G. L., et al. *The William Lowell Putnam mathematical competition: Problems and solutions 1965-1984.* MAA, 1985.
- Kedlaya, K. S. *The William Lowell Putnam mathematical competition 1985-2000: Problems, solutions, and commentary.* MAA, 2002.

Finally, there are many challenging problem books which can serve well for our purposes too.

- Aliprantis, Ch. D.; Burkinshaw, O. *Problems in real analysis: a workbook with solutions*. Academic press, 1999.
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3 Several Examples and How to Approach Them

Every mathematical problem requires a different approach but we find the following general scheme reasonable.

1. Pick a problem or theorem that you find important and interesting from the mathematical point of view.
2. Try to find an elegant proof which is as elementary as possible.
3. Search the mathematical surroundings of your problem – prerequisites, corollaries, applications, etc.
4. Search the historical background of your problem – find information about the author (if he is known), sketch the history of the problem itself (if you chose a famous long-open problem) or at least outline the importance of the field the problem belongs to.
5. If there is an anecdote connected to your problem, do not be afraid to use it!

In the remaining part of our article we introduce two outlines of lectures where we successfully used the described method.

3.1 Example 1 – Functional Equations and Hilbert’s 3rd Problem

Lecture slides of the lecture can be found at [2]. We start with a well known notion of polyhedron. From the vast number of possible definitions we choose the following.

Definition 3.1 (Polyhedron). A convex subset in $\mathbb{R}^3$ is called a *polyhedron* if it can be expressed as a union of finitely many closed tetrahedra.

Problem 3.2. *Hilbert’s 3rd Problem Given any two polyhedra of equal volume, is it always possible to cut the first into finitely many polyhedral pieces which can be reassembled to yield the second?*

Surprisingly, the answer is negative. Students at technical universities very rarely happen to meet negative assertions of this kind. It is a shame since it were negative results that worked as breaking points in the history of mathematics.

1) Initial Observations. The conjecture is easily understandable. An analogous question can be positively answered in dimension 2 (Wallace-BolyaiGerwien

theorem). Although its practical consequences are questionable it is a real mathematical problem has occupied minds of great mathematicians for a long time. It is surprising that there exists a proof again that is more-or-less elementary and its principal ideas can be comprehended even by high-school students.

2) Mathematical Surroundings. The proof requires "tricks" from several significant parts of mathematics: Functional equations (Cauchy functional equation), Axiom of choice and Hamel basis (fundamental elements of set theory, yet very exciting and relatively accessible) or the theory of invariants (so called Dehn's invariants are constructed in the proof).

3) Historical Background. Hilbert's problems form a list of twenty-three problems in mathematics published by German mathematician David Hilbert in 1900. The problems were all unsolved at the time, and several of them were very influential for 20th century mathematics. Hilbert's 3rd problem was solved 1900 by his student Max Dehn. Axiom of choice was a bone of contention in the mathematics of early 20th century and was formalized by Ernst Zermelo. Both M. Dehn and E. Zermelo were German mathematicians who suffered hard times during the Second World War and their life stories posses Hollywood-like qualities.

4) Anecdotes/Quotations.

1. One of Hilbert's students stopped showing up to classes. On inquiring the reason, Hilbert was told that the student had left the university to become a poet. Hilbert: "I can't say I'm surprised. I never thought he had enough imagination to be a mathematician."
2. Hilbert's tomb: "Wir müssen wissen. Wir werden wissen."
3. In the world without Axiom of choice the set of real numbers $\mathbb{R}$ is a countable union of countable sets!

3.2 Example 2 – Fix-Point Theory and Borsuk-Ulam Theorem

Lecture slides of the lecture can be found at [3]. Fix-point theorems play a large role both in applied and pure mathematics. We begin with a definition of a unit sphere and unit ball.

Definition 3.3 (of S^{n-1} , B^n). In $\mathbb{R}^n$, $n > 0$, we denote a unit sphere by $S^{n-1} := \{\mathbf{x} \in \mathbb{R}^n : \|\mathbf{x}\| = 1\}$ and a unit ball by $B^n := \{\mathbf{x} \in \mathbb{R}^n : \|\mathbf{x}\| \leq 1\}$.

Now, we are able to introduce the main topic of the third sample lecture:

Theorem 3.4. *Borsuk-Ulam Theorem* Given $n > 0$ and a continuous mapping $f : S^n \rightarrow \mathbb{R}^n$ there exists a point $x \in S^n$ such that $f(\mathbf{x}) = f(-\mathbf{x})$.

1) Initial Observations. The statement is easily understandable and one can imagine what it says. The version for $n = 1$ is easy to proof. It has many attractive consequences (Brouwer's fix point theorem, Cake-cutting theorem, Sandwich theorem, Necklace theorem, etc.). There is no completely elementary proof known, yet there exists a proof with transparent structure and only a few "black boxes" (and hence accessible for freshmen at technical universities).

2) Mathematical Surroundings. It is strongly connected to the fix points theory (the Banach theorem, etc.). Some basic notions of topology are also necessary. Several concepts of discrete mathematics can be employed. The theorem is connected with the Borsuk's conjecture which is also interesting and very real mathematical problem.

3) Historical Background. The story of Borsuk-Ulam theorem belongs to the golden age of Polish mathematics – lifetime of Steinhaus, Banach, Mazur, Ulam, Schauder, Sierpiński, Kuratowski, Borsuk, Tarski, etc. Lives of all these world-famous mathematicians were violently interrupted by the Second World War and many of them were fascinating personalities.

4) Anecdotes/Quotations.

1. History of the "Scotish café" and Polish mathematics in general is very inspiring.
2. Ulam: "Whatever is worth saying, can be stated in fifty words or less." (see [5]), "It is most important in creative science not to give up. If you are an optimist you will be willing to 'try' more than if you are a pessimist." (See [4, p. 55].)
3. Mazur's problem (prize was a life goose!) can be an inspiration for all young mathematicians. It is also an opportunity to introduce famous mathematician Per Enflo, a solver of the problem and a gifted pianist.
4. "As a mathematician, Banach was self-taught. He did not study mathematics. In 1916, a very important event took place. Hugo Steinhaus, an outstanding mathematician, then already well known, spent some time in Kraków. Once, during his evening walk at the Planty Park in the centre of Kraków, Steinhaus heard the words 'Lebesgue integral'. In those times it was a very modern mathematical term, so Steinhaus, a little surprised, started to talk with two young men who were speaking about the Lebesgue measure. These two men were Banach and Otto Nikodym. Steinhaus told them about a problem he was currently working on, and a few days later Banach visited Steinhaus and presented him a correct solution." (See [1, p. 2].)

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MODELLING OF LIQUIDUS AND SOLIDUS SURFACES IN TERNARY ALLOY SYSTEMS USING POLYTHERMAL SECTIONS

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Abstrakt: Databáze fázových diagramů poskytují nespočet údajů o ternárních systémech v podobě izotermických a polytermických řezů. Jen málokteré však poskytují kompletní izotermickou projekci ploch likvidu a solidu, které umožní výpočet rovnovážných rozdělovacích koeficientů v procesu tuhnutí slitin. Software pro modelování ternárních systémů, umožňuje pomocí polytermických řezů vymodelovat celkovou projekci ploch likvidu i solidu.

Abstract: The alloy phase diagram databases provide lots of data of ternary alloy phase systems in form of isothermal and polythermal sections. Only few of them give complete isothermal and polythermal surface projections, which make it possible to calculate the equilibrium distribution coefficients in the solidification process. Software used for modelling of ternary alloy systems allows using polythermal sections for modelling of complete isothermal projections of liquidus and solidus surfaces.

1 INTRODUCTION

At crystallization of binary alloys a transition from the liquid state to the solid state takes place, it means, e.g. that the melt starts to form crystals from the liquid phase until the entire melt solidifies. In this temperature range of crystallization a dependence of the equilibrium exists between the liquid and solid phases which are represented by tie-lines. The tie-line is a horizontal join of the beginning of crystallization (liquid contains there still 100% melt) and the end of melt crystallization (solid contains there already 100% of solid phase). With existence of the crystallization interval the surfaces of liquid and solid phases connected by tie-lines are in equilibrium, but each of them has different chemical composition. With changing of the liquid and solid phase composition and decreasing of temperature at the interface crystal - melt redistribution of impurities occurs, and it is a measure of the equilibrium distribution coefficient k_{oA}^B impurity of B component in the base matrix of A component (1), where X_{SB} is concentration of the impurity element in the solid state and X_{LB} is concentration of the impurity element in the liquid state.

Therefore, the equilibrium distribution coefficient (1) is used at preparation of single crystals, refining of substances and at micro-alloying [1].

$$k_{oA}^B = \frac{X_{SB}}{X_{LB}} \quad (1)$$

In contrast to binary alloy systems there is an equilibrium relationship between the liquid and solid phases which is described by curves of liquidus and solidus, in ternary alloy systems the equilibrium is describe using the whole surfaces of the liquidus and solidus. The ASM International, MSI and SGTE databases of the phase diagrams contain thousands of records about ternary alloy systems in form of isothermal and polythermal sections, but whole isothermal projections of liquidus and solidus surfaces are known only in several cases. For this reason a special software has been developed and tested, which allows modelling of the complete ternary isothermal projection of ternary alloy systems using only polythermal sections.

Polythermal sections represent vertical sections through a three-dimensional ternary alloy system, where the amount of one of component can be constant and the amount of the other two components may be different, or it may change in a certain ratio (Fig. 1).

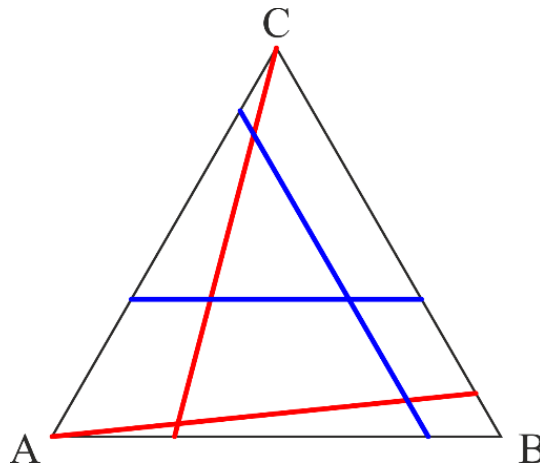


Fig. 1 Schematic representation of polythermal sections in the ternary system *A-B-C*

2 SOFTWARE FOR MODELLING TERNARY ALLOY SYSTEMS

Software for modelling of ternary systems use Matlab interface, which allows numerical calculations, modelling, plotting and simulations [2]. Graphical user interface (GUI) of the software is divided into two windows (Fig. 2). The Input form is used for loading of the processed data, modelling of ternary alloy systems, modelling of 2D and 3D projection of isothermal and polythermal sections or modelling of tie-lines and calculation of the equilibrium distribution coefficients. Data form is used to create data files or to edit the input data.

The calculation of liquidus and solidus surfaces in the software Matlab was performed using the method B-spline surfaces that are formed on the based of B-spline curves, the

direction of which is modified with use of controlling points. The control points are used like anchor points but they do not touch the actual curve [3].

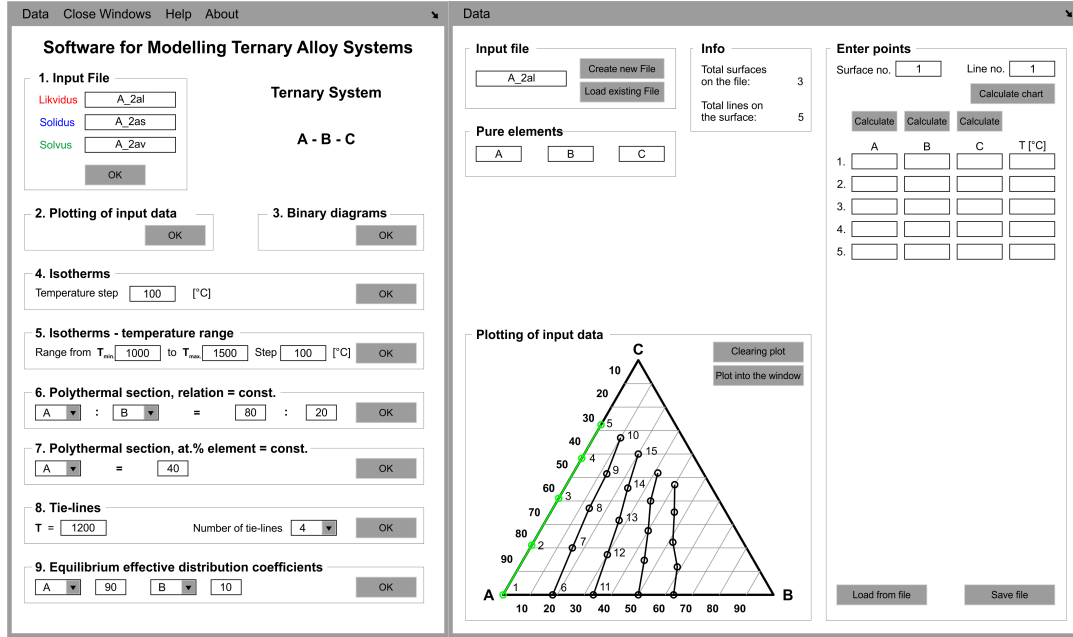


Fig. 2 GUI software; “Input form“ on the left, “Data form“ on the right

Mathematically can be B-spline curve expressed by the equation (2):

$$S(u) = \sum_{i=0}^n N_{i,p}(u) \mathbf{P}_i \Leftrightarrow \begin{cases} x = \sum_{i=0}^n N_{i,p}(u) x_i \\ y = \sum_{i=0}^n N_{i,p}(u) y_i \end{cases} \quad (2)$$

where $\mathbf{P}_i = (x_i, y_i) \in \mathbb{R}^2$, $i = 0, \dots, n$ which are control points of the polygon (control points) and $\{N_{i,p}(u)\}_{i=0}^n$ are p th-degree of B-spline basis functions define on the knot vector (3) and bases for $p = 0$ defined by equation (4) [4].

$$U = (\underbrace{0, \dots, 0}_{p+1}, u_{p+1}, \dots, u_n, \underbrace{1, \dots, 1}_{p+1}) \quad (3)$$

$$N_{i,0}(u) = \begin{cases} 1, & u \in \langle u_i, u_{i+1} \rangle \\ 0, & u \notin \langle u_i, u_{i+1} \rangle \end{cases} \quad (4)$$

Bases for $k = 1, \dots, p$ are defined by quotation (5) [4].

$$N_{i,k}(u) = \frac{u - u_i}{u_{i+k} - u_i} N_{i,k-1}(u) + \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i-1}} N_{i+1,k-1}(u) \quad (5)$$

B-spline surface then formed network of control points of B-spline curves, where $\mathbf{P}_{ij} = (x_{ij}, y_{ij}, z_{ij}) \in \mathbb{R}^3$, $i = 0, \dots, n$, $j = 0, \dots, m$, two knot vectors

$U = (0, \dots, 0, u_{p+1}, \dots, u_n, 1, \dots, 1)$ and $V = (0, \dots, 0, u_{p+1}, \dots, u_m, 1, \dots, 1)$, which form the univariate B-spline functions $\{N_{i,p}(u)\}_{i=0}^n$ and $\{N_{j,p}(v)\}_{j=0}^m$. B-spline surface is possible expressed as equation (6) [4].

$$S(u, v) = \sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u) N_{j,p}(v) \mathbf{P}_{ij} \Leftrightarrow \begin{cases} x = \sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u) N_{j,p}(v) x_{ij} \\ y = \sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u) N_{j,p}(v) y_{ij} \\ z = \sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u) N_{j,p}(v) z_{ij} \end{cases} \quad (6)$$

3 EXPERIMENTATION AND RESULTS

Our method was tested experimentally for the modelling of isothermal projections of liquidus and solidus surfaces using polythermal sections in the ternary alloy systems Ag-Au-Pd and Al-Sn-Zn.

The ternary system Au-Ag-Pd forms the continuous series of solid and liquid solutions and it is composed of three binary systems with quasi-ideal solubility of the components in both liquidus and solidus phase. From the database of the ASM International were selected three polythermal sections lying on the medians of three sides of the system. Using the web application WebPlotDigitizer were selected data on the liquidus and solidus curves in the form of the concentration at the temperature steps of 50 °C. It was done in binary systems and in polythermal sections [5]. In the “Data form” were processed the input data and then were modelled the isothermal sections of liquidus (red isotherms) and solidus surfaces (blue isotherms) for $T = 970 \div 1470$ °C at the temperature isotherm step of 100 °C (Fig. 3).

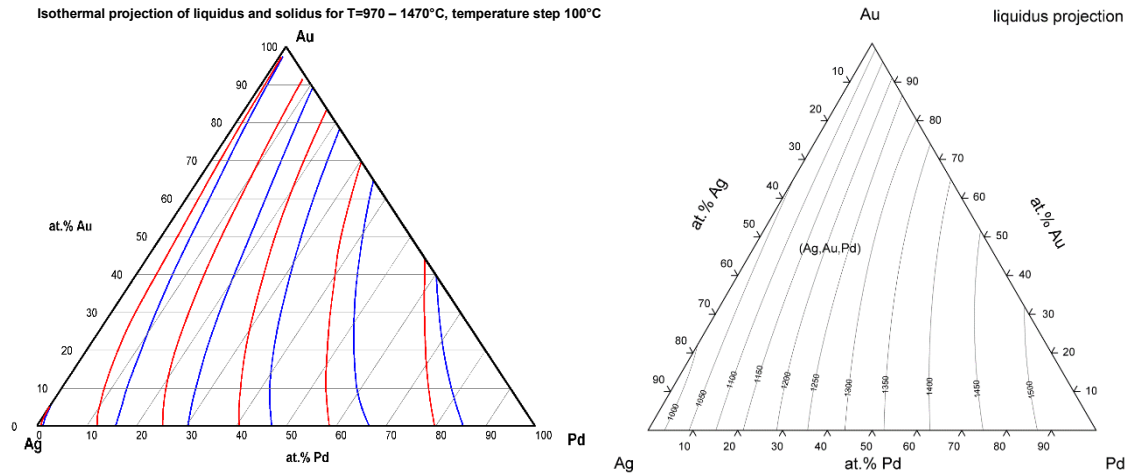


Fig. 3 Isothermal projection of the Ag-Au-Pd ternary system (on the left) and of original system from the ASM International database (on the right) [6]

At the second ternary system we chose the Al-Sn-Zn diagram with the eutectic reaction on all binary sides and with ternary eutectic reaction within the ternary system (Fig. 4). The ASM International database contains five polythermal sections published by the

authors of Sebaoun A., Prowans S. a Jares V. [7]. Polythermal sections are distributed equally over the entire surface in the ternary system.

The data from the binary phase diagrams and polythermal sections were taken using the same method as in the previous case. After modelling of the isothermal liquidus section, it was found that the temperatures of polythermal section published by Sebaoun A. did not match the temperatures of polythermal sections at the places, where they crossed each other. Wrong polythermal section was not applied and new isothermal projections were modelled for $T = 200 \div 650$ °C with the isothermal temperature step of 50 °C, and only from four polythermal sections (Fig. 4).

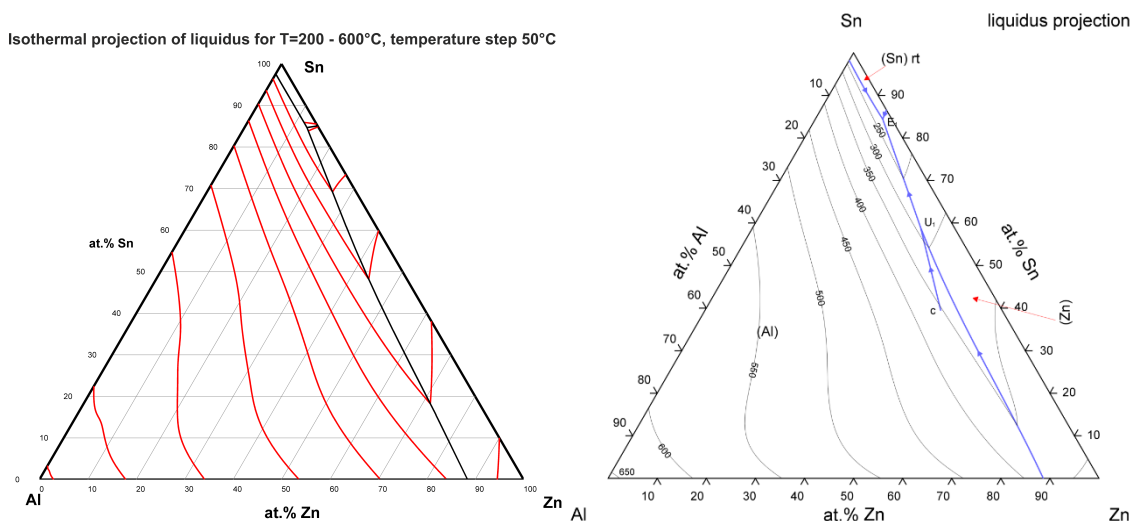


Fig. 4 Isothermal projection of Al-Sn-Zn (on the left) and original system from the ASM International database (on the right) [8]

4 CONCLUSIONS

The software for modelling of the ternary alloy systems using B-spline surfaces method appears to be a powerful tool for modelling a complex isothermal solidus and liquidus projection, from which it is possible to calculate the equilibrium distribution coefficient of individual elements during the equilibrium crystallization process. The basic prerequisite is the availability of the required quantity of polythermal sections and homogeneity of data extracted from them.

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THE USE OF TARGET COSTING FOR QUALITY AND CLIENT-ORIENTED PRODUCTION ENGINEERING

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Abstract: Measurement of product quality perceived by clients is an important factor in maintaining a firm's competitive edge. Managers at all levels are faced with questions regarding bringing quality and user-oriented information into quantitative problem solving and decision-making process. In this paper we (i) describe the methodology of target costing used in accounting, (ii) present value analysis, which is a part of target costing that enables incorporating clients' perception of product attributes into cost management process, and (iii) suggest the use of accounting method in production engineering. Author stresses that production engineering will benefit from incorporating methodology of target costing as it might build a bridge between technical aspects of decision process and market-oriented strategy of modern enterprises.

Introduction

Although production engineering has ended up conventional division between economics and engineering¹, still engineering is concerned with alternative methods of achieving a given productive goal, while economics takes these methods as given, and analyzes the interrelations among productive ends and the factors used to achieve them. In practice, difference between the engineering and economic approach to production goals has limited the practical application of economic theory to engineering. Also engineering studies are rarely formulated in such a way that economists can easily use them.

The purpose of this paper is to suggest target-costing method for production engineering. The method may be of value in clarifying and ranking products' attributes that are important to clients, and in planning and redesigning manufacturing costs. Although it will not solve all the problems arising from the division of engineering and economics, it may bridge the gap between these two.

¹ Chenery H.B. (1949) Engineering Production Functions, „The Quarterly Journal of Economics”, Vol. 63, No. 4, pp. 507-531.

Schwartz L. (1988) The New Management Science, „Interfaces”, Vol. 18, No. 6, pp. 61-64.

The idea of target costing

The target cost is a financial goal for the full cost of a product. It is derived from estimates of selling price and desired profit. According to target costing methodology², selling price of a product is constrained by market price and is determined by analysis across all functions in the firm as well as along the entire industry value chain. Target costing starts with the desired level of profit based on firm's strategy and financial goals. The management of the company sets profit goal taking into account return on sales or return on assets. On the other hand, selling price is determined by competitive end-use market prices. In contrast with traditional cost pricing, product costs do not influence estimated selling price.

Because of the above, target costing is mostly applicable in early stages of product life cycle. Value engineering activities are applied throughout the preproduction process to ensure that the target cost can be met. If these efforts fail, the product is not introduced because managers recognize there is little potential for major cost reductions once production begins.

Clients' perception of product attributes and its incorporation into management decision-making system

In recent years, product quality has become an important issue for managers of all companies as it is seen to be the most important factor in maintaining firms' competitive edge³. In order to assure required quality, in today's unstable business environment and fast changing technologies, company must be designed to innovate product in a way that satisfies customers' needs. Target costing is a cost management system where accounting and client-oriented marketing, as well as engineering functions overlap. In this article we stress that product quality is an attribute important not only manufacturing departments, but also to marketing and management of a business. We propose to implement target costing into production engineering, as it is a management system that might use value engineering in the process of setting financial goals. The idea of value engineering was presented in the academic literature describing its origins and difference between traditional evaluations of products that is based on costs⁴.

In this paper, we model consumer perception of cell phone quality. An important determinant of its quality is a comfort of day-to-day usage. In turn, quality of usage is affected by a series of engineering attributes as the size, quality of a battery, WiFi connection etc. By manipulating the internal features of the product, samples reflecting different levels of these engineering attributes can be created for market research.

The four engineering attributes (tab. 1) were identified as a result of marketing research among potential client group. Client respondents were selected based on her/his ability to provide information about the product. Assuming that each respondent is a sole decision maker, it is very likely that clients' input into value engineering is of crucial importance.

First, respondents are asked to indicate important features of cell phone that are considered when buying a product. After sorting attributes into groups, respondents assign certain numerical values to each attribute in order to reflect their perception. Table 1 presents each attribute described by the importance for quality perceptions of the respondents. It can be

² Gagne M.L., Disenza R. (1995) Target Costing, „The Journal of Business & Industrial Marketing”, Vol. 10, No 1, pp. 16-22.

³ Narasimhan Ch., Sen S. (1992) Measuring Quality Perceptions, „Marketing Letters”, Vol. 3, No. 2, pp. 147-156.

⁴ Value Engineering. Technical Manual. School Facilities Development Procedures Manual, State Department of Public Instruction, Washington, 1981.

Kee R. (2010) The Sufficiency of Target Costing for Evaluating Production-Related Decisions, „International Journal of Production Economics”, Vol. 126, No. 2, pp. 204-211.

Prewysz-Kwinto P. (2010) Rachunek kosztów docelowych, CeDeWu.pl, p. 109.

stated, that based on marketing analysis, table 1 presents general agreement on what constitutes a high quality sample product.

Product attributes	Ranking [%]
Screen size	30%
WiFi connection	25%
Battery working time	25%
Guarantee service	15%
Available colors	5%
TOTAL	100%

Table 1: Ranking of product attributes.

After the marketing analysis, engineers are asked to decompose product structures and set costs on individual manufacturing parts. In the example of a sample cell phone, screen and case, battery, WiFi connection devices were listed. As target costing aims at full cost of a product, guarantee service expenses should also be included, if after-sale service is granted to customers. The following multi-step procedure requires that each component is characterized by production costs or purchase price. Table 2 presents that each manufacturing part requires materials' usage and overhead costs. The total cost of a cell phone is 206 and it is comprised of 4 elements. WiFi devices are the most expensive among manufacturing parts of a cell phone (41,74%). Battery costs are second (33,98%), while guarantee service is the least expensive (4,86%).

Manufacturing part	Materials used	Overhead costs	TOTAL	Percentage of costs
Screen and case	16	24	40	19,42%
Battery	20	50	70	33,98%
WiFi devices	40	46	86	41,74%
Guarantee	-	10	10	4,86%
TOTAL	76	130	206	100%

Table 2: Manufacturing parts and their costs.

An important managerial question is the relationship between the attributes defining the quality and the underlying manufacturing costs to achieve the quality. Quality index can be quantified by comparing attributes with manufacturing parts required to achieve certain product parameter. Table 3 presents relationship between clients' expectations concerning quality and the importance of each manufacturing part in production process regarding certain attribute.

	Screen and case	Battery	WiFi devices	Guarantee	TOTAL
Screen size	60%	40%			100%
WiFi connection			80%	20%	100%
Battery working time		90%		10%	100%
Guarantee service				100%	100%
Available colors	100%				100%

Table 3: Quality index calculations (part 1).

Engineers with production process knowledge should be responsible for data input in the above diagram. Each function of a product should be matched with manufacturing part required for high quality of each attribute. Table 3 shows that WiFi devices are the most important manufacturing parts (80%) in order to ensure good quality of WiFi connection, and screen size depends on manufacturing part called 'Screen and case' and on the kind of battery

used in the product. 20% or less show weak relations between manufacturing part and product attributes, while 80-100% indicate strong relationships between those two. Table 3 clearly presents strong and weak connections between clients' attributes and manufacturing parts. It can be used not only for product designing but also in production innovation processes.

Second part of value engineering requires combining data from table 3 and from table 1. Multiplication of clients' rankings by the percentage showing the importance of each manufacturing part (table 3) allows ranking product components using client-oriented approach. Table 4 reveals that the most important manufacturing part is a battery when judging by clients' preferences (34,5%). Screen and guarantee service present almost the same level of importance (22,% and 23%).

	Clients' ranking	Screen and case	Battery	WiFi devices	Guarantee
Screen size	30%	30 * 60 = 18%	30*40 =12%		
WiFi connection	25%			25*80 =20%	25*20 =5%
Battery working time	25%		25*90 =22,5%		25*10 =2,5%
Guarantee service	15%				15*100 =15%
Available colors	5%	5*100 =20%			
TOTAL	100%	23%	34,5%	20%	22,5%

Table 4: Client-oriented ranking of manufacturing parts. Quality index calculations (part 2).

The last step in calculating quality index requires division the client-oriented ranking of manufacturing parts by the percentage of costs. The amount obtained from division represents quality index (table 5). The index informs whether costs associated to a specific manufacturing part are adequate to clients' ranking based on attributes of a sample product. If the index value is close to 1, it shows that the amount of costs is adequate to the value client obtain when using a certain feature of a product. Index value below 1 identifies costs that should be reduced as they are too high when compared to value representing to a customer. On the other hand, index value above 1 presents that high value for the customer was achieved by small amount of costs. The best relationship between customer value and costs associated with certain functionality can be observed in the case of battery, as value index amounts 1,02.

Manufacturing part	Percentage of costs (from table 2)	Client-oriented ranking of manufacturing part (from table 4)	QUALITY INDEX
Screen and case	19,42%	23%	1,18
Battery	33,98%	34,5%	1,02
WiFi devices	41,74%	20%	0,48
Guarantee	4,86%	22,5%	4,63
TOTAL	100%	100%	

Table 5: Quality index calculations (part 3).

Table 5 shall be a starting point for redesigning a sample product or for innovation planning. It recognizes the best opportunity to reduce costs, improve quality, and enhance manufacturability of new and existing product. According to value index, the costs of WiFi devices are too high when compared to customer's valuation of product attributes. The changes may involve different departments engaged in manufacturing process as well as outside suppliers. At the end, compromises and trade-offs by manufacturing engineers, marketing employees, accountants, and managers should produce a target cost that is close to the original profitability goal.

Benefits of target costing usage in production engineering

As a result of value engineering, each attribute of a product becomes the set of cost objectives and provides the basis for the costing system. It also enables to compare actual manufacturing cost and the target cost for each product's functions. Alternatives are identified in order to bring each function's actual cost estimate to its target cost. Very detailed set of cost tables can be created to compare alternatives. Target costing and value engineering can integrate the client-oriented product attributes, quality manufacturing process, cost management, and price calculations in a systematic way.

Quality and client oriented target costing requires team with members from the various functions of the company. Marketing is in charge of market analysis and attributes' data collection, while manufacturing engineers provide knowledge regarding different types of manufacturing technologies or alternative materials. Finally, management and accountants provide information on the cost effects of the proposed functional modifications. Cooperation between marketing, accounting and production should improve the overall product development process.

Implementing target costing with quality and client oriented value engineering will enhance management decision-making process and will help to design product according to market needs, clients' acceptability and manufacturing feasibility.

Conclusions

Faced with increasing global competition, many firms find that standard cost systems are relic while target costing is a key strategic tool to cost reduction and cost control. Target costing is dynamic, constantly pushing for improvement. Just like today's business environment. Because of the above, target costing, and value engineering should be considered, if manufactured product is subject to diversification, company experiences shorter product life cycles or system for reducing costs should be developed.

Target costing and value engineering require access to many kinds of information found in various departments, so cooperation must be promoted. As a result, an engineer and management would thereby gain a clearer conception of the financial position in an industrial establishment under the present highly changing and customer-oriented production conditions.

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CAN DECISIONMAKING PROCESS IN CLINICAL MEDICINE BE SUPPORTED BY A COMPUTER-BASED EXPERT SYSTEM?

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Abstrakt: Jednym z podstawowych zadań współczesnej informatyzacji organizacji jest wsparcie procesu podejmowania rutynowych, ale i wysoko komplikowanych oraz opartych na heterogenicznych źródłach danych, decyzji. W tym celu konieczna jest budowa bazy wiedzy na potrzeby systemu eksperckiego wspomagającego proces podejmowania decyzji. W artykule przedstawiono problematykę źródeł informacji w medycynie klinicznej oraz jej formalizacji na potrzeby opracowania bazy wiedzy umożliwiającej wspomaganie decyzji.

Abstract: The fundamental goal of modern information technology is to support decision making process in organizations, for both routine and highly complex problems, based on heterogeneous data sources. The main objective of the paper is to present the construction concept of knowledge base for an expert system designed to support the decision making in clinical medicine. Formal interpretation of various sources of information in clinical medicine is discussed and its potential use in the design of knowledge database.

Contemporary medical knowledge is both extensive and scattered, it is also standardized and redundant – as a result of ongoing development and scientific and technical progress. Medicine is still regarded as an art of science, however this point of view has significantly changed over the past ages. Introduction of uniform therapeutic indications and treatment approach has led to standardized medical care. The ongoing process in healthcare policing has improved treatment outcomes and overall patient safety, decreased the incidence of malpractice, and introduced cost-effectiveness and economic approach into healthcare. Progress and improvement of medical care has resulted in demand for strict obedience of guidelines and medical standards by healthcare professionals. Of note, although very helpful, current standards of care do not refer to the most complicated medical cases. It is still expected though, that clinicians will maintain their practices in accordance with novel techniques, methods, and treatment approach. This can be executed through integration of information from various sources and areas of medicine. Medical professionals are supported in their clinical daily routine by increased delivery of scientific papers, new versions of guidelines and recommendations, and updated risk scales. Although the information received from various sources is complimentary, uniform presentation of it requires complex data processing.

Computer programs combining medical standards and guidelines as well as risk scales and procedure cost evaluation could have significantly supported medical professionals in optimal treatment selection and become an information tool to patients. Modern healthcare evolves into personalized medicine with individual treatment scenarios, therefore it is necessary to investigate how to combine generally accessible patient databases to develop the decision making tool.

Design of specialized computer programs which support medical professionals in proper diagnosis, selection of treatment strategy and therapy is hindered by lack of formalism that would recognize noncompliant, multiple sources of variable data.

The research project may be presented both as a theoretical as well as practical problem. In theory, the essence of the project would be to study the possibility of knowledge arrangement and formalism in highly complex domain – clinical medicine. Knowledge engineers and medical experts could design a research model including the process of field knowledge acquisition and update including guidelines and recommendations. In practical phase the hypotheses on feasibility of field knowledge formalism in clinical medicine for the future application in decision making process would be verified. The mutual scientific goal can be stated as follows: is it possible to formalize knowledge in contemporary clinical medicine? The practical approach should be designed to create scientific pathways in the future through application of the knowledge base developed on the formal model. This process shall be followed by design of intelligent system supporting decision making in medicine.

Elementary sources of field knowledge in contemporary medicine comprise of medical guidelines, registries, risk scales, and local databases as well as peer-reviewed publications. Medical guidelines are issued based on limited sources of information, which are mainly historical, i.e. previously published. Other sources of information include: results of randomized, prospective, multicenter clinical trials and other, including metaanalyses, as well as expert opinions invited to participate in work groups to establish the rules of conduct in specific types of clinical scenarios. The above mentioned form into medical guidelines (recommendations) for particular clinical diagnoses, which are limited to majority of cases and do not cover every possible diagnostic and therapeutic scenario. Another limitation of the guidelines is extrapolation of uniform therapeutic assumptions for entire patient population

whereas source data have been obtained from selected groups of patients in individual clinical studies.

Healthcare professionals also exploit medical registries that combine well-organized clinical facts and data including cases where medical treatment was conducted in accordance with the guidelines as well as scenarios not described by recommendations where therapy was selected based on clinical experience of the medical team. Risk scales refer to large databases originating from volunteering units, without external validation of this dataset in many cases, and have been designed to predict early mortality and morbidity of the reference class of patient population.

Development of formal model of field knowledge in contemporary medicine seems feasible with available sources of knowledge. The term 'feasible' means 'possible in current circumstances'. This will be verified through the following response: 'Does the fact that any formal model in this domain has not been developed so far mean that it is still too difficult? Does the problem result from knowledge engineering? Does the problem result from sources of field knowledge? Is it possible to overcome these hurdles?'.

Expected project outcome would be development of research method for acquisition and storage of field knowledge in clinical medicine that might be utilized in other domains (medical and other) of similar knowledge source complexity.

Development of intelligent systems supporting decision-making process requires creating knowledge bases. Decisions made in clinical medicine result in a series of events in the whole economical environment of medical center. Each patient undergoing medical procedure is at risk of temporary or long-lasting health impairment or even death. Selected medical therapy requires use of particular medical equipment and device. In some cases, as a consequence, it may cause temporary modifications of center's activity, including hiring additional staff, extra costs and spending of (mostly) public money. The primary goal of this approach is patient survival and recovery, in case of active employees – return to normal work.

Unfavorable medical decisions are usually undertaken because of lack of extensive and current medical knowledge, mistaken analysis of various factors or due to ignorance of medical checklist. Since not every clinical scenario could have been described in medical guidelines, it can be hypothesized that a properly designed search engine would mine through available registries and locate similar case study and present it for assessment with the proper risk scale. It is impossible to present all possible consequences of medical decisions for the society. Several complications should be listed from an individual perspective as follows: death, long-lasting health impairment or disability. The above mentioned outcome is very frustrating for all members of the medical team. Each medical complication creates not only an organizational hurdle for the medical center but is typically associated with increased costs of treatment, which are not reimbursed. Patient rights are also well-defined in legal acts which not only complicate the daily medical routine, but also give way to compensation demands against the staff and medical center. Polish social insurance system requires that each new beneficiary becomes additional resource consumer for entire society. The same rule applies to medical treatment costs that exceed standard therapy reimbursement thus directly influencing the hospital economic balance.

These medical motivations should result in collaboration between medical professionals and knowledge engineers in the process of design and development of field knowledge base in contemporary healthcare. Thorough analysis of theoretical foundations for the process of design of rule formalism based on knowledge engineering will develop research methods describing knowledge acquisition and assembly. Researchers should intend to verify feasibility of field knowledge formalism in such a complex domain through experimental use

of various methods and techniques of knowledge acquisition. These studies will allow for practical testing of knowledge engineering in undiscovered domain of knowledge formalism.

The formalism model developed in the project will not only provide information for knowledge engineers, it may also give way for the following, practical studies. The backbone of the system, based on medical guidelines, will be supplied with registries and economic data. This process may create interactive health dictionary providing information and prognoses on epidemics and social trends, and most importantly, will define an unbiased and thorough medical knowledge data set. Healthcare providers, payers, patients and insurers will gain general access to the clinical aspect of the system. The described development perspectives present its pioneering characteristics – there aren't any comparable or similar advisory systems in healthcare so far. In 2012 initial formalism concept of field knowledge in cardiac surgery has been published. There is a whole range of international journals in medicine and technology that intend to publish newest innovation and achievements in healthcare and are interested in results of the study. Working prototypes, apart from presentation of limitations of introduced solutions, open new research areas on application of knowledge engineering in clinical medicine.

Positive study results are going to give way to several research areas. Basic studies are going to utilize the developed methods to design models applicable in other domains (not only medical). Alternatively, product implementation leads to design of unique expert tool for contemporary medicine that is going to be applicable to users worldwide.

Negative study results will be published to share experience with other teams approaching similar goals. Development and implementation of expert tools supporting decision making process in healthcare has become a necessary step to warrant further evolution in medicine, where progress is being quoted as increased efficacy of therapy, decreased morbidity, and well maintained level of socio-economic costs.

Successful performance of the project requires transparent and continuous exchange of expectations, thoughts, and needs between two independent study participants: programmers skilled in knowledge engineering and medical professionals who provide the system with their proficiency in medical guidelines, standards, rules of conduct, and clinical routine.

There is no research model available to adapt for the proposed project: so far no medical guidelines have been translated into programming rules, medical registries are not assembled as knowledge base, nor have economic results of medical centers been associated with risk modelling. Extensive and specialistic field knowledge is going to be translated into knowledge base for the first time in this project.

Design of the formal model that will address the above described problem requires creation of unique methodology where characteristics of scientific domains involved will be reflected both in terms of knowledge engineering as well as complexity of the domain. The research methodology will accommodate study results on verification of knowledge engineering methods applied for the first time in clinical medicine. Adequacy of applied research methods will be verified and unique methodology is going to be developed in this project as well.

Conclusions:

1. It is feasible to develop a formal model of field knowledge in clinical medicine based upon medical guidelines and standards. The formal model would develop knowledge base to perform metaanalysis of medical recommendations and assess risk associated with selected medical procedures.

2. It is feasible to develop a formal model of field knowledge in medicine for specific geographic region with application of medical registry data mining. Further the formal model of specific knowledge would compare guidelines and recommendations with actual clinical daily practice in the selected area.

3. It is feasible to develop a formal model for economic knowledge that would include procedure costs and estimated length of hospital stay. Further the formal model would support decision making process from the perspective of cost-effectiveness in contemporary medicine.

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Modelowanie rynku energetycznego ze szczególnym uwzględnieniem roli węgla kamiennego – część I

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Abstract: *Many of the established principles of the energy sector is now subject to change. Significant exporters become importers, and the state has long been recognized as major exporters are simultaneously leading centers of global growth in demand. The proper connection of strategy with technology shows that the relationship between economic growth energy demand and emissions of carbon dioxide (CO₂) from the energy sector can be relaxed. The growing importance of oil and gas from unconventional sources and renewable energy sources (RES) changes the current understanding of the balance of energy resources in the world. Awareness of the dynamics of energy markets is crucial for policymakers trying to reconcile economic energy and environmental issues. Those who correctly predict the development of the global energy sector can get an advantage while those who did not manage to run the risk of taking the wrong strategic decisions and investment (according to the World Energy Outlook (WEO-2013). Additionally, large differences in regional energy prices cause debate about the role of energy in stimulating or inhibiting of economic growth.*

Wprowadzenie

Polska jest największym producentem węgla kamiennego w Unii Europejskiej, stanowi ponad 50% produkcji unijnej, przy czym w przypadku węgla energetycznego jest to około 59%, natomiast węgla koksowego około 39%. Rola polskiego węgla kamiennego w Unii Europejskiej zależeć będzie od polskich producentów. Jedynie utrzymanie odpowiedniego poziomu kosztów pozwoli na konkutowanie na wspólnym rynku Unii Europejskiej z węglem importowanym przez kraje UE z innych kierunków oraz konkutowanie z innymi nośnikami energii.

Zużycie energii pierwotnej w 27 krajach Unii Europejskiej kształtuje się na poziomie 2,4 mld ton jednostek paliwa umownego, z czego[11]:

- 37% energii pierwotnej pochodzi z ropy naftowej,
- 24% z gazu ziemnego,
- 18% z węgla (kamiennego i brunatnego),

- 14% to energia jądrowa,
- 7% z odnawialnych źródeł energii.

Przy utrzymującym się na wysokim poziomie cenach nośników energii pierwotnej, a szczególnie ropy i gazu, rola węgla jako nośnika energii jest bardzo istotna dla bezpieczeństwa energetycznego. Istnieją znaczne rezerwy dotyczące możliwości opracowania i wdrożenia nowych efektywnych technologii spalania, nowych technologii przerobu węgla na paliwa płynne oraz produkcji ekologicznych sortymentów węgla o wysokiej jakości. Wobec powyższego węgiel powinien zacząć być postrzegany inaczej niż dotychczas, a więc nie tylko jako paliwo nadające się do spalania. Zawierania na Ukrainie znacząco uświadomią europejskim decydom jak ważne są własne źródła energii, których import nie jest uzależniony od nikogo. Dane Eurostatu pokazują jasno że za rok 2012 kraje UE są zależne od importu surowców energetycznych i energii w 53,3%. Dania, jako jedyne państwo UE ma nadwyżkę 3,4% i nie jest uzależniona od importu surowców. Polska na tle krajów europejskich nie wypada źle i z poziomem 30,7% zależności plasuje się wśród krajów najmniej zależnych. Do państw z mniejszą zależnością energetyczną niż Polska można wyróżnić Czechy (25,2%), Estonia (17,2%), Rumunia (22,7%) i Szwecja (28,7%). Holandia plasuje się na tym samym miejscu co Polska. Największe gospodarki w UE mają duży stopień zależności energetycznej i do takich państw można zaliczyć: Niemcy (61,1%), Francję (48,1%) i Wielką Brytanię (42,2%). Można znaleźć także kraje, które niemal całkowicie są uzależnione od importowanych surowców energetycznych, jak: Luksemburg (97,4%), Belgia (74%), Hiszpania (73,3%), Włochy (80,8%). Głównym zatem celem polityki energetycznej Unii Europejskiej jest jak największe wykorzystanie swoich rodzimych surowców energetycznych. Rozwój odnawialnych źródeł energii w wielu krajach UE został zwolniony, a jednym z powodów są właśnie wysokie koszty, a także stabilność pracy OZE, która jest uzależniona od warunków pogodowych. Sądono, że elektrownie jądrowe to lek na całe zło, ale warto przypomnieć wydarzenia jakie rozegrały się w Japonii po ataku tsunami, więc energetyka jądrowa będzie nadal budzić spore kontrowersje społeczne i jej rozwój może okazać się niemożliwy. Analizując sytuację rynku energetycznego i mając w zamyśle ograniczenie importowanego gazu i ropy, a jednocześnie wykluczenie budowy elektrowni atomowych, to jedynym wyjściem jest sięgnięcie po swoje rodzime surowce paliw kopalnych, a Unia Europejska ma ich sporo. Jednym z podstawowych surowców, a i flagowych w Polsce jest węgiel kamienny. Trzeba także podkreślić, że takie państwa jak: Polska, Czechy, Rumunia i Bułgaria, które to czerpią energię z własnych źródeł są w dużo mniejszym stopniu

niezależne energetycznie w UE. To obrazuje jak ważne jest prognozowanie wielkości sprzedaży węgla, aby producenci mogli zaspokoić popyt na ten surowiec.

Model prognozowania sprzedaży węgla kamiennego z użyciem pól losowych

Analizowano statystycznie miesięczną wielkość sprzedaży węgla kamiennego od roku 1995 do roku 2013. Badanie empiryczne rozpoczęto od weryfikacji hipotezy H_0 zakładającej, że ciąg obserwacji jest niezależny od czasu, wobec hipotezy alternatywnej będącej zaprzeczeniem hipotezy zerowej. Stosując test stacjonarności wykorzystujący współczynnik korelacji rang Spearmana, wyznaczono wartość statystyki empirycznej, którą porównano z wartością krytyczną [5], [6]:

$$r_s = 1 - \frac{6 \sum_{t=1}^n [t - d(e_t)]^2}{n(n^2 - 1)} = 1 - \frac{6 \times 3713689}{228(228^2 - 1)} = -0.8801 \quad [1]$$

$$r_s = \frac{u_{1-\alpha}}{\sqrt{n-1}} = r_s(\alpha = 0.05; n = 228) = \frac{u_{1-\alpha}}{\sqrt{n-1}} = \frac{1.645}{\sqrt{227}} = 0.1092 \quad [2]$$

gdzie:

$d(e_t)$ – ranga wartości e_t , czyli jej pozycja w uporządkowanym niemalejąco szeregu czasowym ($1 \leq d(e_t) \leq n$),

n – liczba obserwacji (długość szeregu czasowego),

Bezwzględna wartość empiryczna przekraczała wartość krytyczną, zatem należy przyjąć, że wielkość sprzedaży węgla kamiennego na rynku krajowym jest procesem niestacjonarnym, zatem dalsza analiza będzie przeprowadzona z wykorzystaniem pól losowych.

Proces stochastyczny $Y(t)$, będący wielkością sprzedaży węgla kamiennego, można zaliczyć do rodziny zmiennych losowych zależnych od parametru $t \in \mathbb{R}$, określonych na pewnej przestrzeni probabilistycznej Π . Polem losowym nazwano funkcję przyporządkowującą każdemu punktowi obszaru k -wymiarowego, gdzie $k \in \mathbb{N}$ zmienną losową. Należy zaznaczyć, że rozważane będą dyskretne procesy i pola losowe. W przypadku procesów stochastycznych parametr t przyjmuje wartości ze zbioru liczb całkowitych, a realizacją takiego procesu jest szereg czasowy. Jeżeli podstawą rozważań będzie m procesów losowych wówczas otrzymano pole losowe [12]. Jedną z najważniejszych charakterystyk procesu stochastycznego i pola losowego jest funkcja kowariancji. Informuje ona o sile związku korelacyjnego pomiędzy dwiema zmiennymi losowymi $X(t_1)$ oraz $X(t_2)$.

Funkcję [12]:

$$C(t_1, t_2) = E\{[X(t_1) - E\{X(t_1)\}][X(t_2) - E\{X(t_2)\}]\} \quad [3]$$

nazwano funkcją kowariancji. Szczególną uwagę należy poświęcić takiej funkcji kowariancji, dla której [12]:

$$C(t_1, t_2) = C(t_1 - t_2) \quad [4]$$

a która posiada między innymi własność [12]:

$$C(t_1 - t_2) = C(t_2 - t_1) \quad [5]$$

która wynika z określenia stacjonarności procesu [12]:

$$\Delta(B\{X_t\}) = \text{const } C(t_1, t_2) = C(t_1 - t_2) \quad [6]$$

Proces stochastyczny, który nie spełnia tego warunku nazwano procesem niestacjonarnym.

Predykcją nazwano procedurę wnioskowania w przyszłość na podstawie odpowiedniego modelu matematycznego. Kolejne dwa terminy teorii predykcji zostaną określone nieco inaczej aniżeli spotyka się to w przedmiotowej literaturze, jednakże zgodnie z tym, co proponuje Z. Hellwig w pozycji [12].

Przez predyktor należy rozumieć każdy funkcjonal mający dwie własności:

- Każdemu modelowi opisującemu kształtowanie się zmiennej prognozowanej przypisuje się zmienną losową,
- Przypisaną zmienną losową można traktować jako prognozę.

Przepis analityczny predyktora można przedstawić w postaci [12]:

$$\varphi = \varphi_T[G(Y)] \quad [7]$$

gdzie:

φ – operacja, którą należy wykonać w celu wyznaczenia prognozy,

G – model kształtowania się zmiennej prognozowanej,

Y – zmienna prognozowana,

T – okres, na który sporządza się prognozę,

Zatem przez pojęcie prognozy należy rozumieć zmienną losową stanowiącą wynik procesu wnioskowania w przyszłość. Predykcja omawiana w tej pracy ma charakter empiryczno-ekstrapolacyjny. Oznacza to, że podstawą wnioskowania w przyszłość jest obserwacja prawidłowości ruchu zmiennej prognozowanej w przeszłości, co prowadzi w rezultacie do budowy odpowiedniego modelu odzwierciedlającego ten ruch [12]. Na podstawie skonstruowanego modelu dokonuje się ekstrapolacji, czyli rzutowania w przyszłość zaobserwowanych prawidłowości ruchu zmiennej prognozowanej. Jednakże proces prognozowania w przyszłość nie jest idealny i rzeczywiste realizacje zmiennej odchylają się od postawionych prognoz, powodując powstawanie błędów. Należy zauważyć również, że wielkość, którą prognozuje się jest również zmienną losową. Dla wnioskowania

w przyszłość co do faktycznej jej realizacji można stosować różne predykatory. Dowolny predyktor ze zbioru predyktorów możliwych do zastosowania można oznaczyć przez φ . Określając następującą wielkość [12]:

$$U = Y_T - \varphi \quad [8]$$

gdzie wiadomo, że φ jest funkcjonalem - i to funkcjonalem losowym, stąd też różnica U , którą nazwano błędem predykcji, jest również funkcjonalem losowym.

Wprowadzenie wag korekcyjnych do predyktora, a ściślej do modelu predykcyjnego nie wyczerpuje możliwości podniesienia precyzji stawianych prognoz, albowiem stosując różne metody predykcji w oparciu o tę samą zasadę predykcji otrzyma się różne stopnie efektywności prognozowania. Wyłania się zatem problem wyboru metody predykcji. Aby móc rozwiązać ten problem powstaje konieczność sformułowania kryterium, według którego będzie można ocenić każdą metodę z dowolnej klasy modeli Ξ . Jeżeli przez A oznaczono miernik będący normatywem wybranego kryterium, to zadaniem jest znalezienie takiego predyktora φ_{eff} z dowolnej klasy modeli Ξ , który będzie spełniał następujący warunek [12]:

$$A(\varphi_{\text{eff}}) \rightarrow A_{\text{dop}} \quad [9]$$

Oznacza to, że wartość miernika A w przypadku zastosowania predyktora φ_{eff} jest wystarczająco korzystna, albowiem mieści się w granicy dopuszczalnej wartości miernika A_{dop} założonej przez prognozującego a priori. Taki predyktor nazwano predyktorem efektywnym [12].

Niektóre z metod zakładają, że zmienna objaśniająca modelu φ_{eff} jest zmienną zdeterminowaną i wówczas rozkład zmiennej prognozowanej jest generowany przez rozkład składnika losowego ξ . Najczęściej przyjmuje się, iż rozkład ten jest rozkładem normalnym. A to pozwala już na konstrukcję odpowiedniego przedziału prognozy. W przypadku, jednak gdy nie dysponuje się informacją o rozkładzie zmiennej prognozowanej, proponuje się rozważania następującej konstrukcji. Szukana jest wielkość δ_{Ω} . Należy określić różnicę [12]:

$$\Delta = \varphi_{\Omega} - \delta_{\Omega}^P = \varphi_{\Omega}^P - \delta_{\Omega}^P \quad [10]$$

gdzie φ_{Ω}^P wartość rzeczywistej zmiennej prognozowanej w odległości Ω .

Różnica ta jest również zmienną losową.

Rozważania teoretyczne, które zaprezentowano wykorzystano do zbudowania modelu prognostycznego wielkości sprzedaży węgla kamiennego na rynku krajowym, który jest szczegółowo opisany w artykule „Modelowanie rynku energetycznego ze szczególnym uwzględnieniem roli węgla kamiennego – część II”.

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Modelowanie rynku energetycznego ze szczególnym uwzględnieniem roli węgla kamiennego – część II

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Abstract: Transformation coal market, which began in the 90s and it was largely due to a reduction in the energy intensity of production processes. It was noted at that time large fluctuations in the demand for coal. It should be noted that, in relation to the substitution of energy carriers coal still has a low price, availability, and the introduction of the mining process to produce "clean" coal, adapted to the requirements of the European Union regarding the emission of pollutants into the atmosphere. The other hand easy access to other energy sources such as oil, natural gas, etc.. Were the main causes of "overproduction" of coal. This forces the use of extractive industries advanced techniques identify potential sales of coal, analyzes the elasticity of demand with respect to price, ratings behavior of competitors and audience reaction to their behavior. It is therefore necessary to determine the ability of mining front, which meet lower and lower, but still variable demand for hard coal.

Wprowadzenie

Sprzedaż węgla kamiennego energetycznego jest ściśle związana z jego wykorzystaniem w sektorze energetycznym. Prawie 65% węgla zużywa się w świecie do wytwarzania energii elektrycznej. Jak łatwo zauważyć w sprzedaży tego surowca występuje wyraźna sezonowość. Efektywne prognozowanie wielkości sprzedaży węgla kamiennego wymaga uwzględniania występujących w nich wahań sezonowych. Model, który zaproponowano do prognozowania wielkości sprzedaży węgla kamiennego polega na wyznaczeniu wskaźników sezonowości dla poszczególnych faz cyklu. W analizie wahań sezonowych wyróżniono cztery etapy:

1. Wyodrębnienie liniowej tendencji rozwojowej,
2. Eliminację tendencji rozwojowej z analizowanego szeregu czasowego,
3. Eliminację wahań przypadkowych
4. Obliczenie czystych wskaźników sezonowości

Model, który może być stosowany w przypadku szeregów czasowych zawierających tendencję rozwojową, wahania sezonowe i wahania przypadkowe to model Wintersa. Wahania sezonowe nakładają się na trend w sposób addytywny, zatem równania modelu są następujące [18, s.149]:

$$\begin{cases} \hat{y}_t = \frac{1}{l} \sum_{i=1}^l y_{t-i} & \text{dla } t = 1, \dots, l, \\ \hat{y}_t = \alpha(\hat{y}_t - d_{t-l}) + (1-\alpha)(\hat{y}_{t-1} + c_{t-1}) & \text{dla } t > l, \end{cases} \quad [1]$$

przy czym

$$\begin{cases} c_t = \frac{1}{l} \sum_{i=l+1}^{2l} y_t - \frac{1}{l} \sum_{i=1}^l y_t & \text{dla } t = 1, \dots, l, \\ c_t = \beta(\hat{y}_t - \hat{y}_{t-1}) + (1-\beta)c_{t-1} & \text{dla } t > l \end{cases} \quad [2]$$

oraz

$$\begin{cases} d_t = \frac{1}{l} \sum_{i=1}^l (y_t - \hat{y}_t) & \text{dla } t = 1, \dots, l, \\ d_t = \gamma(\hat{y}_t - \hat{y}_t) + (1-\gamma)d_{t-1} & \text{dla } t > l. \end{cases} \quad [3]$$

gdzie:

d_t – ocena wskaźnika sezonowości w okresie t ,

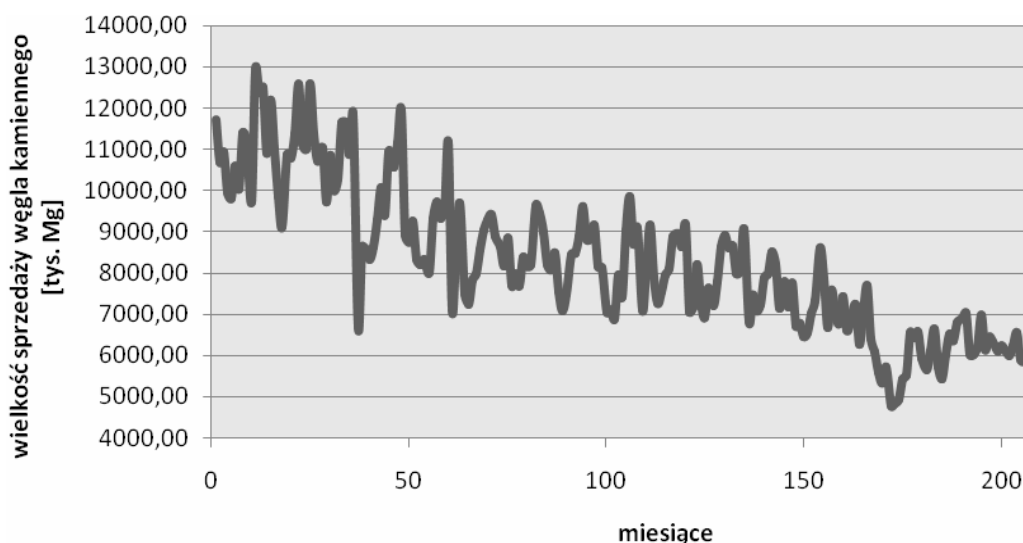
l – liczba faz w cyklu,

α, β, γ – stałe wygładzania ($\gamma \in (0,1)$).

Prognozę oblicza się z zależności [18, s.150]:

$$y_t^P = \hat{y}_n + h c_n + d_{n-l+(h+1)} \quad [4]$$

Wielkość sprzedaży węgla kamiennego w latach 1995 – 2013 ilustruje wykres nr 1. Ocena wzrokowa wykresu wskazuje, że w badanym szeregu czasowym występuje trend malejący, można również zaobserwować występowanie wahań sezonowych.



Rys. 1 Rzeczywista wielkość sprzedaży węgla kamiennego w latach 1995-2013, opracowanie własne, źródło opracowanie własne

Parametry strukturalne oszacowano stosując metodę najmniejszych kwadratów. Założono hipotezę H_0 , że nie ma liniowej tendencji rozwojowej oraz hipotezę alternatywną H_1 , że występuje tendencja rozwojowa o charakterze liniowym. Wielkość sprzedaży węgla kamiennego można opisać równaniem liniowym o postaci:

$$y = -23.76x + 10802 \quad [5]$$

Należy sprawdzić, czy znaleziony wzorec jest istotny statystycznie. W celu weryfikacji hipotezy, że zmienna objaśniana jest dobrze opisana modelem teoretycznym wykorzystano parametryczny test współczynnika rang Pearsona. Wartość krytyczna została odczytana z tablic rozkładu t - Studenta dla n-2 stopni swobody (n=228) oraz poziomu istotności $\alpha=0,05$ i wartość ta wynosi [1]:

$$t_{\alpha} = 1,9704 \quad [6]$$

Jeżeli:

$$|t| \leq t_{\alpha} \quad [7]$$

to nie ma podstaw do odrzucenia hipotezy H_0 co oznacza, że nie udało się udowodnić statystycznie istotnego trendu liniowego [1].

Jeżeli:

$$|t| \geq t_{\alpha} \quad [8]$$

to odrzuca się hipotezę H_0 na korzyść hipotezy alternatywnej H_1 co oznacza, że udało się udowodnić statystycznie istotny trend liniowy w rozważanym ciągu czasowym.

Sprawdzeniem hipotez jest statystyka empiryczna t określona wzorem [1]:

$$t = \frac{r \cdot \sqrt{n-2}}{\sqrt{1-r^2}} \quad [9]$$

Z otrzymanych obliczeń wyznaczono statystyki służące do badania istotności parametrów modelu:

$$t(a) = \frac{|-23,76|}{0,94} = 25,36 - \text{parametr istotny} \quad [10]$$

$$t(b) = \frac{|10801,78|}{123,71} = 87,32 - \text{parametr istotny} \quad [11]$$

Przeprowadzono również ocenę istotności współczynnika determinacji R^2 korzystając z testu Fischera - Snedecora, gdzie wartość krytyczną F_{α} odczytano dla $m_1=1$ i $m_2=228-1-1=226$. Wartość ta dla poziomu istotności $\alpha=0,05$ wynosi $F_{\alpha}=0,8233$. Porównując empiryczną statystykę $F=643,29$ z wartością krytyczną F_{α} , należy odrzucić hipotezę zerową, która

zakładała brak stochastycznej zależności zmiennych. Oznacza to, że dopasowanie modelu liniowego jest poprawne.

Po eliminacji tendencji rozwojowej analizowanego ciągu czasowego określono sezonowość metodą wskaźników addytywnych.

Wartości początkowe do zbudowania modelu Wintersa określono w następujący sposób:

- $\hat{f}_{1s}$ - średnia z pierwszego roku, czyli z roku 1995,
- c_{1s} - różnica średnich w drugim i pierwszym roku,
- $d_t (t = 1, 2, \dots, 228)$ – wyznaczone na podstawie całego szeregu czasowego, są to średnie różnice wartości zmiennej prognozowanej i wygładzonej wartości trendu liniowego opisanego równaniem nr 1.

Wyboru parametrów wygładzania dokonano wykorzystując metodę optymalizacji simplex. Poszukiwano minimalnej wartości średniokwadratowego błędu ex post prognoz wygasłych.

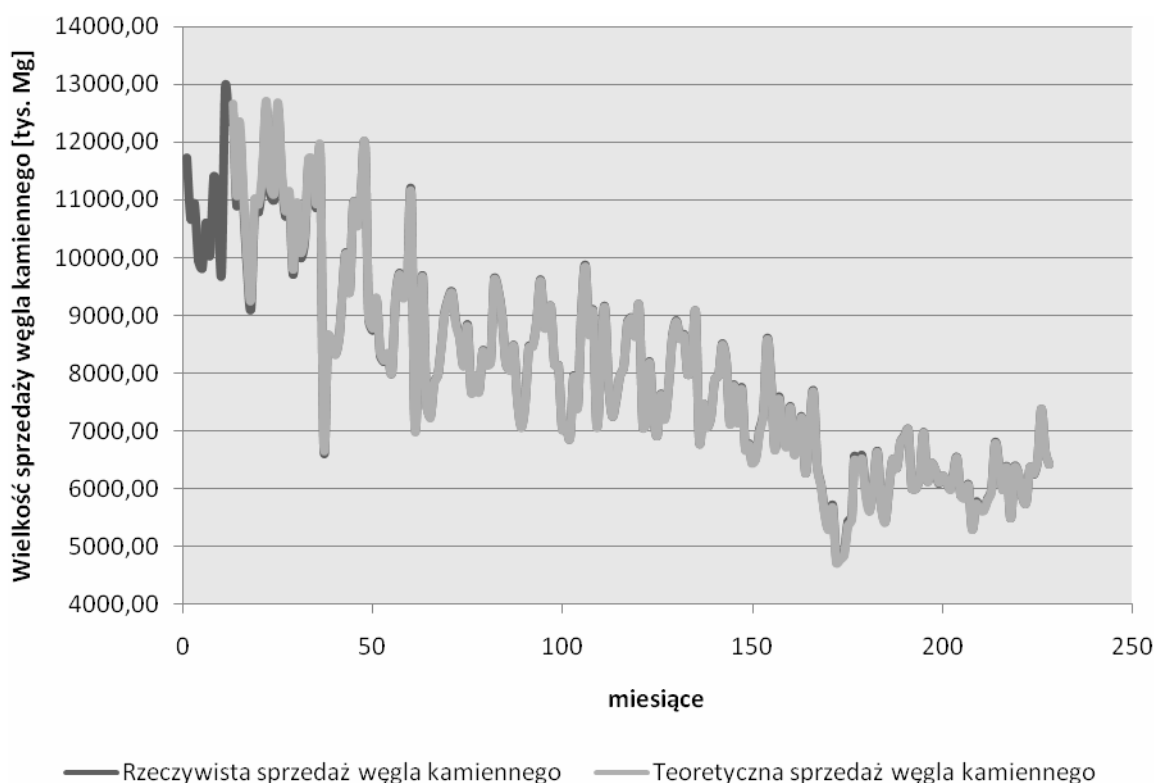
Teoretyczny model wielkości sprzedaży węgla kamiennego ma następującą postać równań:

$$\hat{f}_{t-1} = 0,1(\hat{f}_{t-1} - d_{t-1,t}) + (1 - 0,1)(\hat{f}_{t-2} + c_{t-2}) \quad [12]$$

$$c_{t-1} = 0,06(\hat{f}_{t-1} - \hat{f}_{t-2}) + (1 - 0,06)c_{t-2} \quad [13]$$

$$d_t = 1(\hat{f}_{t-1} - \hat{f}_{t-1}) \quad [14]$$

Średniokwadratowy błąd prognoz wygasłych dla tak zaprojektowanego modelu wynosi 682 Mg.



**Rys. 2 Teoretyczny model wielkości sprzedaży węgla kamiennego w latach 1995-2013,
źródło opracowanie własne**

Kontynuując analizę statystyczną zbudowanego modelu przeprowadzono badanie składnika losowego $\xi(t)$. Właściwości tego składnika powinny być identyfikowane poprzez analizę reszt szeregu czasowego. Reszty zostały ustalone zgodnie z zależnością:

$$u(t) = y(t) - y_t^F \quad [15]$$

Analiza reszt, pozwalająca na określenie dobroci modelu teoretycznego, powinna przebiegać zgodnie z następującym algorytmem [2]:

- (a) Weryfikacja hipotezy głoszącej, że obserwowany ciąg reszt jest stacjonarny, którą można zweryfikować współczynnikiem korelacji rang Spearmana,
- (b) W przypadku stwierdzenia, że nie ma podstaw do odrzucenia hipotezy zakładającej stacjonarność reszt, obliczenie średniej wartości reszt,
- (c) Jeżeli zdarzy się, że średnia reszt okaże się różna od zera, to kolejnym etapem powinna być weryfikacja hipotezy stwierdzającej, że obliczona średnia jest nieistotnie różniącą się od zera; tu można zastosować badanie wykorzystujące statystykę t - Studenta,
- (d) Kolejną fazą badania powinna być weryfikacja przypuszczenia, że rozrzut reszt jest stały w czasie; można to zrealizować np. badając wariancję w pierwszej i drugiej połowie próby i weryfikując hipotezę głoszącą, iż wartości obu wariancji są takie same, wykorzystując statystykę F-Snedecora.
- (e) W przypadku pozytywnego wyniku testu statystycznego F-Snedecora, obliczenie odchylenia standardowego reszt dla całej badanej próby,
- (f) Weryfikacja hipotezy o braku autokorelacji pomiędzy wartościami resztowymi.

Model teoretyczny, który określono powyżej, użyto do prognozowania krótkoterminowego (dwanaście miesięcy wprzód) wielkości sprzedaży węgla kamiennego na rynku krajowym. Tak zbudowany model nie może zostać wykorzystany do prognozowania średnio, czy długoterminowego, gdyż

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BUDOWA MODELU W OPARCIU O METODĘ DYNAMIKI SYSTEMOWEJ (SD) Z WYKORZYSTANIEM OPROGRAMOWANIA VENSIM NA PRZYKŁADZIE PROBLEMU „ŁOWCA-OFIARA“

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Abstrakt: W artykule zaprezentowano sposób budowy modelu w oparciu o metodę Dynamiki Systemowej, a następnie omówiono pakiet symulacyjny Vensim za pomocą którego przeprowadzono symulacje na znanym z literatury przedmiotu modelu „łowca-ofiara“.

Słowa kluczowe: Dynamika Systemowa, model, symulacja

Abstract: The article shows how to build the model based on the method of System Dynamics, and then discusses the Vensim simulation package with which the simulations were performed on well-known from the literature model of "predator-prey".

Key words: System Dynamics, model, simulation

1. Wprowadzenie – metoda Dynamiki Systemowej (SD)

Nazwy „Dynamika systemowa“, „Dynamika Systemów Zarządzania“ czy też „Industrial Dynamic“ wskazują na najważniejszy aspekt prezentowanej metody, mianowicie ujęcie „dynamiki“ otaczającego świata, odwzorowanie jej w modelu i późniejszą symulację. Autorem metody jest prof. Joy W. Forrester i jego współpracownicy z Massachusetts Institute of Technology (MIT) [1]. Zakres wykorzystania metody Dynamiki Systemowej jest praktycznie ograniczony tylko wyobraźnią. W 1970 r. na zlecenie Klubu Rzymskiego zajęto się dynamiką świata [2]. Dynamika Systemowa stosowana jest do modelowania i symulacji systemów ekonomicznych, społecznych, socjologicznych, środowiska naturalnego po medycynę. Metoda SD znalazła swoje miejsce również w zarządzaniu, gdzie wykorzystuje się tak zwane „symulatory lotów“ dla menadżerów.

Kluczowymi pojęciami w omawianej metodzie są:

- *System*, czyli pewna skomplikowana całość złożona z wielu powiązanych ze sobą podsystemów i wchodząca w relacje ze środowiskiem, w którym się znajduje [2].
- *Myślenie systemowe* wywodzące się z teorii systemów będące koncepcją, zasobem wiedzy i narzędzi, które ciągle rozwijane pozwalają wyjaśniać zjawiska systemowe i skutecznie na nie wpływać. Myślenie systemowe przez Senge’a nazwane zostało piątą dyscypliną, kamieniem węgielnym organizacji uczącej się [3].
- *Struktura systemu* jest schematem wzajemnych powiązań pomiędzy składnikami systemu.

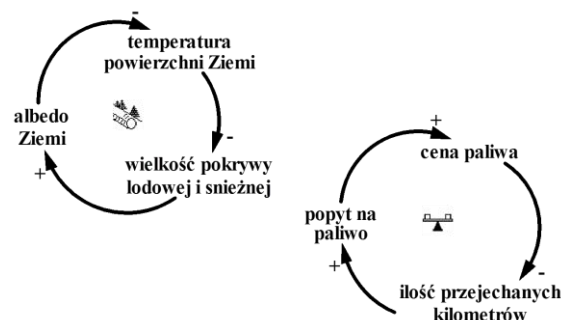
Z kolei elementami myślnia systemowego są:

- *Sprężenie zwrotne wzmacniające* (intensyfikujące), zwane również dodatnim (w ekonomii to tak zwany efekt kuli śnieżnej), charakteryzuje się tym, że wzmacnia własne działanie (rys.1).

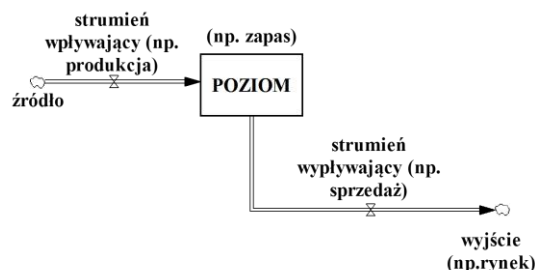
- *Sprężenie zwrotne równoważące* (stabilizujące), zwane również ujemnym charakteryzuje się tym, że utrzymuje system w pożądanym stanie (rys.1).
- *Opóźnienia*, które mają ogromny wpływ na zachowanie systemu. Należy uzmysłowić sobie fakt, iż w funkcjonowaniu systemów zmiany stanów współzależnych elementów mogą zachodzić równocześnie, ale na ogół w systemach rzeczywistych występuje przesunięcie w czasie czyli opóźnienie.

Podstawowymi narzędziami myślenia systemowego są:

- *Proste struktury dynamiczne* (sprężenia zwrotne wzmacniające i równoważące).
- *Poziomy*, których równania opisują gromadzenie się w systemie pewnych ilości określonych mediów np. ilość gotówki na koncie bankowym [zł], ilość osobników danej populacji [osobnik] itp.
- *Strumienie*, których równania wyrażają sposób w jaki regulowane jest ich natężenie w systemie np. w przypadku konta bankowego strumieniem wpływającym będzie strumień gotówki zasilający konto bankowe [zł/msc] (wypłata, dywidenda, odsetki), strumieniem wypływającym będzie strumień wydatków miesięcznych [zł/msc].
- *Diagramy systemowe* (diagramy pętli sprzężeń).
- *Archetypy systemowe* czyli pewne ogólne struktury sprzężeń pojawiające się w różnych systemach, generujące taki samo zachowanie. Zidentyfikowano 10 archetypów systemowych i nadano im nazwy wskazujące na zachowanie jakie generują („granice wzrostu“, „szkodliwe lekarstwa“, „sukces dla odnoszących sukces“, „dryfujące cele“, „eskalacja“, „przerzucenie brzemienia“, „wzrost i niedoinwestowanie“, „tragedia współużytkowania“, „przypadkowi przeciwnicy“, „zasada atrakcyjności“).



Rys.1 Pętle sprzężeń zwrotne [źródło własne]



Rys.2 Prosty model z jednym poziomem i dwoma strumieniami w konwencji Dynamiki Systemowej

2. Języki symulacyjne

Pierwszym językiem symulacyjnym dedykowanym metodzie SD był SIMPLE (R. Bennett 1958r.). Po publikacji „Industrial Dynamic“ [1] pojawił się kolejny język opracowany na potrzeby modelowania symulacyjnego DYNAMO w którym określonych

było siedem typów równań wykorzystywanych w metodzie. W trakcie kompilacji równania były sortowane w odpowiednią sekwencję obliczeniową co ułatwiało pisanie programu. Działania arytmetyczne wykonywane były zgodnie z kolejnością działań w matematyce. Szeroki zakres operacji w modelu można było wykonywać przy użyciu wewnętrznych instrukcji Dynamo. W ostatnich latach pojawiło się wiele programów komputerowych wspomagających programowanie modeli symulacyjno-dynamicznych. Autorka korzysta obecnie z pakietu oprogramowania Vensim firmy Ventana Systems Inc, którego pierwsza wersja pojawiła się w 1985 roku. Najnowsza wersja Vensim Ple 6.3 jest bezpłatną wersją przeznaczoną na potrzeby edukacyjne i do badań własnych. Jest to wersja wystarczająca dla początkujących i uczących się Dynamiki Systemowej, w stosunku do pozostałych wersji komercyjnych ma ograniczenia funkcji programu, narzędzi do budowy modeli, uproszczone menu i dialogi jednak z punktu widzenia nauki zasad budowy modeli i ich symulacji ograniczenia te nie mają znaczącego wpływu. Vensim [4] daje olbrzymie możliwości modelowania, jest zintegrowanym środowiskiem budowy, rozwoju, analizy i zastosowań modeli SD. Posiada „szkicownik“, za pomocą którego można łatwo i szybko stworzyć strukturę modelu. Ścisłe połączenie „szkicownika“ z edytorem równań znacznie ułatwia zapis równań dla opracowanego modelu. W programie tym w wersji DSS istnieje możliwość optymalizacji. Za pomocą tego programu można tworzyć laboratoria nauki, gry systemowe oraz „symulatory lotów dla menedżerów“.

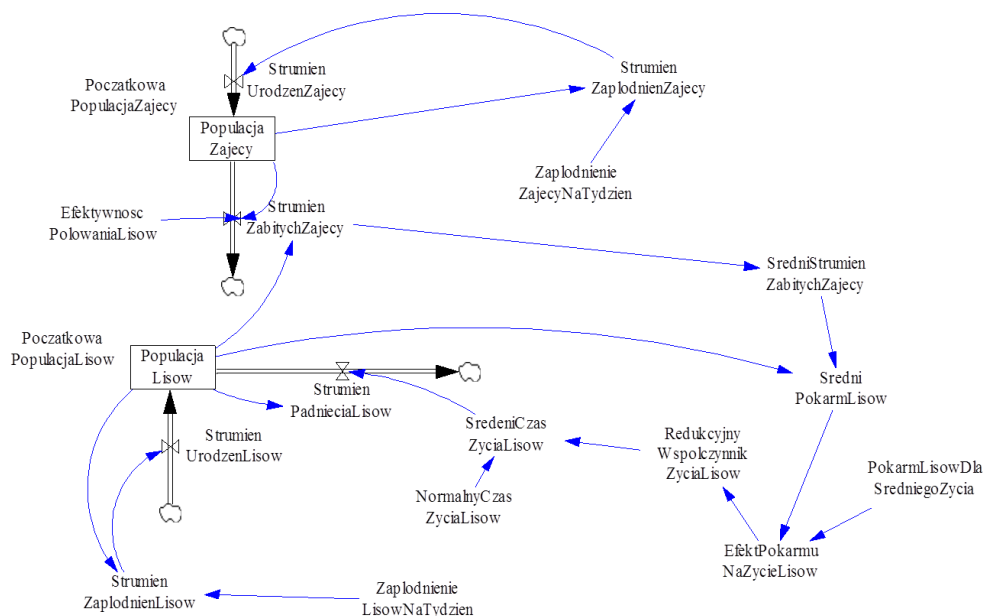
Modele w VENSIM DSS (wersja komercyjna) można łączyć ze sobą, można też kopiować część jednego modelu do innego modelu. Dokonując zmiany nazwy zmiennej w modelu zmiana ta automatycznie dokonuje się również we wszystkich równaniach, gdzie zmienna ta występowała. Program Vensim DSS posiada moduł umożliwiający dwa rodzaje symulacji, moduł umożliwiający optymalizację oraz analizę wrażliwości.

3. Model „łowca-ofiara“, symulacje na modelu

Na rysunku 3 przedstawiono ogólną strukturę modelu „łowca-ofiara“. W modelu populacja lisów to „łowcy“, a populacja zajęcy to „ofiary“. Sprzężenia w omawianym systemie odwzorowują wzajemne związki między obu populacjami zaś zakłócenia wewnątrz systemu ujęto w postaci zmian parametrów.

Model matematyczny „łowca-ofiara“.

- (1) $PopulacjaZajecy(t+dt) = PopulacjaZajecy(t) + (StrumienUrodzenZajecy(t) - StrumienZabitychZajecy(t))$
- (2) $StrumienUrodzenZajecy(t) = DELAY3(StrumienZaplodnieniaZajecy, 4)$
- (3) $StrumienZaplodnieniaZajecy(t) = PopulacjaZajecy(t) * ZaplodnienieZajecyNaTydzien$
- (4) $StrumienZabitychZajecy(t) = PopulacjaLisow(t) * PopulacjaZajecy(t) * EfektywnoscPolowaniaLisow$
- (5) $SredniStrumienZabitychZajecy(t+dt) = SredniStrumienZabitychZajecy(t) + dt * (StrumienZabitychZajecy(t) - SredniStrumienZabitychZajecy(t)/2)$
- (6) $SredniPokarmLisow(t) = SredniStrumienZabitychZajecy(t) / PopulacjaZajecy(t)$
- (7) $PopulacjaLisow(t+dt) = PopulacjaLisow(t) + (StrumienUrodzenLisow(t) - StrumienPadnieciaLisow(t))$
- (8) $StrumienUrodzenLisow(t) = DELAY3(StrumienZaplodnieniaLisow, 7)$
- (9) $StrumienZaplodnieniaLisow(t) = PopulacjaLisow(t) * ZaplodnienieLisowNaTydzien$
- (10) $StrumienPadnieciaLisow(t) = PopulacjaLisow(t) * SredniCzasZyciaLisow(t) + 1$
- (11) $SredniCzasZyciaLisow(t) = RedukcyjneWspolczynnikiZyciaLisow(t) * NormalnyCzasZyciaLisow$



Rys.3 Ogólna struktura modelu „łowca-ofiara“ [5]

- (12) $\text{RedukcyjnyWspolczynnikiZyciaLisow}(t) = \text{TABLE}(\text{EfektPokarmuNaZycieLisow}(t))$
- (13) $\text{EfektPokarmuNaZycieLisow}(t) = \frac{\text{SredniPokarmLisow}(t)}{\text{PokarmLisowDlaSredniegoSreumienia}}$

Na gotowym modelu przeprowadzono liczne symulacje. Vensim umożliwia otrzymanie wyników w postaci tabeli oraz wykresu. Na rysunku 4 pokazano oscylacje populacji lisów i zajęcy przy początkowych wartościach 20 lisów i 300 zajęcy. Z kolei na rysunku 5 początkowa wartość populacji lisów 20, zaś zajęcy 600. W tabeli ujęto wartości pozostałych parametrów w modelu. Horyzont symulacji wynosił 300 tygodni.

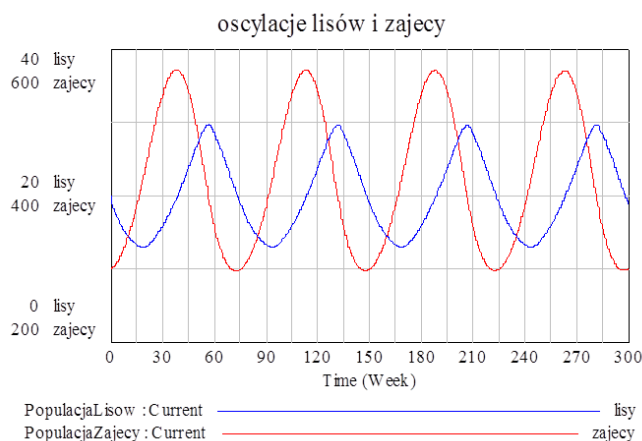
Tabela 1 Wartości parametrów w modelu „łowca-ofiara“

PARAMETR	WARTOŚĆ
ZapłodnienieZajęcyNaTydzień	0,1 [1/tydzień]
EfektywnoscPolowaniaLisow	0,005 [(1/tydzień)/lisa]
ZapłodnienieLisowNaTydzień	0,05 [1/tydzień]
NormalnyCzasZyciaLisow	[120 tygodnie]
PokarmLisowDlaNormalnegoZycia	5 [(zajęcy/tydzień)/lisa]

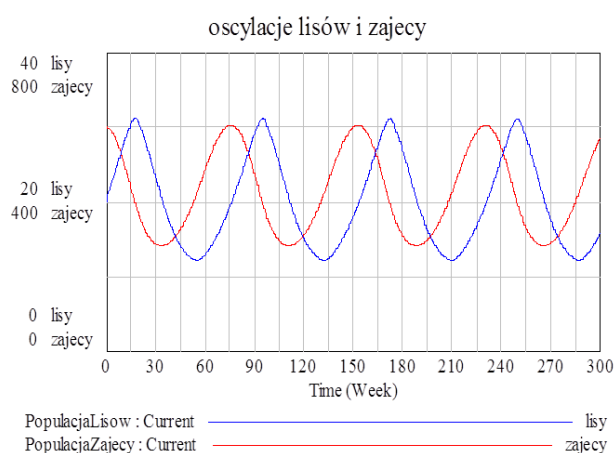
Podsumowanie

Możliwości modelowania są w praktyce nieograniczone, należy jednak zdawać sobie sprawę z pewnych ograniczeń tego typu modelowania w szczególności jeśli chodzi o ekosystemy. Jednym z ograniczeń jest uśrednianie wielkości w modelu. Ograniczeniem jest również konieczność posiadania sporej wiedzy o problemie, możliwość pominięcia istotnych czynników podczas budowy modelu, błędy zaokrągleń, a także bazowanie na uproszczeniach, które wynikają z przyjętej konwencji modelowania. W modelu populacje lisów można podzielić na kolejne populacje uwzględniając etap rozwoju osobników i ich różnorodne zapotrzebowanie na pokarm oraz zdolność rozmnażania, co z pewnością ma wpływ na długość ich życia. Agregacja i uśrednianie są wypadkowymi celu analizy, modelowania i symulacji. Należy zwrócić jednak uwagę, że w praktyce długookresowa

obserwacja zachowania się ekosystemów w czasie jest niemożliwa bądź ograniczona, zaś symulacje komputerowe umożliwiają skompresowanie czasu i przestrzeni w eksperymencie wirtualnym i pozwalają na realizację procesu uczenia się w trakcie eksperymentu bez szkody dla systemu.



Rys. 4 Oscylacje populacji lisów i zajęcy przy początkowych wartościach 20 lisów, 300 zajęcy



Rys. 5 Oscylacje populacji lisów i zajęcy przy początkowych wartościach 20 lisów, 600 zajęcy

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USE OF QUALITY MANAGEMENT TOOLS IN IDENTIFYING DEFECTS OF THE FINAL PRODUCT AND THE REASON FOR THEIR OCCURRENCE IN A HOUSEHOLD APPLIANCE MANUFACTURING COMPANY – CASE STUDY

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Abstrakt: Nowadays, companies are forced to search for new solutions which would lead to an increase in efficiency. This is a result of, among others, growing competition on the market. Companies can compete either with the price or quality of their products. Many of them decide to attract clients with the quality of the goods they offer. However, to ensure that high quality, organisations are forced to use certain methods and tools of quality management. This article presents an example of basic tools of quality control, such as the Pareto analysis and the cause-and-effect diagram, being implemented in order to identify the most common causes of returns and conduct an analysis of the causes of their occurrence. These two tools used together make it possible to identify the foremost reasons of the client's dissatisfaction, which allows us to effectively eliminate the most common returns of damaged and faulty goods. The case studied in this article took place in a household equipment manufacturing company located in Silesia

Abstract: Przedsiębiorstwa w dzisiejszych czasach zmuszone są do poszukiwania rozwiązań prowadzących do większej efektywności działania. Wynika to między innymi z nasilającej się konkurencji występującej w gospodarce rynkowej. Konkurować przedsiębiorstwa na rynku mogą ceną bądź jakością produkowanych wyrobów. Wiele z nich decyduje się na przyciąganie klientów jakością oferowanych produktów. Aby jednak zapewnić wysoką jakość wyrobów, organizacje zmuszone są do stosowania odpowiednich metod i narzędzi zarządzania jakością. W artykule zaprezentowano przykład zastosowania podstawowych narzędzi zarządzania jakością takich jak Analiza Pareto i Diagram przyczynowo-skutkowy do identyfikacji najczęstszych powodów reklamacji i analizy przyczyn ich wystąpienia. Oba te narzędzia stosowane łącznie pozwalają na identyfikację najistotniejszych powodów

wystąpienia niezadowolenia klienta, co pozwala w sposób efektywny wyeliminować najczęstsze reklamacje. Analizowany przypadek w artykule wystąpił w przedsiębiorstwie branży AGD mieszczącej się na Śląsku.

1. Introduction

Modern company diagnostics are based on the evaluation of efficiency of all the operations involved in the processes of manufacturing products and providing market services, as well as the right conclusions being drawn from such an evaluation, which can be useful in further actions [1, 2, 3]. The diagnosis of an organisation forces us to split the company into a series of consecutive and simultaneous processes, which overlap and influence each other to a varying degree [2, 3]. The assessment of individual areas of the organisations activity can be based upon the various losses which occur in them. We can divide the losses according to their different functional importance to the company and then propose various courses of action as well as the utilisation of certain methods and tools in order to improve the functioning of the company [4,5]. Many different types of losses can be distinguished. The main ones include, among others: wastes of time, overproduction and a mismatched scale, waste of resources, improper work organisation, bad communication and insufficient, incomplete information, etc. [6]. The aim of the continuous improvement actions is to minimise and reduce the identified losses, defects and imperfections occurring in the organisation, which allows for the company to function better and for the quality of its products and working conditions to increase, resulting in a greater satisfaction of its clients and employees [7]. The analysis of the final product with the use of quality management tools presented in this article was conducted in a household appliance manufacturing company based in Silesia and owned by an Italian concern, which produces cooker hoods. The company offers in its product line a variety of cooker hoods, namely: built-in, island and chimney hoods, as well as ones intended to be built over. The facility currently employs 263 employees, 81 of which are white-collar workers and 182 work directly at the production line. The production process takes place in three departments: the mechanical department, the engine production department and the assembly department [8].

2. Identification of the major defects created in the process

As part of the evaluation of the current state and a critical analysis of the manufacturing process in the company, actions aimed at gathering data pertaining to the defects that occur during the process were collected and an analysis and evaluation of the causes for returns of the company's products in the studied period has been conducted. The data necessary at this stage has been gathered on the basis of, among others, a direct observation at the production hall, an analysis of the company's documentation disclosed by the quality management department, including the quality control reports, and discussions with the company's employees. Apart from the defects occurring currently in the production process, the number of defects which occurred over the course of 2013 has also been determined.

The conducted research indicates that the biggest number of defects has been reported in the assembly department. There are 3627 of such defects, which accounts for 64% of all the defects detected in the production process. A total of 1636 defects have been detected in the mechanical department and 366 in the engine production department, comprising respectively 29% and 7% of all the defects in the production process detected in all three production departments. At this point we can already see a very important problem in the company, i.e. the high number of product defects detected during the production process in the mechanical

and assembly departments. It is important, because the downtimes, repairs and the necessity to perform further operations which occur due to these defects all affect the timely delivery of orders, but also their quality and the overall work atmosphere in the company. The defects collected and their number will serve as a basis for further actions connected with the aim of this work.

Due to the high number of defects detected in the production process in the mechanical and assembly departments, actions were taken, aimed at determining the problems important to the company and shortcomings which directly affect the aforementioned state. In order to accomplish this, the Pareto-Lorenz analysis was used, which made it possible to justifiably isolate the defects which, due to their frequent occurrence, affect the imperfection of the production process to the highest degree.

The analysis has shown clearly that 80% of the defects in hood housings are caused by the following defect groups:

- Improper welds of the hood housing elements,
- Mistakes in the order of bending, damage to the sheet metal during bending,
- Low quality of materials received from the supplier,
- Improper quality of operation in the mechanical department, damage to the paint coating.

In the assembly department, on the other hand, 80% of the defects were caused by:

- Improper paint coating of the hood,
- Components which were improperly assembled, damaged or not suited to a given model.

Based on the Pareto-Lorenz analysis of the production process defectiveness in the company, defects and groups of defects have been determined, which should be reduced or eliminated in the first place in order to increase the efficiency of the production process.

Defects created in the course of the production process of the cooker hoods in this company are the cause of many rejections of the products in the final stage of their production, i.e. the assembly. During the analysis, a certain situation could be observed, in which the cause of rejection of the product during the final inspection, apart from the defects created during the production process, was also very often the defects formed much earlier, in the mechanical department at the stage of hood housing production. These are the defects to the paint coatings of the hoods. This problem is significant and thought-provoking, as the defects to the paint coating which were formed in the mechanical department should have been detected much earlier, before the defective housings entered the assembly department.

3. An analysis of the causes of returns using the Pareto-Lorenz tool

Taking the above reflections and results into consideration, outside returns have also been analysed in order to gain a better understanding of the causes of defects, using the Pareto-Lorenz tool. The input data necessary for the analysis has been obtained from the return documentation of the client. The period analysed was the year 2013.

The results have been presented in Table 1.

Table 1. Pareto-Lorenz analysis of the causes of returns in the company.

Causes of returns	Number of occurrences of the defect in the studied period	Quota of the type of defect in the overall number of returns (%)	Cumulated quota of the type of defect in the overall number of returns (%)
Paint coating defects	546	35.94	35.94
Noisy rotor	343	22.58	58.53
Damaged hood wiring	255	16.79	75.31
Malfunction of the control panel	194	12.77	88.08
Malfunction of the hood lighting	97	6.39	94.47
Other	84	5.53	100.00
Total number of returns	1519		

Summarising the analysis, we can conclude that 80% of the returns of the final product in the examined company are mostly caused by three main factors, i.e. paint coating defects, noisy rotor and damaged hood wiring. The analysis has confirmed that the foremost problem in the company is the formation of defects to the paint coating.

4. An analysis of the causes of paint coating defect formation using the Ishikawa cause-and-effect diagram

Special attention should be paid to the defects of the paint coating, which are the cause of return of the final product in 36% of the cases, identified using the Pareto-Lorenz analysis. In order to identify the causes of paint coating defect formation, the Ishikawa cause-and-effect diagram tool has been used – Fig. 1.

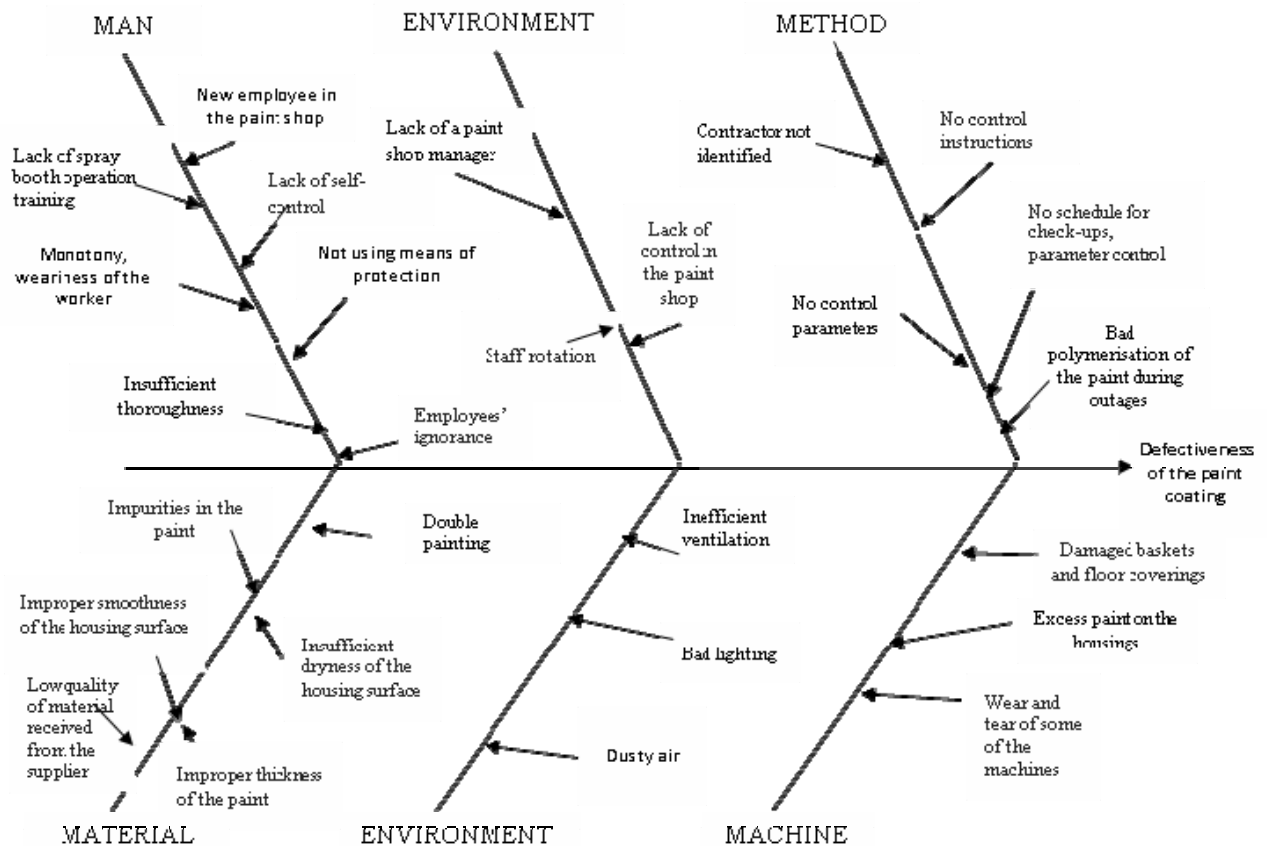


Fig. 1. A cause-and-effect diagram for the identification of the paint coating defects.

Taking into consideration the data obtained from the conducted Ishikawa analysis, a further Pareto-Lorenz analysis has been conducted in order to determine the major causes of the paint coating defects. The results of the analysis have allowed us to identify the most significant problems, which should be eliminated in the first place in order to reduce the number of returns from the clients due to damaged paint coating by 80%. These problems are:

- a lack of organised control in the paint shop,
- ignorance and low qualifications of the production line workers,
- parameters of the paint coating control method not specified,
- means of protection not used by the workers,
- insufficient self-control on the workers' part,
- impurities in the paint and low quality of operations prior to painting the housings in the mechanical department.

Summary

If they are to improve the quality of the offered products, the quality management tools have to be used repeatedly, which will cause the problem to be analysed more and more thoroughly, leading to the discovery of its most basic causes. The presented case study shows that the interchangeable use of Pareto-Lorenz and Ishikawa tools allows us to identify the major problems and the causes of their occurrence.

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WYBRANE ASPEKTY TEORII GIER

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Streszczenie:

Teoria gier bada zachowanie się w warunkach konfliktu interesów. Rozwój tej dziedziny matematyki spowodował, że znalazła ona zastosowanie w różnych obszarach działalności człowieka i różnych dziedzinach nauk, takich jak socjologia, zarządzanie, psychologia, biologia, informatyka i inne. Artykuł zarysowuje podstawowe zagadnienia teorii gier i zastosowanie jej w innych naukach.

Podstawowe zagadnienia z zakresu teorii gier

Obszarem zainteresowań teorii gier jest podejmowanie optymalnych decyzji w warunkach nieokreśloności. Teoria gier wywodzi się z gier hazardowych i taką też posługuje się nomenklaturą. Do podstawowych pojęć należą: zbiór graczy, zbiór reguł, zbiór strategii, zbiór wyników oraz wypłata.

1. Gra jest dowolną sytuacją konfliktową, w której uczestnik (gracz) działa w celu maksymalizacji swoich korzyści.
2. Zbiór graczy $D=\{P_i\}$, gdzie każde P_i (dla $i=1, 2, 3 \dots n$) oznacza gracza
3. Zbiór reguł gry R to zasady obowiązujące wszystkich graczy
4. Zbiór strategii S_i – plany działania gracza P_i , uwzględniające wszystkie możliwe sytuacje
5. Zbiór wyników W
6. Wypłata $u_i(w)$ dla każdego gracza P_i i dla każdego wyniku w ze zbioru W . Funkcja $u_i(w)$ określana jest również jako funkcją wypłaty lub funkcją użyteczności. Oznacza ona wielkość korzyści, jaką odniesie gracz, gdy uzyska określony wynik [4].

Podział gier przedstawia się następująco [4]:

- gry o sumie stałej oraz o sumie zmiennej,
- gry sprawiedliwe (kiedy wartość oczekiwana wypłaty jest taka sama dla każdego z graczy) oraz niesprawiedliwe (kiedy nie jest spełniony warunek gry sprawiedliwej i najwyższa wygrana jednego gracza przewyższa najwyższą wygraną innego gracza),
- gry dwuosobowe oraz gry wieloosobowe,
- gry, w których agent (gracz) podejmuje decyzję o wyborze strategii bez wiedzy o decyzjach przeciwników, oraz gry, w których dysponuje taką wiedzą,
- gry, w których gracz dysponuje informacją o wypłacie wszystkich graczy oraz gry, w których takiej wiedzy nie ma,
- gry o skończonym czasie rozgrywki, oraz gry o czasie nieskończonym.

Gry można również sklasyfikować w oparciu o takie kryteria [5]:

- ze względu na kolejność podejmowania decyzji: gry strategiczne oraz w postaci rozwiniętej (ekstensywnej),

- ze względu na posiadaną przez agenta wiedzę: gry z kompletną informacją i gry z informacją niekompletną,
- ze względu na możliwość zawiązywania koalicji: gry koalicyjne (kooperacyjne), gry niekoalicyjne,
- ze względu na ilość dostępnych strategii: gry skończone oraz nieskończone,
- gry statyczne oraz ewolucyjne,
- ze względu na sposób dokonywania posunięć: gry z posunięciami równoczesnymi (symultaniczne) oraz z posunięciami naprzemiennymi (pozycyjne).

Założenia dotyczące gracza mówią o jego racjonalności, tzn. że przejawia następujące cechy:

- gracz jest w stanie określać możliwe wyniki w grze,
- gracz jest w stanie określać decyzje, które prowadzą do zamierzonych wyników,
- gracz podejmuje działania, które zmierzają do osiągnięcia najbardziej pożądanego wyniku, w zależności od posunięć i wyborów pozostałych agentów.

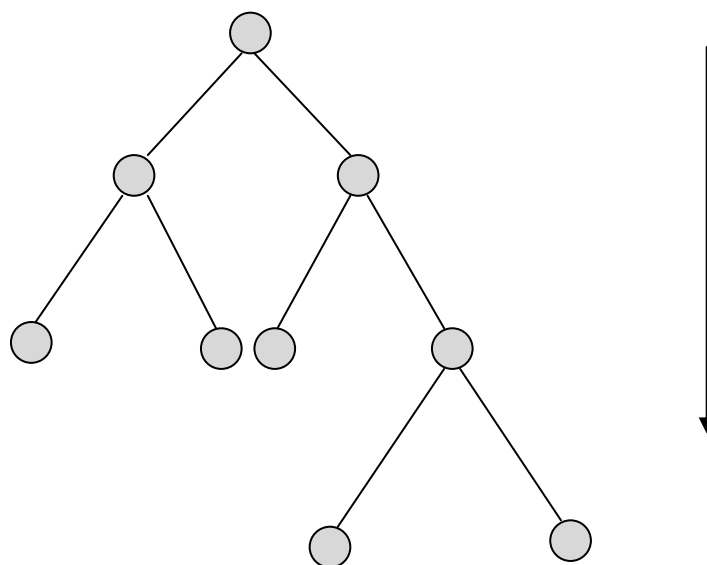
Najprostszym rodzajem gier są tzw. gry macierzowe. Są to gry dwuosobowe. Wiersze macierzy reprezentują strategie jednego z graczy, kolumny – drugiego. Wnętrze macierzy to wyniki gry w zależności od decyzji podjętych przez agentów [7].

Każdy z graczy stara się wyszukać strategię dominującą – strategię, która jest zawsze lepsza od strategii pozostałych graczy. Jeżeli obydwaj gracze mają swoje strategie dominujące, wynik gry stanowi para tych strategii. Strategie zdominowane – takie, dla których zawsze istnieją u przeciwnika lepsze strategie, można wyeliminować z gry.

Grami dwuosobowymi o sumie 0 są sytuacje, w których przegrana jednego z graczy jest wygraną drugiego.

Prostota cechuje gry z pełną informacją. Gracz wykonuje swoje posunięcie, analizując każdą z sekwencji reakcji, będących konsekwencjami danego działania. Następnie określone zostaje przez gracza rozwiązanie mające dla niego największą wartość, po czym następuje sekwencja działań, mających doprowadzić agenta do założonego celu. Analiza działa wstecz, poczynając od zakładanych wyników danej sekwencji. Jest to tzw. indukcja wsteczna.

Graficznym jej przedstawieniem jest drzewo. Drzewo stanowi alternatywę macierzowego przedstawienia sytuacji [4]. Przykładowe drzewo gry pokazano na rysunku 1.



Rys. 1. Przykład drzewa [w oparciu o 4]

W celu przewidzenia wyborów dokonywanych przez agentów dokonuje się analizy strategii stron. Poszukiwany jest stan równowagi, wyznaczający rozwiązanie gry. Jednym z podstawowych pojęć związanych z teorią gier jest **równowaga Nasha**. Równowaga Nasha opisuje posunięcia graczy, których strategie gry są optymalizowane, przy uwzględnieniu wyborów przeciwników. Równowaga Nasha określa taki plan działania, kiedy żaden gracz nie może poprawić swojej sytuacji działając samodzielnie. Tak osiągniętej równowagi gracze nie mają interesu naruszać – ma szansę być trwałą. Nie zawsze taką jest, ponieważ zawsze znajdują się inne rozwiązania, polepszające sytuację agenta kosztem innych graczy [6].

Teoria gier jest narzędziem opisującym zachowanie graczy w sytuacji konfliktu oraz kooperacji – pozwala formalizować zjawiska i interakcje zachodzące pomiędzy ludźmi lub pomiędzy ludźmi i innymi podmiotami [5]. W specyficznych sytuacjach graczem może być podmiot „bezosobowy”, jak np. niszczycielskie siły natury, które pełnią w określonych sytuacjach istotną rolę, ale nie są zainteresowane wygraną [6].

W ostatnich latach wiele dziedzin nauk zaczęło czerpać ze zdobyczy teorii gier. Sposób matematycznego sformalizowania zjawisk znalazł zastosowanie w socjologii, biologii, antropologii, ekonomii, informatyce i innych gałęziach nauk.

Teoria gier w zarządzaniu

Matematyczne modelowanie problemów decyzyjnych nie jest prostym wyzwaniem. Przyczyny małej efektywności leżą nie w braku metod optymalizacyjnych, ale w niewłaściwej ocenie, którą strategię w warunkach niepewności zastosować [2].

Odkrycia behawioralnej teorii gier włączyły do badań psychologię, co sprawiło, że teoria gier stała się użyteczna społecznie. Zastosowanie teorii gier w zarządzaniu spowodowało pewne przeinterpretowanie definicji owej teorii. Działanie strategiczne w myśl teorii gier jest sztuką znalezienia drogi współpracy, nawet jeśli działania pozostałych graczy powodowane są interesami, a nie życzliwością [1]. Podejmowanie decyzji w otoczeniu, w którym również stale są dokonywane wybory jest skomplikowane – na decyzje gracza mają wpływ wybory pozostałych agentów. Zdarzają się sytuacje, które nie mają właściwego rozwiązania, a zastosować można jedynie strategię częściowo lub w sposób niedoskonały rozwiązujące problem.

Teoria gier w informatyce

W informatyce modelowaniu podlegają interakcje komputera z otoczeniem.

Teoria gier w biologii ewolucyjnej

W ramach biologii ewolucyjnej jako gracze przyjmowane są poszczególne gatunki lub geny. Przy regułach określonych przez naturalną selekcję gracze uzyskują tym większą wypłatę im więcej potomków zostawią [4]. W ewolucyjnej teorii gier nie są rozważane wybory poszczególnych osobników, ale gra różnych strategii przeciwko sobie. Bada ona zmiany rozkładu różnorodnych strategii w populacji w kolejnych grach.

Teoria gier w ekonomii

Powiedziane zostało powyżej, że u graczy zakłada się zachowanie racjonalne. Zwłaszcza w ekonomii założenie to jest szczególnie istotne – kierownicy zarządzający organizacjami, mając na uwadze interes – zachowują się racjonalnie. W takich okolicznościach możliwe jest zbudowanie modelu matematycznego, który nasycony danymi, oszacuje wszystkie możliwości i scenariusze działań uwzględnionych podmiotów, ich reakcje oraz siłę oddziaływania. Efektem jest wyłonienie najbardziej prawdopodobnego wariantu [3].

Zakończenie

Rozwój matematycznej teorii gier badającej zachowanie się gracza w warunkach konfliktu i kooperacji, przy założeniu, że posunięcia agenta wpływają na innych graczy pozwolił na zaimplementowanie jej w innych dziedzinach nauk. Pozwoliło to ująć w karby matematycznego modelu różnorakie sytuacje, których opisywaniem zajmują się min. socjologia, psychologia, informatyka, ekonomika czy biologia.

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APPLICATION OF STATISTICAL METHODS FOR CUSTOMER SATISFACTION ANALYSIS

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Abstract

The paper is devoted to statistical methods to study customer satisfaction. The purpose of this article is to describe the possibility of applying statistical methods to analyse data obtained in the customer satisfaction survey. The article uses the actual data of one of the industrial enterprises, which in the framework of the quality management system conducts annual satisfaction survey action. In the first part of the article describes the process of research and the data. In the second part of the article contains a brief description of the statistical process control - with the use of control charts. In the third part of the article describes the use of statistical control charts to control customer satisfaction.

Introduction

Quality management system based on ISO 9001 requires improvement [1,3]. Further it should primarily be based on information received from the client. A survey among customers is the basic and most popular method of collecting data on customer satisfaction [4,5,6,7]. In order to know the real customer satisfaction, is necessary to apply the correct methods of data analysis. This will allow for a reliable evaluation of customer satisfaction. The results of these analyses will furthermore observe changes in customer satisfaction in a specified time interval.

The article is devoted to the use of statistical methods to analyse data on customer satisfaction. The article includes an example of the use of standard Shewhart's control charts [2] for data analysis of customer satisfaction. Example based on the results of customer satisfaction surveys carried out in one of the large industrial enterprises, providing services to other businesses (B2B).

Customer satisfaction survey

In each year, the company in question, send a written questionnaire to 20 selected major customers. In surveys, customers are asked to assess several aspects of

cooperation. Customers evaluate each aspect on a scale of 1 (negative evaluation) to 5 (positive assessment). The survey covers aspects of:

- A - assessment of the timeliness of services,
- B - an overall assessment of the quality of services,
- C - assessment of cooperation during the service,
- D - assessing the fulfillment of expectations,
- E - assessment of response to complaints (evaluation of the express customers who filed complaints),
- F - assessment of the competence of personnel.

The evaluation results obtained customer satisfaction survey for 2006-2013 are presented in figures 1 and 2. Without the use of control charts is difficult to assess the stability of the quality of services offered by the analyzed company.

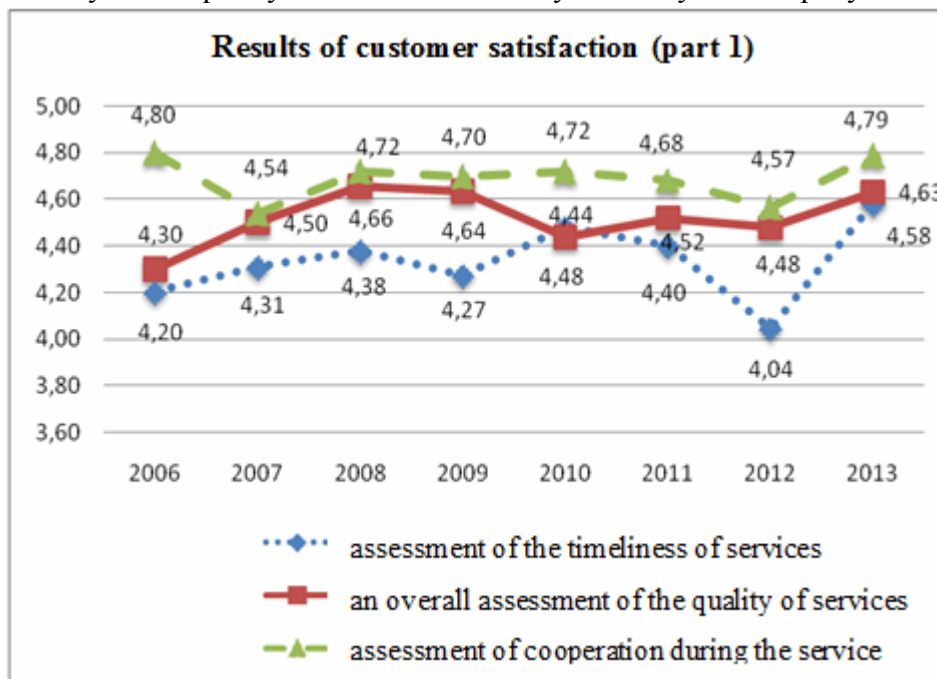


Fig 1. Figure 2: Results of customer satisfaction (part 1). Source: Own.

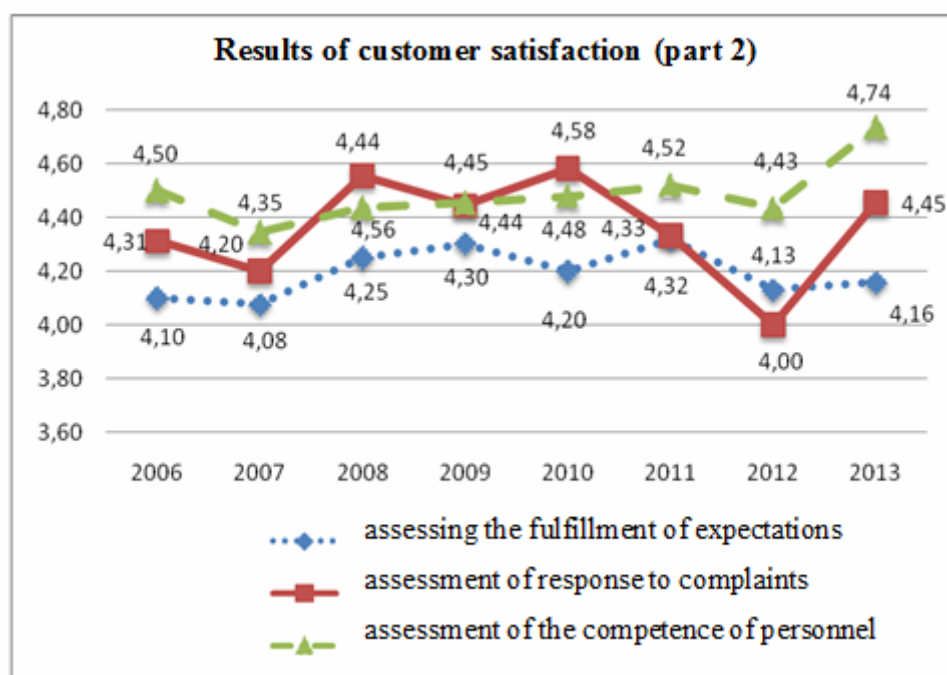


Fig 2. Figure 2: Results of customer satisfaction (part 2). Source: Own.

An example of the use of control charts

The use of control charts is described in detail in the PN-ISO 8258 + AC1. For the analysis of data on customer satisfaction has been selected pair of two charts: X and R. It was established that parameter, affixed on the card X is the value of CSI - Customer Satisfaction Index. CSI value was calculated for each year as the arithmetic average of the ratings received from 20 customers. To simplify the evaluation of each customer is calculated each year as the arithmetic average of the ratings 3 aspects:

- A - assessment of the timeliness of services,
- B - an overall assessment of the quality of services,
- C - assessment of cooperation during the service.

The actual value of customer satisfaction obtained by the analyzed company are given in table 1. To calculate the control limits using the formulas given in table 2.

Table 1. Results of the evaluation of customer satisfaction. Source: Own.

Customer	i=2006	i=2007	i=2008	i=2009	i=2010	i=2011	i=2012	i=2013
CSI _{-1,i}	4,3	4,7	4,3	4,7	5,0	4,3	4,0	4,3
CSI _{-2,i}	4,0	3,7	4,7	4,7	4,0	4,0	4,3	5,0
CSI _{-3,i}	4,7	4,0	4,7	4,7	4,7	4,0	4,0	5,0
CSI _{-4,i}	4,7	4,0	5,0	4,3	4,7	4,7	4,7	4,7
CSI _{-5,i}	4,3	4,3	4,0	4,7	4,7	5,0	4,3	4,0
CSI _{-6,i}	5,0	4,3	4,3	4,3	4,7	4,3	3,3	5,0
CSI _{-7,i}	4,7	5,0	4,7	4,3	4,3	3,7	4,3	4,7
CSI _{-8,i}	4,7	4,7	5,0	5,0	5,0	4,3	5,0	3,7

CSI - _{9,i}		4,3	4,0	3,7	4,0	4,7	4,0	4,7	5,0		
CSI - _{10,i}		3,7	4,3	3,7	4,3	5,0	4,7	4,0	4,3		
CSI - _{11,i}		3,7	4,3	5,0	4,3	5,0	3,3	4,7	5,0		
CSI - _{12,i}		4,3	5,0	5,0	4,3	4,3	5,0	5,0	3,7		
CSI - _{13,i}		4,3	4,7	3,7	4,7	5,0	4,0	4,0	5,0		
CSI - _{14,i}		4,7	4,0	5,0	5,0	5,0	5,0	4,0	5,0		
CSI - _{15,i}		5,0	4,7	5,0	4,3	4,0	5,0	4,0	4,7		
CSI - _{16,i}		5,0	4,7	4,7	5,0	4,7	5,0	5,0	5,0		
CSI - _{17,i}		4,0	4,3	4,3	4,7	3,7	5,0	4,7	5,0		
CSI - _{18,i}		4,3	4,7	4,7	4,7	4,3	4,3	4,3	5,0		
CSI - _{19,i}		4,3	4,7	4,3	4,7	5,0	5,0	5,0	4,7		
CSI - _{20,i}		4,7	4,3	5,0	5,0	3,7	5,0	4,7	4,7		
CSI- <i>i</i>	$\bar{X}_i$	4,433	4,417	4,533	4,583	4,567	4,483	4,400	4,668	$\bar{\bar{X}}$	4,511
R- <i>i</i>	$\bar{R}_i$	1,3333	1,3333	1,3333	1,0000	1,3333	1,6667	1,6667	1,3333	$\bar{\bar{R}}$	1,375

Table 2. Patterns of control limits. Source: PN-ISO 8258+AC

Chart	Central line – CL	Control lines : LCL and UCL
$\bar{X}$	$\bar{\bar{X}}$	$\bar{\bar{X}} - A_2\bar{R}; \bar{\bar{X}} + A_2\bar{R}$
R	$\bar{\bar{R}}$	$D_3\bar{\bar{R}}; D_4\bar{\bar{R}}$

Calculations for the chart R (N=20):

$$CL=(\bar{\bar{R}})=1,375$$

$$UCL=(D_4*\bar{\bar{R}})=2,18; (D_4=1,585)$$

$$LCL=(D_3*\bar{\bar{R}})=0,57; (D_3=0,415)$$

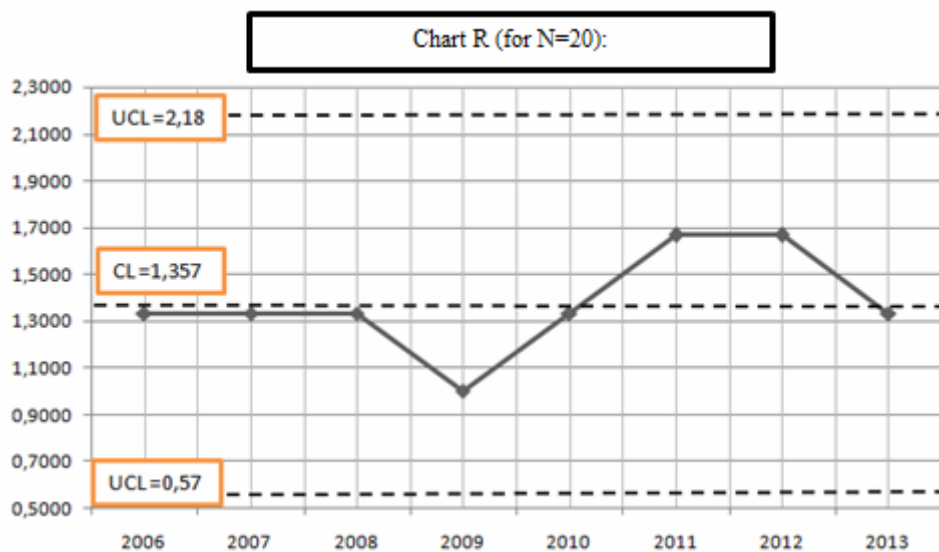


Fig.3. Chart R for auditedcompany. Source: Own

Calculations for chart X (for N=20)

$$CL=(\bar{\bar{X}})= 4,511$$

$$UCL=(\bar{\bar{X}}+A_2*\bar{R}) = 4,76; \quad (A_2=0,18)$$

$$LCL=(\bar{\bar{X}}-A_2*\bar{R}) = 4,26$$

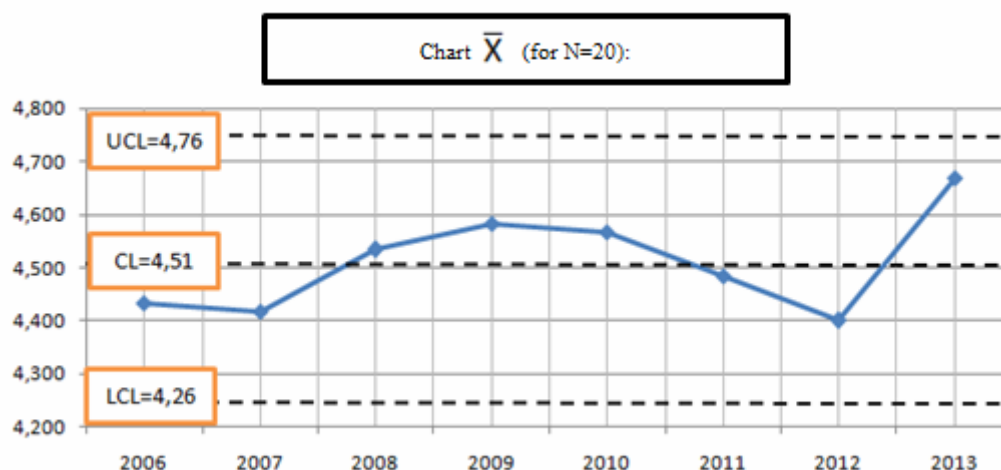


Fig.4.Chart X for auditedcompany. Source: Own.

Summary

Adopted assumptions for the construction of control cards and used the actual results of customer satisfaction surveys show that X-R charts is a useful tool for the analysis of customer satisfaction. The use of selected control charts will monitor the levels of customer satisfaction and proactively respond to the worrying indications of deterioration in the quality of services.

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SCHNAKENBERG REACTION-DIFFUSION SYSTEM WITH SIGNORINI BOUNDARY CONDITION

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Abstract: We consider Schnakenberg reaction-diffusion system of activator-inhibitor in one dimension with Signorini and Neumann boundary conditions. We will show discretization and numerical solution of the system by Newton method.

1 The formulation of problem

We will study differential equations:

$$\begin{aligned}d_1 u'' + g(a - u + u^2 v) &= 0 \\d_2 v'' + g(b - u^2 v) &= 0\end{aligned}$$

with boundary conditions:

$$\begin{aligned}u'(0) &= v'(0) = 0 \quad (\text{Neumann}) \\u'(1) &= 0 \quad (\text{Neumann}) \\v(1) &\geq 0, \quad v'(1) \geq 0, \quad v(1)v'(1) = 0 \quad (\text{unilateral Signorini})\end{aligned}$$

where $d_1, d_2, g, a, b \in \mathbb{R}$ are constant and $x \in \langle 0, 1 \rangle$. The function u is called activator and v is called inhibitor.

The unilateral Signorini condition can be reformulated:

$$v(1) - \max\{0, v(1) - \varrho v'(1)\} = 0$$

where $\varrho > 0$ is constant.

If following condition is satisfied

$$v(1) - \varrho v'(1) \geq 0$$

then unilateral Signorini condition has form:

$$\begin{aligned} v(1) - \max\{0, v(1) - \varrho v'(1)\} &= 0 \\ v(1) - (v(1) - \varrho v'(1)) &= 0 \\ v'(1) &= 0 \quad (\text{Neumann}) \end{aligned}$$

else

$$\begin{aligned} v(1) - \max\{0, v(1) - \varrho v'(1)\} &= 0 \\ v(1) - 0 &= 0 \\ v(1) &= 0 \quad (\text{Dirichlet}) \end{aligned}$$

2 Discretization

The discrete form of differential equations and Neumann boundary conditions will be obtain by using formulas of numerical differentiation, see [2].

The discretization of unilateral Signorini condition:

$$v_n - \max\left\{0, v_n - \varrho \frac{v_n - v_{n-1}}{h}\right\} = 0$$

If

$$v_n - \varrho \frac{v_n - v_{n-1}}{h} \geq 0$$

then

$$\frac{v_n - v_{n-1}}{h} = 0 \quad (\text{Neumann})$$

else

$$v_n = 0 \quad (\text{Dirichlet})$$

3 Newton method

Lets denote $\vec{y} = (\vec{u}, \vec{v})$.

The differential equations with boundary conditions lead to equations:

$$F(\vec{y}) = \vec{0}$$

where $F : \mathbb{R}^{2(n+1)} \rightarrow \mathbb{R}^{2(n+1)}$.

The initial approximation $\vec{y}^{(0)}$ is choosen. Next approximation $\vec{y}^{(k+1)}$ is computed:

$$F^0(\vec{y}^{(k)}) \vec{y}^{(k+1)} = F^0(\vec{y}^{(k)}) \vec{y}^{(k)} - F(\vec{y}^{(k)})$$

where $F^0(\vec{y})$ is Jacobian of $F(\vec{y})$.

The method stops when the condition is satisfied:

$$\frac{\|F(\vec{y}^{(k)})\|}{\|\vec{y}^{(k)}\|} < \varepsilon$$

where ε is a given accuracy.

4 The example

We will show the example for parameters: $d_1 = 0.075$, $d_2 = 2.7359$, and $g = 10$, $a = 0.1$, $b = 0.9$, $\varrho = 1$.

The system has one trivial solution:

$$u = 1, \quad v = 0.9$$

and nontrivial solutions. Following four nontrivial solutions was found by using different initial approximation in Newton method.

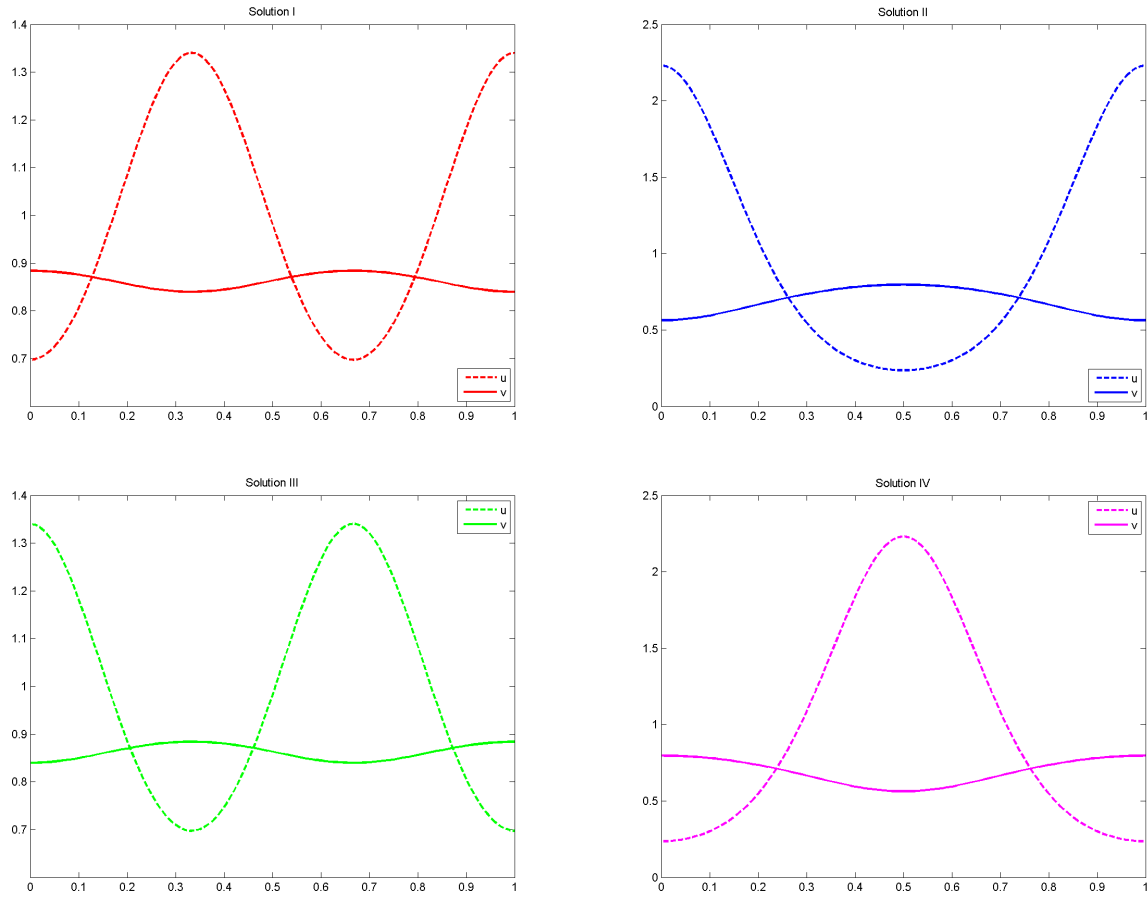


Figure 1: The nontrivial solutions

5 Bifurcation

We will show the example for bifurcation parameter d_2 . Values of d_1 , g , a , b , ϱ are the same as in previous example.

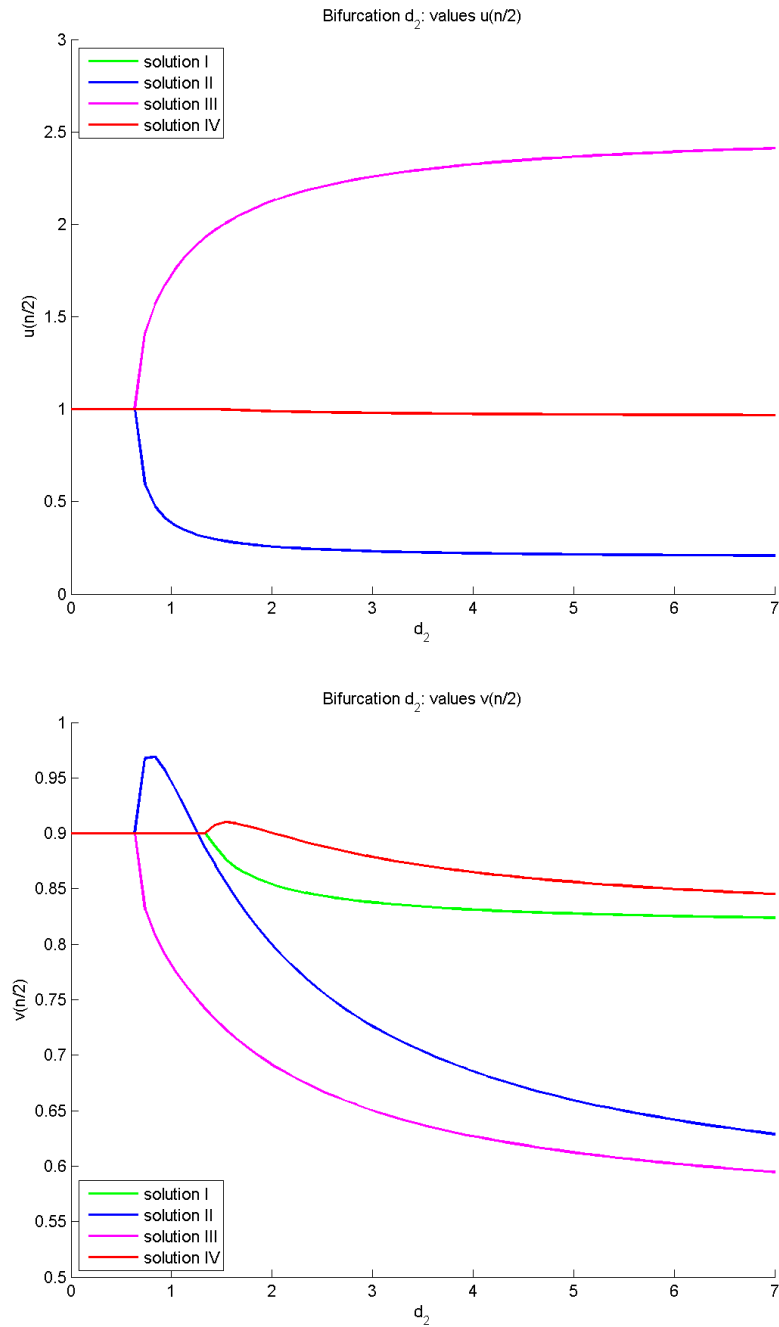


Figure 2: Bifurcation of d_2

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APPLICATION OF RELIABILITY THEORY TO THE SYSTEM MAN-MACHINE IN PROCESS OF WORK

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Abstrakt: This article presents an attempt of synthesis methods for human reliability assessment and methods for the assessment of the reliability of a technical object in the man-machine system. The paper presents indicators of human reliability proposed by psychologists as measures of the probability of human error-free operation. Also presented a proposal to take into account indicators of human reliability in determining the reliability of the human-machine system using mathematical description used in the theory of technical reliability.

Key words: human reliability, technical reliability, reliability indicators

1. Introduction

Reliability theory is a discipline where the language is based on two areas of Mathematics: probability theory and mathematical statistics. A common feature of human reliability and reliability of other objects is the need to define certain requirements (expectations). An important difference between these two concepts is the interactivity between the sender and the recipient, i.e. the requirements. the ability to exchange information about both the object itself (the man) and the environment.

A study on human reliability was what most often use psychological theories about the less general, referring to behaviors that qualify as bugs, stumbling and failure. Such specific theories are: the theory of the tendency to accidents and errors. The basis of these two statistical theory is the analysis of events and the basic concept of this analysis is the likelihood of adverse events (injuries and errors). Psychological theory of reliability is derived directly from these two strands of research. Earlier, regarding the tendency to accidents and later, concerning the analysis of the errors and their causes as a function of poor construction and equipment. These two different strands of research, in which otherwise are considered causes of adverse events, were joined at a time when it was considered that none of these currents does not lead to final settlements. The

subject of the analysis, it was decided to make the man-technical object as a whole [1].

The reliability of the technical object is a property determined by the probability of fulfilment of requirements. A. Kiliński [2] calls for consideration of the other features of the site which are close to the characteristics of reliability. It is the durability of the object, that is, its ability to maintain a significant ownership. This characteristic is relative, depends on the terms of use of the object, operation, storage, transportation, maintenance and repair. Sustainability can be measured by the length of time during which it will retain certain properties within specific limits their changes under certain conditions of existence. So understood durability is called sometimes life, sometimes use. Reliability can also be understood as the probability of success, i.e.. meet the requirements specified by the object or as the probability that during the $(0, t)$ changes to object ownership does not exceed specified limits, under certain conditions, the existence of the object [2].

2. Description of the human reliability.

In order to determine the probability that it is necessary to know about errors (their number and types). Human error is said to be in two contexts: when looking for causes of accidents, and in the context of the analysis of human activities in technical systems. In order to determine the causes of accidents errors shall be treated in a different way than in the analysis of errors in connection with the necessity of proper allocation of functions between man and machine and design technical systems. In the first case, it is parsed as a cause of the accident, the second as a result of the improper design of the devices with which it works man [3,10].

Among the many proposed by researchers classification mistakes made by man, the systematic errors and accidental seems most appropriate. Probabilistic approach (a description of the probability of error) is typical for the research of human reliability in connection with risk analysis, and human error is treated as a fault or disturbance in the functioning of the machine. Number of errors of man and machine is the starting point for calculating the reliability coefficient of the whole layout. For the so-called. operator error can distinguish types of errors as follows: lack of proper steps when the signal Act of late, the Act made in good time, but not completed or done instead of another act of redundant, resulting from the chaotic activity, Act premature, spontaneous activity, without a signal from the outside, rather than abstain from activity to premature action, action offsets to the desired or inaccurate [4].

Error data collection is the foundation for building a human reliability measure. The aim of this action is to predict the reliability of the system, in which a man has a specific role. Among the long list of error-prone mechanisms at work a man deserves special attention mechanism for decision making. According to the authors of the publication [5] the creators of the theory of signal detection, the behavior of the operator can be described mathematically. In theory the operator commits the two main types of errors: undetected (non-detection object), false alarm (signal detection, who does not appear). A man is treated here as a dysfunctional detector signals. It builds in his

nervous system two timetables activity. One of them refers to a probability of acceptance of the object, the other to the likelihood of rejection of the object. The degree of separation of these two schedules is a measure of the sensitivity of the perceptual junior operator in anticipation of the signals or inspecting. The sensitivity of the controller can be described mathematically [6].

3. Indicators of human reliability.

In the study of human reliability tried to use the indicators developed in the field of technical sciences based on probability theory. Their use requires the identification of such size characterizing the work of the man as the average time between two shortcomings at work (errors), the overall number of mistakes made in a given stretch of time, the percentage of correctly performed the tasks in a given stretch of time. According to B. F. Łomowa [7] the main quantitative indicators of human reliability are: rate of accuracy, preparedness, restitution and news (the adequacy of action time operator). The accuracy rate is the probability of a correct work of the operator, which can be determined with respect to the operations, and to the entire algorithm steps. This indicator is expressed by the formula:

$$P_j = \frac{N_j - n_j}{N_j},$$

where: P_j -probability of error-free operation, N_j -the total number of operations performed, n_j is the number of mistakes. The restitution index is associated with the possibility of self-control and making amendments to the Act in question. This indicator expresses the probability improve committed by operator errors :

$$P_{pop} = P_k \cdot P_{wyk} \cdot P_p,$$

where: P_{pop} -probability of inclusion to the operator to act when you see the error in order to correct, P_k is the probability of sending a signal by controlling mechanism, P_{wyk} is the probability of being detected by the operator input device controller, P_p - probability of correcting erroneous operation repeated their performance.

The news index (the adequacy of action time operator) shall be due to the fact that the correct but made in the wrong time action does not lead to the goal. Often to perform specific operations shall be specific. His crossing is considered to be an error. Indicator of the adequacy of the temporary operator is likely to carry out the tasks at the time $\tau < t_l$. It is expressed by the formula:

$$P_a = P\{\tau < t_l\} = \int_0^{t_l} f(\tau) d\tau,$$

where: P_a -probability news, $f(\tau)$ is a temporary feature of the task by the operator. This formula is applicable if the t_l is constant. In some cases, shall also be so called. standby indicator the operator as the probability of inclusion to the operator to work at any time. It is expressed by the formula:

$$K = 1 - \frac{T_0}{T},$$

where: K-ready indicator, T_0 -off time like operator with work, his absence for the period in question at the workplace, T is the total time the operator [1].

4. Reliability of man-machine system.

A kind of a man as a part of the subsystem is, on the one hand, the possibility of making mistakes, on the other hand, the ability to learn and to practice. The machine as part of a subsystem is, on the one hand, the ability to correct, repetitively work, on the other hand, the inability to self-steering operations of the machine (a lack of ability to learn). Characteristics of man and machine as a different system elements affect the reliability of the system. Using the above mentioned indicators of reliability in relation to man, keep in mind that each feature is not constant, but varies over time and is subject to changes in the environment and in the human being. In determining the reliability of a man, in any case, you must choose a specific factor in the most characteristic of the type of operations and has the greatest impact on the achievement of the aim pursued in front of the operator. With each of these factors is related to the human-machine system state and for each of these states, you must specify the specific importance of the reliability index of the operator. If we accept that the states may be: $i_1, i_2, \dots, i_n$, in each of them the reliability of the operator accepts the states: $P_{op1}, P_{op2}, P_{op3}, \dots, P_{opn}$. For example, in a time interval $0 - t_1, t_2 - t_3, t_4 - t_5$ (Fig. 1.) the man-technical object is able to $i = 4$. This condition is subject to factors affecting the reliability of the operator's work, which at that time equal to P_{op4} . With the operation of other factors the layout is located in other States, and each of them corresponds to a specified value the reliability of the operator. Taking into account the adopted earlier assumptions, the value of the probability of faultless work the operator is given by:

$$P_{op} = \sum_{i=1}^n P_i \cdot P_{op \mid i}.$$

where: P_i is the probability that the i-this state of the $P_{op \mid i}$, and the probability of

faultless work and operator-this condition, n is the number of system states concerned

[1]. The probability of $P_{op \mid i}$ and (conditional) on the work of the operator can be

obtained experimentally [1,7]. While the likelihood of P_i , in many cases, failure to specify using the methods of mathematical theory of mass service. In theory, this

behavior will be examined the technical object in operation, that is, during the period of work (the so-called stability of T). The accuracy of the statistical behavior of each of the elements of the technical layout describes the theoretical distribution random variable T, which establishes the link between the possible values of the variable T and the corresponding probability P. The reliability of the technical object is defined as the chance that it will work well for a specified period of time or as uptime at a specified probability. The measure of reliability is, therefore, the probability of damage-free P_i work the specified object, the operating conditions and adopted at the time of use. Reliability may be designated for items on a different level of complexity: parts, sub-assemblies, assemblies, mechanisms, systems or devices. At each of these levels can be applied the same reliability characteristics, related to each other certain dependencies (the structure of serial, parallel, in serial-parallel and parallel-to-serial) [8,9].

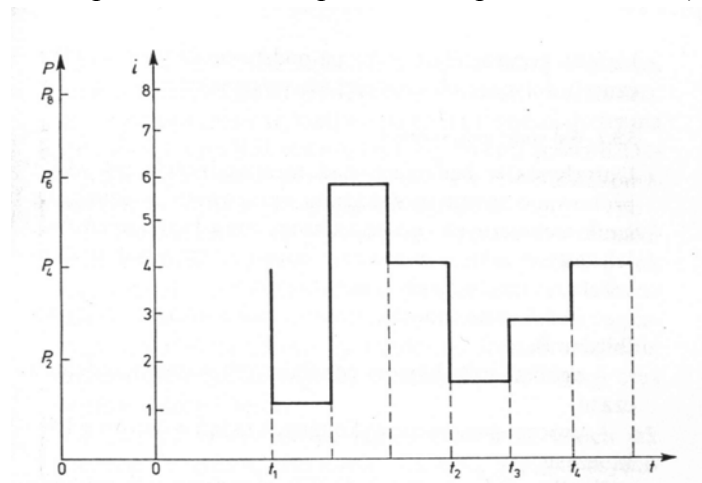


Fig. 1. Dynamics of changes of states-man technical object- example graph [7].

5. Summary

The costs associated with the man because as an integral link with creations techniques may prove to be so high that not enough funds to cover them. While the consequences of human errors should be sufficient justification for the need for their research. A man must be considered as a very specific link system, against which the technical reliability theory is limited and rough use. In determining the reliability of the entire system man-technical object certain conditions must be met. First of all indicators for all of the cells must be the same type. Can be used for the calculation of the mathematical apparatus used in reliability theory with a clear indication of the size of the man among the remaining links in the layout.

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PRAVDĚPODOBNOST A HAZARDNÍ HRY

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Abstrakt: text v jazyce příspěvku, pokud není článek v angličtině

Abstract: text v angličtině

Úvod

Hazardní hra je definována jako hra šancí a náhody, při které je poměr vyplácené částky (výhry) nižší, než který by odpovídal skutečnému, matematicky vypočtenému sázkovému poměru na základě teorie pravděpodobnosti. Hazardní hry zřejmě provázejí lidstvo od počátku jeho existence. Vždyť první hrací kostky byly vyřezány z kostí zvířat a archeologové je datují do doby 40 000 let před naším letopočtem. Hrací kostky ze slonoviny a další artefakty hazardních her byly objeveny v Egyptě, Indii, Japonsku a Číně, pocházejí z období 1 500 let před naším letopočtem. Ačkoliv byl hazard v Řecku zakázaný, tak jak o tom svědčí archeologický průzkum, řeční vojáci hráli v kostky. Z té doby zřejmě pochází tvrzení o šťastném čísle sedm. Součet sedm totiž padá nejčastěji při hodu dvěma kostkami, neboť součet sedm nastává v těchto případech: (1,6), (2,5), (3,4), (4,3), (5,2) a (6,1), kdežto například součet tři můžeme modelovat jen dvěma uspořádanými dvojicemi (1,2) a (2,1). Císař Claudius zakázal v Římě hazard, ale sám si nechal přebudovat svůj kočár tak, aby se mu tam vešel pěkný hrací stůl, v dnešní terminologii bychom ho mohli označit za „gemblera“. Jindřich VIII v 15. století zakázal hazardní hry. Pro ilustraci uveďme ještě pár historických střípků: Ruleta se začala hrát v Paříži už v roce 1796, v roce 1830 už existovalo 420 různých státních loterií ve Spojených státech, Hazard byl považován v Nevadě za těžký zločin v letech 1910 – 1929, v roce 1930 byl hazard v Nevadě legalizován a od té doby na něm herny neustále vydělávají. Díky teorii pravděpodobnosti se dají vybrat jisté prostředky a a také určovat výšky poplatků, aby salóny hazardních her přinášely svým majitelům zisk.

Základní pojmy

Šance (anglicky *Odds*) je pouze jiným vyjádřením pravděpodobnosti. Pokud označíme písmenem P pravděpodobnost, s níž nastane určitý náhodný jev, pak šanci vyjádříme následovně:

$$\check{s} = P : (1 - P) = 1 : x$$

Převrácena hodnota šance se nazývá **skutečný matematický poměr**

$$smp = (1 - P) : P = x : 1.$$

Náhodnou veličinou rozumíme zobrazení z množiny náhodných jevů do jisté číselné množiny. Nabývá-li náhodná veličina spočetně mnoha hodnot, pak ji nazýváme diskrétní náhodnou veličinou, v opačném případě se jedná o spojitou náhodnou veličinu. Počet ok, která padnou při hodu dvěma kostkami, je náhodná diskrétní veličina S nabývající hodnoty 2, 3, 4, . . . 12. Jestliže náhodná veličina X nabývá hodnoty $x_1, x_2, \dots, x_i$, potom součet

$$x_1 \cdot P(X=x_1) + x_2 \cdot P(X=x_2) + \dots + x_i \cdot P(X=x_i)$$

se nazývá **průměrná hodnota**, nebo **očekávaná hodnota**, nebo **střední hodnota**, nebo **matematická naděje náhodné veličiny X** a označujeme ji $E(X)$.

Střední hodnotu výše zmíněné náhodné veličiny S určíme tedy takto:

$$E(S) = 2 \cdot (1/36) + 3 \cdot (2/36) + 4 \cdot (3/36) + 5 \cdot (4/36) + 6 \cdot (5/36) + 7 \cdot (6/36) + 8 \cdot (5/36) + 9 \cdot (4/36) + 10 \cdot (3/36) + 11 \cdot (2/36) + 12 \cdot (1/36) = 7$$

Základní pojmy si vysvětleme na „Minisportce“ a ruletě, jak se hraje v Monte Carlu. Lidé se často domnívají, že užitím počtu pravděpodobnosti mohou najít „recept“, jak vyhrát ve Sportce. Ukážeme, že takové recepty neexistují a ani nemohou existovat žádné racionální postupy. Šance vyhrát ve Sportce je pro všechny stejná. Ovšem existují i takové hry, ve kterých matematika napomáhá myslícímu hráči ovlivnit šanci na vítězství.

Kupón Minisportky obsahuje pouze pět čísel 1, 2, 3, 4 a 5. Hráč na něm zakroužkuje pouze dvě čísla. V případě, že neuhodl ani jedno tažené číslo, nedostane nic, za jedno uhodnuté číslo vyhrává 1 Kč, za dvě 10 Kč. Jaký musí být minimální poplatek v této hře, aby „bankéř neprodělal? Počet uhodnutých čísel určuje náhodnou veličinu U , jejíž střední hodnota musí být rovna minimálnímu poplatku. Vypočteme s jakou pravděpodobností nabývá náhodná veličina U , která má hypergeometrické rozdělení pravděpodobnosti, svých hodnot 0, 1, 10 a určíme její střední hodnotu

$$P(U=0) = \frac{C(2,3)}{C(2,5)} = \frac{\binom{3}{2}}{\binom{5}{2}} = 0,3$$

$$P(U=1) = \frac{C(1,2) \cdot C(1,3)}{C(2,5)} = 0,6$$

$$P(U=2) = \frac{C(2,2)}{C(2,5)} = 0,1$$

i	1	2	3
u_i	0	1	10
$P(U=u_i)$	0,3	0,6	0,1

$$E(U) = 0 \cdot 0,3 + 1 \cdot 0,6 + 10 \cdot 0,1 = 1,6.$$

Aby bankéř neprodělal, ale ani nevydělal, musel by za jeden vyplněný tiket vybírat 1,60 Kč. Pokud bude za jednu hru vybírat 2 Kč, bude mít průměrně z každé hry zisk 0,40 Kč.

V Monte Carlu se ruleta skládá ze 37 shodných výšečí, které jsou označeny čísly 0, 1, 2, ..., 36. Sáží-li hráč 1 jednotku na určité číslo kromě nuly a kulička na něm zůstane stát, činí jeho výhra 36 jednotek, v opačném případě hráč 1 jednotku prohrává. Kasino se honosí tím, že jde o rovné sázky a uvádí, že výplatní poměr kasina je 1 : 1. Pravděpodobnost výhry hráče je však $P(V) = 18:37 = 48,65\%$, a nikoliv 50%. Šance hráče na výhru je

$$\check{s} = (18:37) : (19:37) = 18:19,$$

z čehož je vidět, že kasino je oproti hráči ve výhodě. Skutečný matematický poměr je

$$smp = 19:18.$$

Střední hodnota výhry hráče je

$$E(V_H) = 35 \cdot (1:37) - 1 \cdot (36:37) = -0,027.$$

Při mnohokrát opakovaném sázení ztrácí hráč při každé hře průměrně 2,7% své sázky, což je zisk kasina, který může být vyjádřen jako střední hodnota výhry kasina:

$$z = E(V_K) = 1 \cdot (36:37) - 35 \cdot (1:37) = 0,027.$$

Užitím Markovových řetězců lze dokázat, jak uvádí Kemeny (1972), že ať má hráč jakoukoliv sumu a sází v kasinu, tak nutně bude ruinován, což znamená, že všechny své peníze prohraje, byť by jich měl jakékoliv množství. Aby ztráty nebyly zpočátku tak velké a slast ze hry byla co nejdéle prodlužována, tak byly vymyšleny různé systémy, o kterých se zmíníme v následující kapitole.

Systémy oddalující ruinování hráče

1. Systém využívající Fibocchiho posloupnost 1,1,2,3,5,8,13,...

- Při sázení se využívají rovné sázky, podobně jako u většiny nejznámějších systémů. Hráči obvykle sázejí na svou oblíbenou barvu (červenou nebo černou), stejně dobře lze ale sázet i na ostatní rovné sázky malá-velká a sudá-lichá čísla, které mají výplatní poměr 1:1.
- Začíná se vsazením 1 jednotky. Pokud se vyhraje, výhra se uloží a začíná se znovu s jednou jednotkou. Pokud se prohraje, vsadí se další sázka v počtu jednotek dle Fibonacciho posloupnosti, tedy ve druhém kole 1 jednotku, ve třetím 2 jednotky, ve čtvrtém 3 jednotky, v pátém 5 jednotek atd. Vždy, když se vyhraje, škrtnou se poslední dvě čísla z řady a pokračuje se v sázení. Princip sázení nejlépe ilustruje níže uvedená tabulka.

Kolo (spin)	Sázka v jednotkách	Výsledek sázky	Sázka dle Fibonacciho řady	Bilance (zisk/ztráta)	Poznámka
1.	1 jednotka	prohra	1	-1	další v řadě je 1
2.	1 jednotka	prohra	1 1	-2	další je 2

3.	2 jednotky	prohra	1 1 2	-4	další je 3
4.	3 jednotky	prohra	1 1 2 3	-7	další je 5
5.	5 jednotek	výhra	1 1 X X	-2	škrtneme dvě poslední čísla (2, 3)
6.	2 jednotky	prohra	1 1 2	-4	další je 3
7.	3 jednotky	výhra	1 X X	-1	škrtneme 2 poslední
8.	1 jednotka	výhra	1	0	další je 1
9.	1 jednotka	výhra	konec	+1	série končí, výsledkem je +1 jednotka

2. Labouchere system

- Podstata ruletního systému Labouchere spočívá v rušení sázek.
- Volí se libovolná číselná posloupnost, například to může být šest čísel 1, 2, 3, 4, 5, 6 (stejně dobře ale poslouží jakákoliv jiná posloupnost se sudým počtem čísel, například 5, 5, 5, 5 nebo 50, 100, 150, 200), a vsadí se součet prvního a posledního čísla čili $1 + 6 = 7$ jednotek. Pokud se vyhraje, zařadí se výhra, resp. číslo 7, k sérii, která nyní bude následující 1, 2, 3, 4, 5, 6, 7. Opět se vsadí součet prvního a posledního čísla, tj. $1 + 7 = 8$. Takto se postupuje dále a přidává se výhra po každém vyhraném kole.
- Pokud se spin prohraje, tak naopak první a poslední číslo řady se škrtnou. Stalo-li by se tak v prvním kole, škrtnou čísla 1 a 6. V řadě zůstanou čísla 2, 3, 4, 5 a ve druhém kole se vsadí opět součet prvního a posledního čísla, tentokrát tedy $2 + 5 = 7$ jednotek. Pokud by se i nyní prohrálo, vsadí se zbývajících $3 + 4 = 7$ jednotek. Pokud se nedaří, prohraje se celý kapitál, ale celková ztráta není nikterak zlá: $1 + 2 + 3 + 4 + 5 + 6 = 21$ jednotek.

Při alespoň trochu příznivé sérii lze, při užití systému Labouchere, přičítat výhry a odečítat ztráty. Může si předem určit, při jakém zisku nebo po kolika vyhraných hodech se série zastaví a začne se systém od začátku

3. Systém d'Alembert

Použití tohto systému dokumentuje následující tabulka.

Sázka	Výsledek sázky	Zisk/ztráta jednotlivé sázky	Kumulovaný zisk/ztráta za sérii	Příští sázka (prohra +1, výhra -

				1)
1 jednotka	prohra	-1	-1	+1
2 jednotky	prohra	-2	-3	+1
3 jednotky	prohra	-3	-6	+1
4 jednotky	výhra	8	+2	-1
3 jednotky	výhra	6	+8	-1
2 jednotky	prohra	-2	+6	+1
3 jednotky	výhra	6	+12	-1
2 jednotky	výhra	4	+16	-1
1 jednotka	výhra	2	+18	Série končí, začíná se opět vsazením jedné jednotky

A co Sportka, je to hazardní hra?

Sportka je u nás velmi známá a oblíbená hra, uveďme jen pár postřehů, odpověď na výše položenou otázku ponechejme na čtenáři. Výpočet pravděpodobnosti výher v jednotlivých pořadích je možné najít v jakékoliv středoškolské učebnici, jedná se o hypergeometrické rozložení pravděpodobnosti.

- Vyplácí se procenta jen z poloviny celkově vsazené částky
- 1. pořadí – uhodnutí všech 6 tažených čísel, výhra je 34 % z poloviny všech vsazených peněz a Jackpot, vyhrává se s pravděpodobností 0,00000715%
- 2. pořadí (5 + 1 číslo – dodatkové) – 40 %, 0,0000429%
- 3. pořadí (5 čísel) – 12 %, 0,00184%
- 4. pořadí (4 čísla) – 9 % , 0,097%
- 5. pořadí (3 čísla) – 5 % 1,765%

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POROVNÁNÍ NUMERICKÝCH METOD V MODELOVÁNÍ TERNÁRNÍCH SYSTÉMŮ

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Abstrakt: Ternární diagramy se modelují pomocí výpočetní techniky. Tento příspěvek porovnává dvě metody modelování a popisuje modelování využívající B-splajnové plochy včetně tvorby výpočetního programu. Výpočetní program byl vytvořen v systému MatLab. Příspěvek rovněž zahrnuje příklady použití tohoto programu.

Abstract: Ternary diagrams are modeled by computer technology. This study compares two methods of modelling and describes modelling using B-spline surfaces and creating calculation program. The calculation program was elaborated in the system MatLab. The study also includes examples of using the program.

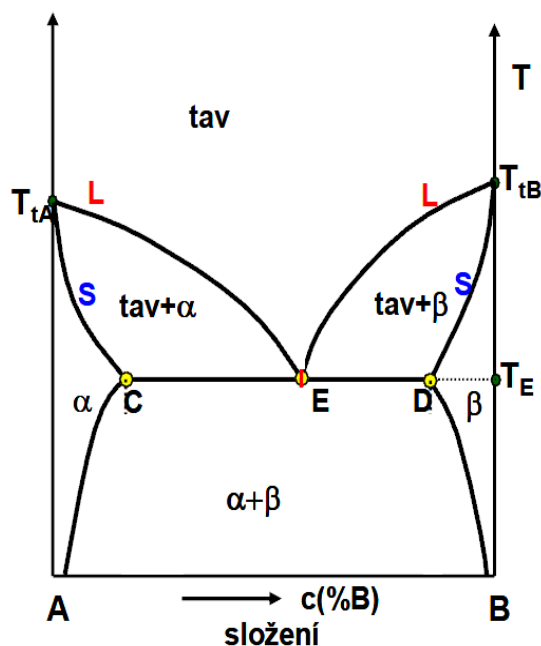
1. Úvod

Jako ternární systém se běžně označuje sloučenina resp. slitina tří prvků. Graficky se pak takový systém obvykle znázorňuje ve formě tzv. fázového diagramu. Tento diagram je velmi důležitý kvůli své vypovídající hodnotě o vlastnostech slitiny s konkrétním chemickým složením. Z vlastností, zjišťovaných z diagramů, jsou nejdůležitější průběh ochlazování slitiny, hodnoty rozdělovacích koeficientů a počet jednotlivých fází, kterými při ochlazování slitina prochází. [1]

Bohužel získání hodnověrného fázového diagramu není triviální úkol. Pro konkrétní bod v diagramu (konkrétní slitinu s pevně daným obsahem jednotlivých prvků) se dá průběh zjistit empiricky. Takových pokusů lze provést několik. To může dobře naznačit průběh jednotlivých ploch v diagramu. Pro jejich přesnou polohu a tvar je ale nejvíce vhodná pomoc výpočetní techniky.

2. Binární fázový diagram

Pro lepší pochopení ternárního fázového diagramu, který je prostorový, se jeho zákonitosti a tvorba vysvětlují na binárním diagramu, který lze zobrazit pouze v ploše (Obr. 1).



Obr. 1 Binární diagram

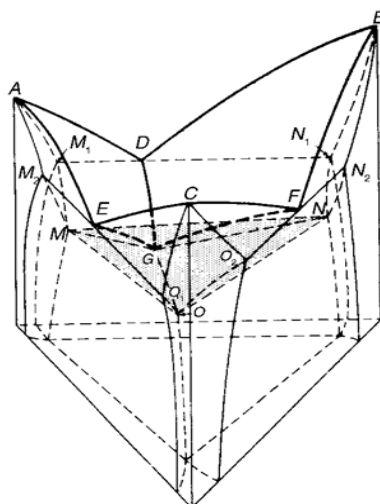
Binární diagram dává do souvislosti složení slitiny ze dvou prvků (většinou v atomárních procentech) a teplotu (většinou ve stupních Celsia). Koncentrace je od 0% do 100%, a tedy svislé osy představují čisté složky. Teplotní rozsah se nastavuje tak, aby pokryl celý průběh ochlazování slitiny – tedy od kapalného stavu (tavenina) po pevný stav (pokojová teplota). V diagramu se pak zkoumají především teplotní a koncentrační rozsahy jednotlivých fází (znázorněny uzavřenými oblastmi). Lze tedy říct, že vlastnosti a strukturu finální slitiny neovlivňuje jen procentuální složení, ale i fáze, kterými prochází při ochlazování a jak dlouho se v té které fázi zdrží. Každý diagram se konstruuje pro určité, předem dané a neměnné podmínky, například tlak.

3. Ternární fázový diagram

Přidáním třetího prvku do slitiny vzniká ternární systém, který se graficky znázorňuje prostorovým ternárním fázovým diagramem (Obr. 2). Koncentrační vodorovné ose odpovídá koncentrační rovnostranný trojúhelník, kde v každém vrcholu je jeden čistý prvek. To vytváří podstavu trojbokého pravidelného hranolu, jehož stěny jsou tvořeny jednotlivými binárními diagramy (A-B, B-C, C-A). Na svislých osách je opět teplota. Jednotlivé fázové oblasti jsou v diagramu rozděleny plochami.

Studium a práce s takovým diagramem je značně obtížná a proto se již v dnešní době nelze obejít bez výpočetní techniky. Při modelaci ploch ternárního diagramu hrozí nepřesnosti dvou typů. Jeden v souvislosti se vstupními daty, kdy se čerpá buď z empiricky získaných dat nebo ze starších izotermických a polytermických řezů. Tato data mohou být nepřesná a je proto vhodné čerpat z více zdrojů.

Druhý typ nepřesností vzniká nesprávnou volbou modelačního postupu. Musí se vybrat takový postup, který nejvíce odpovídá skutečnosti.



Obr. 2 Ternární diagram

4. Aproximace pomocí kvadratických ploch

Jedna z využitelných aproximací ploch ternárního diagramu je jejich namodelování pomocí kvadratických ploch. Plochy lze popsat polynomem druhého stupně (1).

$$t(x_B, x_C) = k_1 x_B^2 + k_2 x_C^2 + k_3 x_B x_C + k_4 x_B + k_5 x_C + t_m^A \quad (1)$$

Kde k_1 až k_5 jsou koeficienty regrese ($k_6 = t_m^A$), x_B a x_C jsou koncentrace příměsí B a C a t_m^A je teplota tání prvku A.

Základním nedostatkem je možnost využití pouze u ploch s nepříliš měnícím se charakterem. To splňují jen nízkolegované slitiny a jednoduché ternární systémy. Kvadratické plochy nevystihují přesně změny v monotónnosti ani změny mezi konvexností a konkávností. Způsobují to omezené možnosti tvarování, kdy se plocha tvaruje pouze pomocí hranic a nezbyvá žádný stupeň volnosti pro střed.

5. Aproximace pomocí B-splajnových ploch

Vhodným řešením se ukázaly B-splajnové plochy, které mají mnoho vlastností, přínosných pro využití v metalurgii – např. interpolace krajních bodů a aproximace bodů uprostřed, dále v místech spojů dokonalá hladkost a v neposlední řadě způsob tvarování plochy pomocí řídicího polygonu dává schopnost měnit finální tvar plochy i uživateli bez hlubší matematické erudice.

B-splajnová plocha je po částech polynomická plocha (každá její část má svůj vlastní konstrukční předpis). Finální tvar plochy ovlivňují jen tři tvarovací nástroje. Je to stupeň plochy, síť řídicích bodů a uzlové vektory, nad kterými jsou definovány báze funkce. Tyto nástroje pak dohromady tvoří B-splajnovou plochu, která je zadána parametricky rovnicí (2).

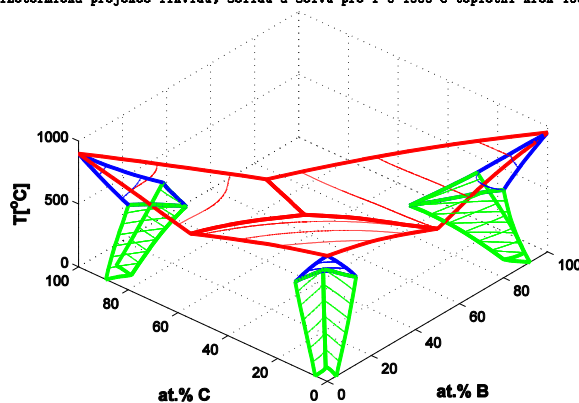
$$S(u, v) = \sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u) N_{j,p}(v) \mathbf{P}_{ij} \Leftrightarrow \begin{cases} x = \sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u) N_{j,p}(v) x_{ij} \\ y = \sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u) N_{j,p}(v) y_{ij} \\ z = \sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u) N_{j,p}(v) z_{ij} \end{cases} \quad (2)$$

Kde $N(u)$ a $N(v)$ jsou báze funkce, p je stupeň plochy a $\mathbf{P}$ je řídicí polygon. [2]

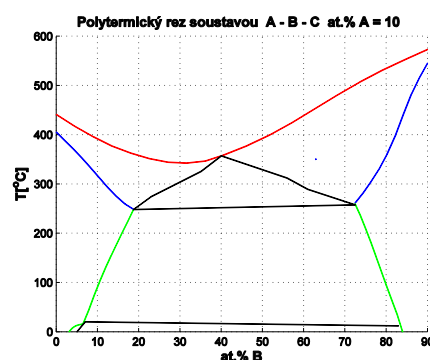
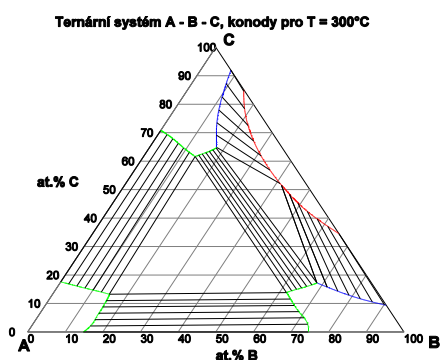
Na základě této numerické metody byl naprogramován výpočetní program v prostředí MatLab „Software pro modelování ternárních systémů slitin“ (evidenční číslo 007/27-03-2012_SW) (Obr. 3). Po zadání vstupních dat je program schopen vykreslit 3D model ternárního systému (Obr. 4) i jeho izotermické a polytermické řezy (Obr. 5). Tento software je plně schopen nahradit a v mnoha aspektech i překonat program, který využíval kvadratické plochy.

Obr. 3 Uživatelské rozhraní

Izotermická projekce likvidu, solidu a solvu pro $T=0-1000^{\circ}\text{C}$ teplotní krok 100°C



Obr. 4 Ternární fázový diagram



Obr. 5 Izotermický a polytermický řez

Poděkování

Tento příspěvek vznikl v rámci řešení projektu: MŠM 6198910013 „*Procesy přípravy a vlastnosti vysoce čistých a strukturně definovaných speciálních materiálů*“.

Také bych rád poděkoval Mgr. Zuzaně Morávkové, Ph.D., za vysokou úroveň programátorské činnosti a vzhledu do metalurgie při tvorbě výpočetního programu.

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Application of Lyapunov Functions in Selected Problems of the Theory of Nonlinear Oscillations

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Abstract: The problem of regularity of dynamical system was examined. The paper presents results concerning the theory of oscillations in the field of linear extensions of dynamical systems.

It is known that the study of the existence of invariant manifold of dynamic system is related to the existence of the Green's function for the linearized system, i.e. linear extension of dynamic system. Speaking more specifically, if the linear extension have a Green's function, which is regular system, then the invariant manifold for heterogeneous extension of a dynamical system can be written in the explicit integral form (see [1]). This gives us the opportunity to examine the smoothness of invariant manifolds. If the homogeneous linear extension has many different Green's functions, which is a strictly weakly regular system, the study of smoothness of invariant manifolds is rather difficult. Therefore, in the monograph [1], proposed complement the linear extension in the form of a triangle to a regular, giving an Green's function for the initial linear extension as an n -dimensional block in a $2n$ -dimensional Green's function.

In [1] is shown that regularity of a linear extension of a dynamic system having the form of:

$$\begin{cases} \frac{dx}{dt} = a(x), & x \in \mathbb{R}^m, \\ \frac{dy}{dt} = A(x)y, & y \in \mathbb{R}^n, \end{cases} \quad (1)$$

is equivalent to the existence of a certain non-degenerated quadratic form whose derivative, with respect to the tested system, is positive definite.

It is assumed that the Cauchy problem $\frac{d\varphi}{dt} = a(\varphi)$, $\varphi|_{t=0} = \varphi$ has a unique solution, denoted by $\varphi_t(\varphi)$ (see [1], [2]). Since the periodic function $a(\varphi)$ is bounded, the solutions $\varphi_t(\varphi)$ will always be defined on the whole axis $\mathbb{R}$, $\mathbb{R} = (-\infty, \infty)$.

Denoting a fundamental matrix of solutions normalised for $t = \tau$, i.e. $\Omega_\tau^t|_{t=\tau} = I_n$, where I_n is an n -dimensional identity matrix of the linear system

$$\frac{dx}{dt} = A(\varphi_t(\varphi))x, \quad (2)$$

with $\Omega_\tau^t(\varphi)$ ($\Omega_\tau^t(\varphi) = \Omega_\tau^t(\varphi; A)$), the following theorem takes place.

Theorem 1. *Let k constant numbers $\Delta_i \in \mathbb{R}$, $i = \overline{1, k}$, that satisfy the inequality below exist:*

$$k \cdot \sum_{i=1}^k \|\Omega_0^{\Delta_i}(\varphi) \cdot P_i(\varphi)\|_0^2 < 1 \quad \forall \varphi \in T_m, \quad (3)$$

where matrices $P_i(\varphi) \in C(T_m)$, $i = \overline{1, k}$ satisfy the condition:

$$\sum_{i=1}^k P_i(\varphi) \equiv I_n, \quad (4)$$

then the system (1) is regular.

Proof of the theorem and other details will be refereed and discussed in the lecture.

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ZERMELO NAVIGATION PROBLEM: SIMULATION OF A 2-DIMENSIONAL SITUATION

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Abstract: In this article we provide several examples of the generalization of the Zermelo navigation problem from flat to Riemannian spaces. We introduce corresponding deformation force arising due to the wind distribution.

1 Introduction

In [2] E. Zermelo deals with classical control problem. In an unbounded plane where the wind distribution is given by a vector field as a function of position and time, a boat moves with constant velocity relative to the surrounding air mass. There is a question: How must the boat be directed in order to come from a starting point 0 to a destination point D in the shortest time?

Geometrically, the problem is to find the deviation of geodesics under the action of a time-dependent vector field.

Let a pair (M, g) be a Riemannian manifold of dimension m , where

$$g = g_{ij}dx^i \otimes dx^j \quad (1)$$

is a Riemannian metric ((g_{ij}) be non-degenerate, symmetric and positive definite matrix). We shall consider a fibred manifold $\pi : \mathbb{R} \times M \rightarrow \mathbb{R}$, where π is the first canonical projection. On $\mathbb{R} \times M$ we use local coordinates of the form (t, x^a) , $1 \leq a \leq m$, where t is the global coordinate on $\mathbb{R}$ and (x^a) , $1 \leq a \leq m$ are local coordinates on M . If $c : \mathbb{R} \rightarrow M$ is curve in M defined in a neighborhood of $0 \in \mathbb{R}$, we denote γ its graph, i.e.

$$\begin{aligned} \gamma : \mathbb{R} &\rightarrow \mathbb{R} \times M, \\ t &\mapsto (t, c(t)), \end{aligned} \quad (2)$$

which is a section of the fibered manifold π . Any section γ of the fibered manifold π can be prolonged to a section $J^1\gamma$ of the fibered manifold $J^1(\mathbb{R} \times M) \approx \mathbb{R} \times TM$, and $J^2\gamma$ of $\mathbb{R} \times T^2M$. Then $J^1\gamma(t) = (t, c(t), \dot{c}(t))$ and $J^2\gamma(t) = (t, c(t), \dot{c}(t), \ddot{c}(t))$.

Let the wind distribution on M be represented by a time-dependent vector field on M , i.e. by a projectable vector field ξ on $\mathbb{R} \times TM$ of the form

$$\xi = \frac{\partial}{\partial t} + \xi^i(t, x^j) \frac{\partial}{\partial x^i}. \quad (3)$$

To analyze the deformations of geodesics consider the variational problem on $\mathbb{R} \times TM$ defined by the kinetic energy in the form

$$\bar{T} = \frac{1}{2} g_{ij} y^i y^j, \quad (4)$$

where $y^i = \dot{x}^i + \xi^i$.

The Euler-Lagrange equations of the mechanical system (4) can be explicitly expressed in the form

$$F_k - \Gamma_{kij} \dot{x}^i \dot{x}^j - g_{kj} \ddot{x}^j = 0, \quad (5)$$

where and Γ_{ijk} are standard Christoffel symbols of (g_{ij}) ,

$$\Gamma_{ijk} = \frac{1}{2} \left(\frac{\partial g_{ji}}{\partial x^k} + \frac{\partial g_{ki}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^i} \right). \quad (6)$$

In [1] we found out that if M is 3-dimensional then the force we can write in the form

$$\vec{F} = \text{rot } \tilde{\xi} \times \vec{v} + \vec{E}, \quad (7)$$

where

$$E_k = \frac{1}{2} \frac{\partial \xi^2}{\partial x^k} - \frac{\partial \tilde{\xi}^k}{\partial t} \quad \text{and} \quad \tilde{\xi}_i = g_{ij} \xi^j. \quad (8)$$

The force F is called deformation force which arise due to the wind distribution ξ .

2 Simulation of a 2-dimensional situation

As examples we provide a solution of the problem for the case $\dim(\mathbb{R} \times M) = 2$.

Example 1.

$$\begin{aligned} g &= dx, \\ \xi &= \frac{\partial}{\partial t} + (t - x) \frac{\partial}{\partial x}. \end{aligned} \quad (9)$$

Equation (5) takes the form $-1 - t + x - \ddot{x} = 0$ and $F = -1 - t + x$.

On Figure 1 is the vector field ξ that was chosen for our simulation. Curves on the Figure 2 represent curves before the “wind” deformation

$$x = c_1 + c_2 t, \quad c_1, c_2 \text{ are arbitrary,}$$

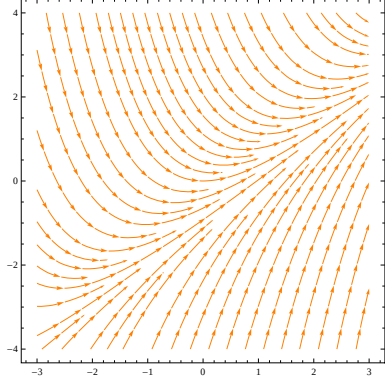


Fig. 1

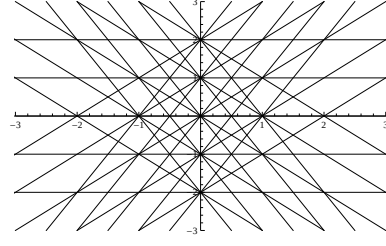


Fig. 2

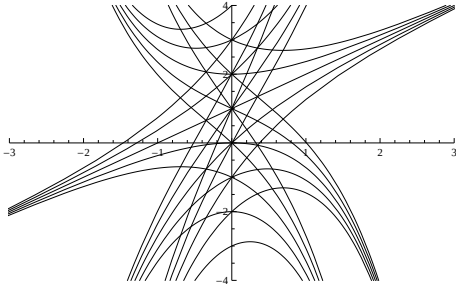


Fig. 3

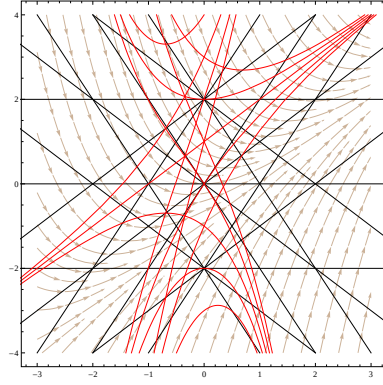


Fig. 4

and next Figure 3 shows curves after “wind” deformation

$$x = 1 + t + c_1 e^t + c_2 e^{-t}, \quad c_1, c_2 \text{ are arbitrary.}$$

Last Figure 4 demonstrates the whole situation where we can see changes on the curves caused by the force F .

Example 2.

$$\begin{aligned} g &= x^2 dx, \\ \xi &= \frac{\partial}{\partial t} + \frac{\cos t}{x} \frac{\partial}{\partial x}. \end{aligned} \tag{10}$$

Equation (5) takes the form $x \sin t - x\dot{x}^2 - x^2\ddot{x} = 0$ and $F = x \sin t$.

On Figure 5 is the vector field ξ that was chosen for our simulation. Curves on the Figure 6 represent curves before the “wind” deformation

$$x = c_1 \sqrt{2t - c_2}, \quad c_1, c_2 \text{ are arbitrary,}$$

and next Figure 7 shows curves after “wind” deformation

$$x = \pm \sqrt{2c_1 + c_2 t - 2 \sin t}, \quad c_1, c_2 \text{ are arbitrary.}$$

Last Figure 8 demonstrates the whole situation where we can see changes on the curves caused by the force F .

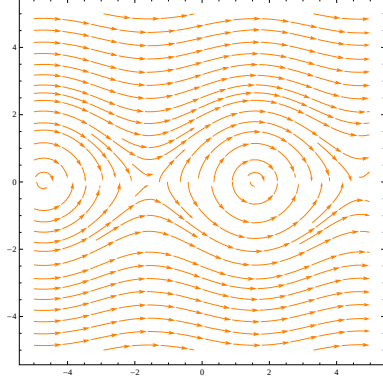


Fig. 5

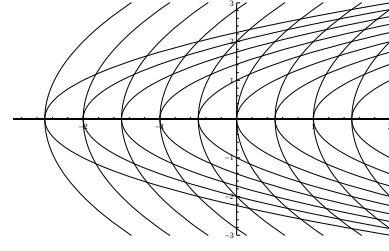


Fig. 6

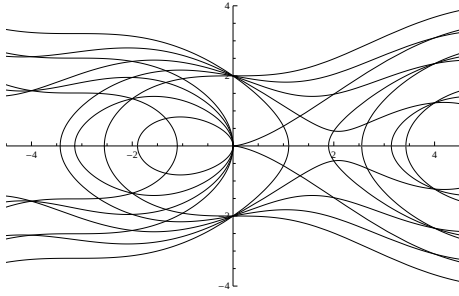


Fig. 7

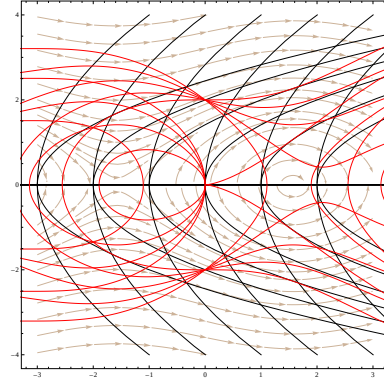


Fig. 8

Example 3.

$$\begin{aligned} g &= x \, dx, \\ \xi &= \frac{\partial}{\partial t} + \frac{x}{t} \frac{\partial}{\partial x}. \end{aligned} \tag{11}$$

Equation (5) takes the form $\frac{5x^2}{2t^2} - \frac{1}{2}\dot{x}^2 - x\ddot{x} = 0$ and $F = \frac{5x^2}{2t^2}$.

On Figure 9 is the vector field ξ that was chosen for our simulation. Curves on the Figure 10 represent curves before the “wind” deformation

$$x = c_1 \sqrt[3]{(3t - 2c_2)^2}, \quad c_1, c_2 \text{ are arbitrary},$$

and next Figure 11 shows curves after “wind” deformation

$$x = c_1 \frac{\sqrt[3]{(t^4 + c_2)^2}}{t}, \quad c_1, c_2 \text{ are arbitrary}.$$

Last Figure 12 demonstrates the whole situation where we can see changes on the curves caused by the force F .

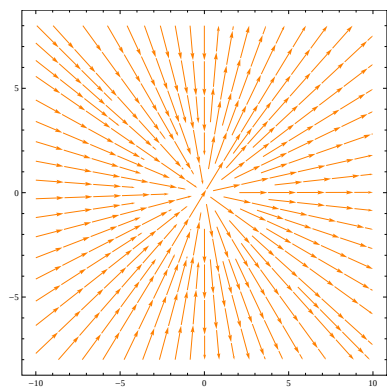


Fig. 9

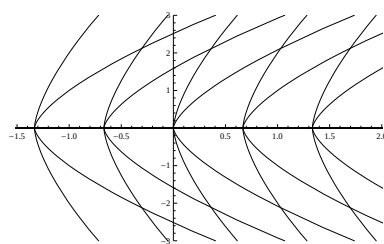


Fig. 10

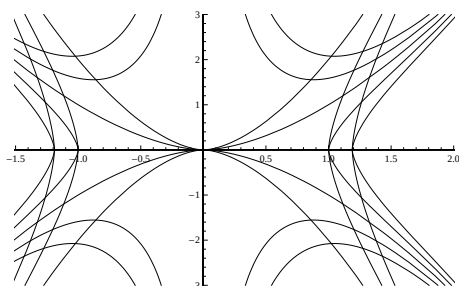


Fig. 11

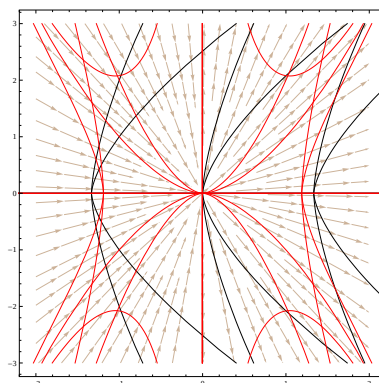


Fig. 12

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ARE WE TEACHING MATH RIGHT?

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Abstract: This paper points out continually decreasing knowledge of mathematics which new students of FCH of BUT have. This paper also thinks about where schools have lost their verbal and practical tasks. It recalls the sad anniversary of the standard ISO-31-11 and ISO 80 000-2. Are we sufficiently dealing with what we want them to learn, but also how they should know it? Has the time come for CAS systems to be implemented to regular lessons of mathematics?

Introduction

Compulsory subject Computer exercises from Mathematics is taught in the second semester of the first year (since academic year 2007/2008) at the Faculty of Chemistry of BUT. It follows the courses Mathematics I and Chemical informatics I that students should have completed during the winter semester. The students here solve individually assigned tasks which are practically focused with the help of mathematical program MATLAB. Students have difficulties with this subject, so optional lectures were added. These allowed students to solve these tasks directly in the exercises. That is when I learned with horror that what I have so far considered careless mistakes, students actually do not know. I am not talking about the college subject matter but I mean the practical mathematical skills that should have been learned at the primary school. How is it possible that student in the summer semester of college can not, for example, find the least common denominator and they do not know how to find it? So they can not multiply the vector $\mathbf{n} = \left(\frac{1}{27}; -\frac{2}{81}; \frac{1}{21} \right)$ by common denominator to get a vector $\mathbf{n} = \frac{1}{567} \cdot (21; -14; 27)$. They can not convert from the left side of the equation to the other. So they can not find the missing coordinate of the point of contact. They can not deal with complex fraction. I am completely fascinated how they can "edit" algebraic

expressions. Their treatment looks like this: $x = a \cdot \sqrt{1 - \frac{\left(\frac{a}{\sqrt{2}} \right)^2}{a^2}}$. Instead of simplifying

the expression to get $x = \frac{\sqrt{2} \cdot a}{2}$. They are able to copy the expression and substitute in other terms, to obtain a "result" to a half of new page. Then they are horrified when they

have to put into this expression, for example a simple fraction $\frac{2}{7}$. How can we give them something so "difficult" to solve? If I request from the students to streamline the expression or partially extract, at first they have no idea what to do. It turns out that they are unable to properly deal with square root, not to mention cube root. And I could go on. How is it even possible that today students do not have these basic numerical skills? Are we teaching math right?

Word problems

In today's society is "cool" when famous personalities highlight their poor results in mathematics. What is the attitude of the general public to mathematics? Excuse me for a bit of nostalgia. When I went to elementary school in every lesson from mathematics teacher had to solve at least one "practical example" with us. We had to write assignment, then underline it using ruler, and then followed a mathematical calculation, again underline and then verbal solution. Did it make any sense? With hindsight, I realize that it had a huge meaning. When parents were at home and looked at their kid's math exercise book, they found tasks that they understood and they could see that their children are learning something that they may need in life. Now, when parents open the math exercise book, for example, eighth-grade student, what do they see? They see some numbers, "letters" and various mathematical symbols, but no "practical example", no verbal task. Why do we not include tasks that reflect everyday situations in teaching?

Example: Pupil stayed in school and got hungry. Next to the school is a small bakery where rolls smell and he would like to have a bite. But he has little pocket money and roll is CZK 2.40. TESCO around the corner has rolls for CZK 1.50 each. Where should he go to buy a roll to save money? The pupil should go to the bakery because it is closer and he would save time. Students can not understand this. So I asked them, how much will he pay in the bakery. He would pay CZK 2 because the price of CZK 2.40 is rounded down. How much will he pay at TESCO? He would pay CZK 2 because the price of CZK 1.50 is rounded up. (In Czech Republic, prices are given in tenths of CZK, however lower denominations than CZK 1 are no longer valid, so price is rounded prior to payment.) So why not save time? And now there are many variations of this task. What if I want two rolls, three, etc? When is it meaningful to go across town to hypermarket, which has a discount?

And what it is like on college? Do we include "application" examples in our lessons? At our faculty Mathematics I is taught 4 hours per week. Two of these 4 hours are lectures and other two are exercises. Given the amount of subject matter I only have 4 hours per semester to teach and practice derivations and the same amount of time to teach and practice indefinite integrals. Is there enough time to include application examples? There is barely time for one very simple task and I think that it is not enough. Situation for combined form students is even worse, because they have only exercises and their total duration is about a third compared to daily form students. How do we handle this situation? We use e-learning and we create distance learning texts. In compulsory subject Physical Chemistry I they solve the differential equations. So I found example, when they have to solve the integral of a rational function while solving differential equation. I included this example immediately to first exercise, because the first thing that has to be done is to find definition domain. In the next exercise they have to perform decomposition of denominator to root factor. In the following exercise decomposition into partial fractions is done and in another exercise students integrate. And finally in last exercise the differential equation including initial conditions is solved. I thought that this will show students why they have to learn mathematics,

where they will need it and why is it necessary not to forget knowledge already gained. And what was the response from students and from colleagues? Students were particularly interested whether these examples will be in exam or not. Teachers were very positive about the inclusion of these examples in teaching.

Mathematical norms and standard

I quote: *"Given the widespread use of text editors on computers for text there is a new task for authors: Ability to independently process technical text. ... Terminological standards or recommendations are not mandatory - there is no punishment for not complying with these standards. But it is a recommendation! We adhere to correct spelling, so we would not be regarded as uneducated, so compliance with various standards and rules should be a matter of professional honour of writers and editors."*^[1]

In reality majority of primary and secondary school teachers do not even know that norm ČSN 01 1001 (from 3rd January 1961) is no longer valid and that in February 1999 the Czech Standards Institute released norm ISO 31-11 entitled "Quantities and units - Part 11: mathematical signs and symbols for use in physical sciences and engineering". This standard is the Czech version of the International Standard ISO 31-11:1992, which has the status of Czech technical standard. In 2012, this standard has been superseded by ISO 80 000-2 with title "Quantities and units - Part 2: Mathematical signs and symbols used in science and technology", which is the Czech version of the International Standard ISO 80 000-2 from 2009. I ask following question. Do teachers of our primary and secondary schools know these standards? Do we teach students to adhere to these standards? How is it that everybody is writing (in Czech language) Easter and Christmas with capital letters and teachers immediately teach students about this sort of changes, but changes in writing mathematical symbols, which occurred 15 years ago, are almost unknown today? Why do the teachers not adhere to this norm? Because even the new materials to school leaving exams are written according to standards from 1961 and students are penalised when they do not use this norm.

I ask, when these changes, which occurred in the world 22 years ago and should have occurred 15 years ago, are going to take effect in our schools. You can argue that new legislation standardization provided by Act No. 142/1991 Coll., The Czechoslovak technical standards, as amended by Act No. 632/1992 Coll. defines technical standards as essentially voluntary documents. The only exception from the voluntary nature of these standards is provisions whose binding was established at the request of government authority having power to issue in the area generally binding regulations. Basically, in 1961, each standard was legally binding and everybody, including teachers, had to observe it, they had to know it and learn by it. The mentioned law ended the validity of the professional standards at 31st December 1993 and legal binding of Czechoslovak state standards at 31st December 1994. Since then, the standards are only "voluntary documents" such as during so-called First Republic (between World Wars). So is there any reason to observe them? For example, because our students are using foreign literature and some of them study abroad. How can they adequately read and understand this literature, when they do not know that $\lg x$ is common logarithm, $\text{lb } x$ is the binary logarithm and for example $\mathbb{N}$ is the symbol used for natural numbers, in which zero is included? How can they correctly write in a text editor such as Microsoft Word that has this standard built-in in its equation editor. When student use different norm there will be issues with output (such as $\tan x$, which is correct, but $\text{tg } x$ is incorrect by new norm). I think that as long as schools do not get quality teaching materials written in accordance with these standards students of the first year of college will have problems to understand assignment written to according to valid norm.

What and how to teach

From colleagues who teach technical subjects I often hear that students need to learn ... and a list of what students should learn in math follows. I think it is not enough to consider only what students should know, but also how the students should know it and how we are going to teach it. This depends on time available to us for teaching. Let me give you an example. We teach students "regression line". To my question: "How should the students know it?" I usually get a one-word answer: "Properly?". In this case I reply: "Are you aware even your PhD students do not know it properly? Try to give one of them a pencil, paper, calculator and 6 points and let them calculate the equation of the regression line. But give them Microsoft Excel and all of them will do it, but what about second regression line in the same graph?" What do I want to say? Let me show you on these examples three levels of knowledge (a kind of extremes).

- a) True knowledge - students must be able to approximate the discrete polynomial function first stage using least square method. They must know when and how to use this approximation, when it is inappropriate. They can calculate the coefficients of the regression line and they can apply this method on polynomial of higher degree. They need a lot of knowledge for this and they can not learn it in 15 min.
- b) Intermediate knowledge - students know that he must approximate the discrete polynomial function of the first stage using least square method and knows when and how to use this approximation and when it is inappropriate. They are unable to calculate the coefficients themselves. They have to use Microsoft Excel or MATLAB, for that. If they can create a graph in Microsoft Excel or MATLAB, they usually do not need so much time for learning that.
- c) Limited knowledge (trained monkey) - students know, where they have to click in Microsoft Excel ("hit points" to put the right mouse button, choose Insert trend line etc.) But they have no idea what are they doing, why are they doing it, when to use it. In this case students are able to "learn" it in 15 minutes.

The ideal situation is of course a), but do we have enough time to teach students this way? At our faculty Mathematics I was taught 8 hours per week for 8 credits, while today it is only 4 hours per week for 4 credits. And what about content? It was expanded; differential equations were included, because the subject Mathematics II is only optional.

What is actual knowledge of students when they come to college? As mentioned earlier actual knowledge of students is much lower than it was in previous years. And what do we want from them? We want them to learn differential calculus during only three exercises and the same for integral calculus. Their skills are almost like level c). Something mechanically learnt. They are unable to apply it in other technical subjects and they forget it as fast as they learn it.

There is obvious question: Is it somehow possible for students to learn the same amount of subject matter in half of the time, when students have a much lower initial knowledge? In daily form of study it may be solved the same way as in the combined form of study. We prepare e-learning distance learning materials for students from which they can study alone step by step. Most of the learning process will be done by students themselves. This has the advantage that everyone can go through lecture notes according to their individual skills, but it also has one big disadvantage: Lack of personal contact with the teacher and teacher's immediate feedback. Students can spend excessively long time studying some portion and because the time is limited, they stay behind. These students can be helped by "remedial courses" or a personal consultation.

Or we can also use, as shown in the example above (regression line), mathematical program, which will help students to solve problems that are suitable for computer-aided instruction. If we start with the premise that the new subject matter must be first explained (lectures MI), then practiced (exercise MI) and only then it can be applied to practical problems and these can be solved using computer. Particularly these types of tasks can be solved using computers:

- a) Routine and tedious calculations (system of equations, determinants, inverse matrix, sorting of data, etc.)
- b) Repeating and deepening the curriculum to other non-traditional procedure (global and local extremes, differential, area, volume body, etc.)
- c) An explanation and clarification of a new concept in conjunction with graphics (gates and angular points, points of discontinuity of the tangent plane surfaces, conic sections and their intersections, etc.)
- d) An impossible task without a computer (non-linear equations, polynomial equations of higher order than the second, approximation and interpolation, numerical algebra and analysis, etc.)

We can achieve through computer-aided teaching of mathematics that for some students mathematics may be "easier" in a certain way. It can even motivate them to study. Then we would not give tasks like "calculate the determinant," but "calculate the volume of a tetrahedron." They still have to calculate the determinant, but first, they must establish it and they must understand the subject matter, because only then can it be applied.

There is obvious question: Are teachers of mathematics and other technical subjects ready for this way of teaching? Around the world, CAS systems are integrated to teaching of all technical subjects and they became an integral part of teaching. This compensates for decreasing level of mathematical knowledge. Even underperforming students are capable of adequately using mathematics in technical subjects. However, this is not happening on our faculty and teachers of technical subject do not integrate computer-aided instruction into their lessons.

Few years ago many teachers disliked calculators and schools have refused for long time to accept and incorporate them into teaching. Nowadays you can hardly find a school in which calculators are not an integral part of teaching math. Currently, when and to what extent to include CAS systems to lessons of mathematics is being decided.

For example, GeoGebra is completely free and online application WolframAlpha is free as well. This allowed for these systems to be used in high schools, but also in elementary schools, where they help in teaching even disabled students who are smart, but lack some mathematical skills. Our students are familiar with these systems and will use them whether we like it or not. We can either capture and use this interest (in these systems) or we can try to suppress it. Students are not going to ask if they can use these systems while doing their homework, and given the presence of smartphones, they can use these systems without us knowing it.

Conclusion

Current knowledge of math that students have is at a very low level. It will not get better, thank to fact, that attitude towards math is not very good and school leaving exam from mathematics is not compulsory. In this paper I tried to show not only the existing problems, but also the possibility of at least partially solving them.

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MEASUREMENT UNCERTAINTY IN THE STUDIES OF ACOUSTIC IN WORKPLACE ENVIRONMENTAL

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Abstract: The publication presents a methodology for estimating uncertainty of measurement on the example of studies of noise exposure in the workplace. The analyzed example involves estimating the measurement uncertainty with regard to assessment errors exposure times. Specified expanded uncertainty U_r (LEX, 8h) specific for a confidence level of 95%. Also included is the fact that very often the number of measurements in each elementary situations is not the same.

1. Introduction

The work environment can be divided into many sources of noise. All equipment, machinery and means of transport are considered as a source of noise and vibration. The level of noise emitted by the device depends on many factors such as the strength and nature of the activity or condition. It is therefore important measurements of noise in the work environment because of the frequently changing the actual situation.

Measurements of noise in the work environment is performed to determine the level of human exposure to the impact of noise at workplaces. The results of measurements refers to the value specified in the regulations and standards in order to determine the occupational risk associated with exposure to noise.

Noise measurements in accordance with the Regulation of the Minister of Health on tests and measurements of harmful factors in the work environment shall be carried out:

- A minimum of once a year, if the results of recently conducted measurements reached the limit values above 0.5,
- A minimum of once every two years, if the results of the measurements recently reached a level above 0.1, but did not exceed the limit values of 0.5,
- In each case the change in the conditions of occurrence of the noise.

Measurements of defining noise in workplaces are carried out by two methods:

- The direct method, which involves continuous measurement of worker exposure to noise and read directly from the meter size, for example. Noise dosimeter or integrating sound level meter. This is an easy method that does not require any complex calculations and can be used

for measuring team with little experience without the risk of committing significant measurement errors in the case of transient noise. The disadvantage of this method is its time-consuming, because the measurement for one job lasts a full shift or longer.

- The indirect method, which relies on the measurement of noise in less than the time of exposure of the employee and the application of mathematical tools to determine the size of describing the noise at workplaces. The main problem in this method is to determine the level of exposure to noise referred to the 8-hour daily working time (LEX, 8h) or workweek (LEX in) for noise, which at the time shift is transient noise.

It can be argued that the most important issue in the measurement of the size of the reliability of the experimental results. For this reliability was full, it is necessary to estimate the uncertainty of measurement. One example of the calculation of uncertainty of measurement is research in the area of acoustic measurements in the workplace. An exemplary object of study is the service station conveyor belts in the main haulage tiered in a coal mine.

2. Study model

In the first stage of the study is to develop a model of a studied phenomenon. The phenomenon of workplace exposure to noise can be characterized as follows:

1. On the basis of news collected by interview and on the job test sets are situations that can be used to characterize worker exposure to noise over the entire work shift.
2. Each situation at the time of its occurrence is strictly related to:
 - Place of residence of the employee (or the point of observation),
 - State of the noise sources (active, not active, a specific mode or type of work) with significant effect on the sound level at which the worker is exposed (ass occurs in the external environment)
 - Duration of the situation,
 - Sound level averaged for the duration of the situation (equivalent level)
3. The set of all the relevant circumstances, suitably selected and referred to, is model of the noise exposure.

3. Presentation of the results of acoustic measurements

In coal the main task is to move the horizontal transport of excavated stuff from cutting runs machines or loading into containers, which allow the movement of excavated material for vertical transportation equipment.

Currently, the most common in underground coal mines to the process conveyor belts are used, depending on the needs of different width and length of the tape.

Working with process-emitting excessive noise makes it difficult to communicate with employees, reduces their productivity, enhances, among others, irritability, often causes a feeling of dementia and leads to severe hearing disorders. The measurement results of the selected measurement of the main haulage tiered coal mines are as follows:

Lp.	Date of measurement	Branch, place of study, performed activities, position of the employee	Duration time i of action T [min]	Duration time of measurement t [min]	Measuringdevice		
			Times exposure $T_e = \sum_{i=1}^n T$ [min]	Total time of the measurement Σt [min]	Equivalent soundlevel A	Maximum soundlevel A	Top soundlevel C
					$L_{A\ eq,T}$ [dB]	$L_{A\ max}$ [dB]	$L_{C\ peak}$ [dB]
Maxiumumconcentration (NDN):			---	---	---	115	135
1		2	3	4	5	6	7
1.		<u>Wall hauling VII on deck 408/1 i 408/2</u>		-			
14/4		Belt conveyor haulage service - P-4					
		- Train passing in both directions (sitting position, 3 measurements)	30	5 5 5	91,6 90,4 90,7	101,3	119,6
		- Operation of the conveyor P-4 while the haulage (standing position, 3 measurements)	240	5 5 5	89,4 89,7 88,8		
		- Cleaning the area of the conveyor P-4 while (standing position, 3 measurements)	90	5 5 5	74,6 75,8 74,3		
		- Other activities – work division, exit and exit, acces to workplace, manual work in the area (standing position, 6 measurements)	90	1 1 1 1 1 1	72,6 76,4 75,8 78,8 77,2 76,1	87,6	102,6
			Te = 450	Σ t= 51			

3. The calculation of uncertainty of measurements (3)

In the case study of noise exposure in the work environment, the test noise (including ultrasound and infrasound noise) is the result of conversion of measurements in different listening situations and exposure times relevant to these situations - you can express using the formula:

$$L_{EX,8h} = 10 \lg \left[\frac{1}{T_o} \sum_{i=1}^m T_i 10^{0,1 L_{eq,i}} \right] \quad dB \quad (1)$$

(1)

where: $L_{EX,8h}$ - the level of noise exposure up to 8 hours working day in dB,

T_i - duration of the situation i ,

$L_{eq,i}$ - equivalent sound level in the situation i ,

T_o - reference time equal to 8 hours (28800 sek.),

m - number of situations i .

The result is a fully credible, specify the uncertainty of measurement. Patterns on the standard uncertainty of complex size LEX, 8h, designated as UC (LEX, 8h), derived from the law of propagation of uncertainty, expresses the relationship:

$$U_c^2(L_{EX,8h}) = \sum_{i=1}^m \sum_{k=1}^{n_i} \left(\frac{\partial L_{EX,8h}}{\partial W_{i,k}} \right)^2 * [U_A^2(W_{i,k}) + U_B^2] \quad (2)$$

where: U_B – type B standard uncertainty,

$U_A(W_{i,k})$ – type A standard uncertainty of noise measurement result of elemental situation „i”,

$$\frac{\partial L_{EX,8h}}{\partial W_{i,k}}$$

sensitivity coefficients which are the derivatives of the function $L_{EX,8h}$ terms of $W_{i,k}$.

Partial derivatives of the function LEX, 8h largest T_i , but also the size of the standard uncertainties of T_i , have not been designated as the time of exposure (exposure) T_i is not obtained by means of measurements or findings made by the laboratory, but they are obtained as a ready input - ie. the information provided by the employer for which the laboratory is not liable. This information is an important part of data making up the model studied phenomenon of exposure.

Follows from these considerations that the relation (2) does not take into account the uncertainty arising from errors assess exposure times. The standard uncertainty of complex UC (LEX, 8h), you can calculate from formula:

$$U_c(L_{EX,8h}) = \sqrt{\sum_i \left[\frac{1}{n_i^2} \sum_{k=1}^{n_i} \left(\frac{T_i * 10^{0,1W_{i,k}}}{T_e * 10^{0,1L_{A,eq,T_e}}} \right)^2 * (S_i^2 + U_B^2) \right]} \quad dB \quad (3)$$

where: $W_{i,k}$ – result of measurement in elemental case „i”

L_{A,eq,T_e} – equivalent noise level in the time T_e ,

T_e – the time for which determined L_{A,eq,T_e} .

The expanded uncertainty U_r (LEX, 8h) specific for a confidence level $P = 95\%$, you can calculate from the relationship:

$$U_r(L_{EX,8h}) = k_p \cdot U_c(L_{EX,8h}) \quad dB \quad (4)$$

where: k_p – coverage factor for a confidence level of 95% i v effective degrees of ease

Because quite often the number of measurements in each elementary situations it is not the same, then instead calculated the effective number of degrees of freedom (to define a common value k_p) is more convenient to make use the formula:

$$U_r(L_{EX,8h}) = \sqrt{\sum_{i=1}^m \left[t_{\alpha,v,i}^2 \sum_{k=1}^{n_i} \left(\frac{E_{i,k}}{E_{A,T_e}} \right)^2 \right] * S_i^2 + U_{B,95}^2} \quad dB \quad (5)$$

where: $t_{\alpha,n,i}$ – constant form resolution t – student resulting from the adopted level of significance α and the amount of degrees of freedom ν in situation i

For example analyzed was calculated level of noise exposure for an 8-hour working day, which is LEX, 8h = 87.1 dB. We also calculated the uncertainty extended U_r (LEX, 8h) specific for a confidence level $P = 95\%$ which is 1.7 dB.

4. Summary

Calculation of uncertainty of measurement for the size of the sound in the work environment, for which we know the patterns of averaging, allows the use of standard statistical tools. However, a necessary condition is that the requirements relating to the definition of static elementary events. It follows from this approach to the calculation for determining the uncertainty of measurement noise level in the work environment.

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NON-PARAMETRIC TEST USED FOR COMPARISON OF PER-OPERATING PARAMETERS OF FEMUR AND PELVIS

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Abstract: In this article the authors demonstrate one of the applications of non-parametric hypothesis test. This application deals with comparing per-operation parameters by operations of the fractures of the femur and the pelvis realized on Department of Traumatology of University Hospital of Ostrava. Focus of this comparing is on three variables (the skia time, the dose of X-rays during the operation and the operating time) in two ways of the operation: the traditional way of the operation skiascopy and the modern way of leading the operation by the navigation by a computer.

Keywords: femur, pelvis, skia time, exposition, skiascopy, navigation, non-parametric hypothesis tests, Mann-Whitney test.

1 Data

Five-years medicine study deals with the operations of the fractures of the femur and the pelvis realized on Department of Traumatology of University Hospital of Ostrava. The operations are practised by one of two possible methods - the traditional skiascopy, where the whole process of the operation is controlled by X-ray machine and the modern navigation of the operation by a computer, where the patient is submitted input X-ray radiation and after it the operation is controlled by the navigation of a computer.

The data are recorded into two input excel files femur.xls and panev.xls with a competent requirement to analyze this two groups separately. In both of the data

files are included three continuous variables, which are then submitted the analysis: an operating time (min), skia time (sec) represented by time of the irradiation of one patient by X-ray radiation and an exposition (Cgy/cm²) represented by the radiation dose of X-ray radiation of one patient.

The essential question of this study is, whether this tree variables are dependent on the type of the operation.

2 Discussion of regularity of the collection of the data

With respect to the fact that the choice of the method by the patient was done stochastically therefor the type of the operation was dependent neither the age nor other parameters, which could be the experience of the surgeon, the type of the fracture, importance of the planned intervention. We can say, that the collection of the data passed regularly although the pilot study contained only a few data.

3 Femur

Into the study of the fractures of the type A were involved 9 patients submitted the skiascopy and 8 patients submitted the navigation by a computer. As for the operations of the fractures of the type B there were 5 patients submitted the skiascopy and 6 patients were submitted the navigation by a computer. By the methods of exploratory data analysis were found several outliers that were left in the data file according to the character of the data, the number of the data and the following choice of the statistical methods.

Medicine expectations

By the navigation were expected shorter skia time, smaller exposition and shorter time of the operation in comparison with the method of skiascopy.

Choice of the methods of hypothesis testing

For the hypothesis testing it was chosen non-parametric Mann-Whitney test of medians for a reason of non fulfilment of the assumption of normality (Shapiro-Wilk test) in all groups.

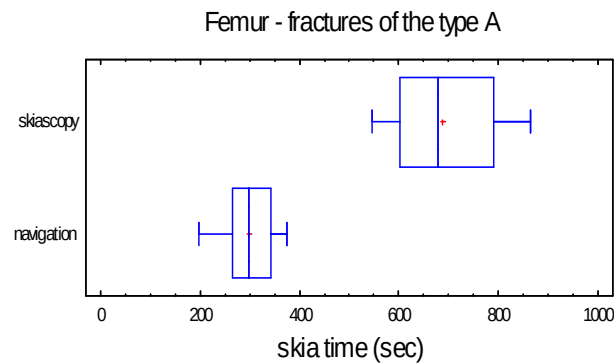
According to the choice of non-parametric test, is not necessary to test homoscedasticity. For the completion it was done Levene's test. There is a statistically significant deviation from homoscedasticity by the group skia time by the fractures of the type A and by the exposition by the fractures of the type B. There was found higher variance at the traditional skiascopy in both cases. The test of homoscedasticity is in this case suitable for the validation of the same variance of both types of the operation. If the new method would have always higher variance, it would mean its temporary instability. This was not proved in our case. The comparison of

the method of the skiascopy and of the navigation is therefor really good for this analysis. All of the hypothesis tests were tested at the 95 percent confidence level.

For the graphical illustration of the interactions of several factors was used multi-factors analysis of variance ANOVA. This method was used on level exploratory data analysis. The method of multi-factors analysis of variance ANOVA on parametric way is not very suitable to use for the evaluation of the principal hypothesis of our problems according to non fulfilment of the assumption of normality.

Results of the hypothesis testing

There is a statistically significant difference between the medians of the skia time and the type of operation at the 95 % confidence level (p-value for the type A is less than 0,001, p-value for the type B is equal 0,008). The new method of the navigation required a shorter time of the radiation exposure of individual person then the traditional method of the skiascopy.



There is a statistically significant difference between the medians of the exposition and the type of operation at the 95 % confidence level (p-value for the type A is less than 0,001, p-value for the type B is equal 0,008). The dose of the radiation is smaller by the navigation then by the skiascopy.

There is a statistically significant difference between the medians of the operating time and the type of the operation at the 95 % confidence level (p-value for the type A is 0,003, p-value for the type B is equal 0,021). The new method of the navigation requires a longer operating time then the traditional skiascopy.

4 Pelvis

Into the study of the fractures of the type B were involved 13 patients submitted the skiascopy and 14 patients submitted the navigation by a computer. As for the operations of the fractures of the type C there were 11 patients submitted the skiascopy and 10 patients were submitted the navigation by a computer. By the methods of exploratory data analysis were found several outliers that were left in the data file according to the character of the data, the number of the data and the following choice of the statistical methods.

Medicine expectations

By the navigation were expected shorter skia time, smaller exposition and shorter time of the operation in comparison with the method of skiascopy.

Choice of the methods of the hypothesis testing

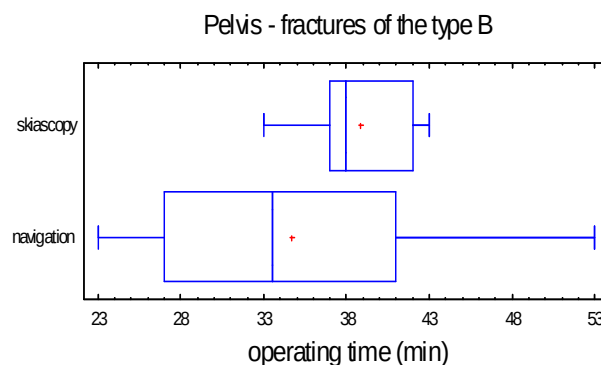
For the hypothesis testing it was chosen for a reason of non fulfilment of the assumption of normality (Shapiro-Wilk test) in all groups non-parametric Mann-Whitney test of medians.

Results of the hypothesis testing

There is a statistically significant difference between the medians of the skia time and the type of the operation at the 95 % confidence level (p-value is less than 0,001 for both of the types B and C). The new method of the navigation required a shorter time of the radiation exposure of individual person then the traditional method of the skiascopy.

There is a statistically significant difference between the medians of the exposition and the type of the operation at the 95 % confidence level (p-value is less than 0,001 for both of the types B and C). The dose of the radiation is smaller by the navigation then by the skiascopy.

There is not a statistically significant difference between the medians of the operating time and the type of the operation at the 95 % confidence level for the fractures of the type B (p-value is equal 0,075). There is a statistically significant difference between the medians of the operating time and the type of the operation at the 95 % confidence level by fractures of the type C (p-value is equal 0,003). The new method of the navigation requires a longer operating time then the traditional skiascopy. The operations of the pelvis of the fractures of the type C require new method navigation shorter operation time that operations done by the traditional skiascopy.



5 Conclusions

In the primary medicine expectations an existence of a statistically significant difference was proved between the skia time by the skiascopy and the skia time by the navigation. Thus the new method of the navigation by a computer proved a statistically significant contraction of the time of the submission X-ray radiation.

Farther an existence of a statistically significant difference was proved between the exposition by the skiascopy and the exposition by the navigation. Thus the new method of the navigation by a computer proved a statistically significant depression of the dose of the radiation against the traditional way of the leading of the operation by skiascopy. The conclusions written above are the same for the operations of the pelvis and of the femur in both cases of the type of the fractures.

By the operating time are the conclusions for the femur and the pelvis different. The new method of the navigation by a computer requires by the operations of the femur a longer operating time then the traditional skiascopy in case of the fractures of both of the types A and B. By the operations of the pelvis by the fractures of the type B was not proved a statistically significant difference between the operating time and the type of the operation. By the fractures of the type C the navigation requires shorter operating time than the traditional skiascopy.

Used software

Statgraphics, version 5

Acknowledgement

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STANOVENÍ REFERENČNÍCH MEZÍ PRO KONCENTRACI OSTEOKALCINU

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Abstrakt:

Za účelem určení referenčních mezí byly analyzovány hodnoty určující koncentraci osteokalcinu naměřenou v plazmě a v séru u třech skupin pacientů – ženy v menopauze (80 pacientů), ženy ve fertilním věku (90 pacientů) a muži (101 pacientů). Dalším cílem analýzy bylo zjistit, zda se hodnoty koncentrace osteokalcinu získané měřením z plazmy významně liší od hodnot získaných měřením ze séra. Naměřené hodnoty byly zaznamenány v databázi MS Excel, k výpočtům byl krom MS Excelu 2010 použit statistický software SPSS verze 18 (PASW Statistics 18.0).

Abstract:

In order to determine reference ranges, osteocalcin concentration in plasma and serum were analyzed in three groups of patients - women in menopause (80 patients), women in fertile age (90 patients) and men (101 patients). Another aim of the analysis was to determine whether osteocalcin concentration in plasma and serum differ significantly. Measured data were recorded in MS Excel database, statistical software SPSS version 18 (PASW Statistics 18.0) and MS Excel 2010 were used for the calculations.

1. Osteocalcin

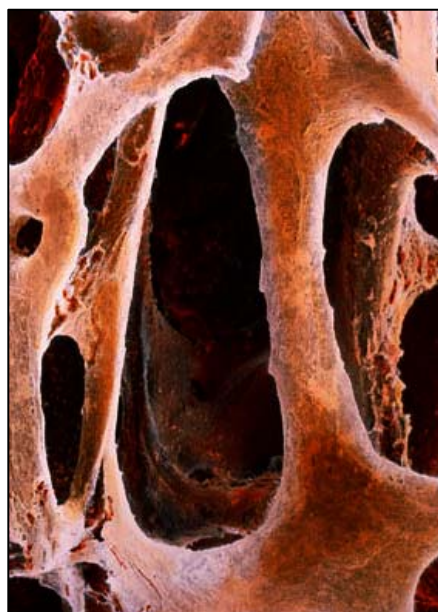
Osteocalcin (OC) je nekolagenní bílkovina, vyskytující se v kostech a zubovině. Je produkován kostními buňkami osteoblasty a podílí se na mineralizaci kostí, výstavbě kostí i zubů a působí také jako hormon stimulující vyšší produkci inzulínu ve slinivce břišní. Často je používán jako biochemický marker pro proces kostní formace. Byla pozorována významná korelace mezi vysokou sérovou koncentrací OC a nárůstem hustoty kostního minerálu, tzv. BMD (z angl. Bone Mineral Density). V mnoha studiích je OC používán jako předběžný marker před podáním anabolických léků.

Zvýšené hodnoty u dospělých pozorujeme v případech kostní hypermodelace (renální osteodystrofie, časná hyperparathyreóza, hyperthyreóza, Pagetova choroba). U Pagetovy choroby může v případě zvýšené mineralizace dojít k částečné vazbě OC na hydroxyapatit a tak k nalezení nižších hodnot OC v séru. Snížené hodnoty OC v séru jsou pozorovatelné v případech hyporemodelace kostní hmoty (hypoparathyreóza, hyperkalcémie u kostních metastáz, případně u dlouhodobé léčby glukokortikoidy). U

primární osteoporózy (např. postmenopauzální forma) je OC výborným markerem k posouzení rychlosti regenerace kostní hmoty (osteoporóza s vysokou nebo nízkou remodelací).



Obrázek 1.1. Nehomogenní struktura skeletu v oblasti pánve u Paretovy choroby



Obrázek 1.2. Kostní tkáň postižená osteoporózou

2. Referenční meze a intervaly

Referenční interval je definován jako interval, ve kterém leží 95% hodnot statistického znaku, naměřených u dostatečně homogenního a početného souboru zdravých jedinců. Krajní body tohoto intervalu - *referenční meze*, tedy určíme jako 2,5. a 97,5. percentil zpracovávaného statistického souboru (neparametrický přístup) nebo jako 2,5. a 97,5. percentil odpovídajícího rozdělení (parametrický přístup). Výjimku tvoří situace, kdy se koncentrace analytu patologicky zvyšují pouze na jednu stranu. Pak se místo výše popsaného postupu, který vede k získání tzv. oboustranného referenčního intervalu, stanovuje referenční interval jednostranný, jehož dolní (resp. horní) referenční mez se volí jako 5. (resp. 95.) percentil. Případný rozdíl v šíři referenčního intervalu získaného parametrickým a neparametrickým postupem přitom není klinicky významný ([1]).

Zdravotní stav jedinců, kteří byli vybráni pro stanovení konkrétních referenčních mezí, musí být co nejpřesněji popsán a zaznamenán. Tito jedinci tvoří tzv. *referenční populaci*.

Referenční intervaly jsou základním podkladem pro hodnocení výsledků laboratorních testů. Představují základní popis vlastností laboratorního testu, mají význam jak u nově zaváděných testů, tak při běžném rozhodování. Umožňují vzájemné porovnání různých postupů stanovení analytu a také různých výrobců diagnostických souprav. Pojem „referenční“ přitom nesmíme zaměňovat s pojmem „fyziologický, normální“, protože umožňuje pouze srovnání hodnocené populace s populací referenční. Obdobně musíme odlišovat pojmy „referenční meze“ a „rozhodovací meze“ (tzv. „cut-off“ hodnoty). Rozhodovací meze slouží k rozhodování o různých lékařských akcích a mohou se od referenčních mezí výrazně lišit. V řadě situací se však musí výsledek měření pacienta porovnat jak s referenční, tak s rozhodovací mezí. Abnormalita, tedy výsledek mimo referenční interval, může upozornit také na neočekávaný patologický stav.

3. Analýza dat

V této studii byla analyzována data shromážděná na Oddělení klinické biochemie Nemocnice ve Frýdku-Místku v letech 2013-2014. Jednalo se o koncentrace osteokalcinu naměřené v plazmě a v séru u skupiny 271 pacientů, čítající 80 žen v menopauze, 90 žen ve fertilním věku a 101 mužů. Žádný z těchto pacientů přitom neměl zdravotní komplikace, které bývají doprovázeny zvýšenou nebo naopak sníženou koncentrací osteokalcinu v plazmě nebo v séru.

3.1. Stanovení významnosti rozdílů koncentrací OC naměřených v plazmě a v séru

Koncentrace osteokalcinu byla u každého z 271 pacientů určena dvěma způsoby – měřením ze séra a měřením z plazmy. Cílem analýzy bylo zjistit, zda je mezi hodnotami OC získanými těmito dvěma postupy významný rozdíl. Základní popisné charakteristiky obou souborů (plazma, sérum) jsou zachyceny v Tabulce 3.1.1:

Tabulka 3.1.1. Koncentrace OC měřené z plazmy a séra

	Střední hodnota	Směrodatná odchylka	Chyba střední hodnoty
Sérum	22,5875	11,04362	0,67085
Plazma	26,3575	12,57865	0,76410

K testování významnosti rozdílů mezi jednotlivými postupy byly z důvodu porušení předpokladu normality použity neparametrické testy, a sice Wilcoxonův test a znaménkový test pro dva závislé výběry. Oba testy shodně zamítly hypotézu o nulové hodnotě mediánu rozdílů párovaných hodnot (viz Tabulka 3.1.2), čímž byla rozdílnost obou technik prokázána. Hodnoty osteokalcinu získané analýzou séra se významně liší od hodnot získaných analýzou plazmy.

Tabulka 3.1.2. Wilcoxonův a znaménkový test pro dva závislé výběry

Nulová hypotéza H_0	Test	p -value	Závěr
Medián rozdílů mezi plazmou a sérem je 0	Znaménkový test	0,000	Hypotézu H_0 zamítáme
Medián rozdílů mezi plazmou a sérem je 0	Wilcoxonův test	0,000	Hypotézu H_0 zamítáme

3.2. Stanovení referenčních mezí pro koncentraci OC

Jelikož bylo potvrzeno, že se hodnoty OC naměřené v plazmě a v séru významně liší, byly referenční meze stanoveny pro každou metodu zvlášť. V obou případech (sérum, plazma) pak byly analýzou rozptylu (ANOVOU) zjištěny významné rozdíly v hodnotách OC u jednotlivých skupin pacientů – ženy v menopauze (80 pacientů), ženy ve fertilním věku (90 pacientů) a muži (101 pacientů), proto byly referenční meze stanoveny zvlášť i pro každou skupinu pacientů.

Protože byly hodnoty v jednotlivých skupinách rozloženy přibližně normálně (testováno jednovýběrovým Kolmogorovým-Smirnovovým testem), byl pro určení referenčních mezí použit parametrický přístup. Pro odhad referenčních mezí a jejich 90%-ních intervalů spolehlivosti bylo použito následujících vzorců ([2]):

dolní mez: $x_D = \bar{x} + x_{0,025} s$,

horní mez: $x_H = \bar{x} - x_{0,025} s$,

dolní mez dolní meze: $x_{DD} = \bar{x} + x_{0,025} s + x_{0,05} \sqrt{s^2/n + x_{0,025}^2 s^2/(2n)}$,

horní mez dolní meze: $x_{HD} = \bar{x} + x_{0,025} s - x_{0,05} \sqrt{s^2/n + x_{0,025}^2 s^2/(2n)}$,

dolní mez horní meze: $x_{DH} = \bar{x} - x_{0,025} s + x_{0,05} \sqrt{s^2/n + x_{0,025}^2 s^2/(2n)}$,

horní mez horní meze: $x_{HH} = \bar{x} - x_{0,025} s - x_{0,05} \sqrt{s^2/n + x_{0,025}^2 s^2/(2n)}$,

kde $\bar{x}$ značí střední hodnotu, s výběrovou směrodatnou odchylku, x_p p -kvantil normovaného normálního rozdělení a n rozsah výběru.

Ve všech případech byla stanovena rovněž chyba konfidenčního intervalu referenční meze, která je vypočtena jako podíl konfidenčního intervalu na šířce referenčního intervalu. Získané výsledky vidíme v následujících tabulkách.

Tabulka 3.2.1. Ženy v menopauze – sérum

Ženy v menopauze – sérum (ze souboru 80 hodnot byla odstraněna 2 odlehlá pozorování)						
počet hodnot:		78				
střední hodnota:		24,06				
standardní odchylka:		9,88				
referenční meze:		4,71 - 43,42				
dolní mez	dolní mez dolní meze	horní mez dolní meze	horní mez	dolní mez horní meze	horní mez horní meze	chyba konfidenčního intervalu
4,71	1,57	7,84	43,42	40,29	46,56	16%

Tabulka 3.2.2. Ženy ve fertilním věku – sérum

Ženy ve fertilním věku – sérum (ze souboru 90 hodnot bylo odstraněno 7 odlehlých pozorování)						
počet hodnot:		83				
střední hodnota:		18,07				
standardní odchylka:		6,29				
referenční meze:		5,75 - 30,39				
dolní mez	dolní mez dolní meze	horní mez dolní meze	horní mez	dolní mez horní meze	horní mez horní meze	chyba konfidenčního intervalu
5,75	3,82	7,68	30,39	28,46	32,32	16%

Tabulka 3.2.3. Muži – sérum

Muži – sérum (ze souboru 101 hodnot byla odstraněna 3 odlehlá pozorování)						
počet hodnot:		98				
střední hodnota:		21,92				
standardní odchylka:		10,44				
referenční meze:		1,46 - 42,38				
dolní mez	dolní mez dolní meze	horní mez dolní meze	horní mez	dolní mez horní meze	horní mez horní meze	chyba konfidenčního intervalu
1,46	-1,49	4,42	42,38	39,42	45,33	14%

Tabulka 3.2.4. Ženy v menopauze – plazma

Ženy v menopauze – plazma (ze souboru 80 hodnot byla odstraněna 2 odlehlá pozorování)						
počet hodnot:		78				
střední hodnota:		27,89				
standardní odchylka:		10,93				
referenční meze:		6,48 - 49,31				
dolní mez	dolní mez dolní meze	horní mez dolní meze	horní mez	dolní mez horní meze	horní mez horní meze	chyba konfidenčního intervalu
6,48	3,01	9,95	49,31	45,84	52,77	16%

Tabulka 3.2.5. Ženy ve fertilním věku – plazma

Ženy ve fertilním věku – plazma (ze souboru 90 hodnot bylo odstraněno 7 odlehlých pozorování)						
počet hodnot:		83				
střední hodnota:		21,21				
standardní odchylka:		7,45				
referenční meze:		6,61 - 35,82				
dolní mez	dolní mez dolní meze	horní mez dolní meze	horní mez	dolní mez horní meze	horní mez horní meze	chyba konfidenčního intervalu
6,61	4,31	8,90	35,82	33,53	38,11	16%

Tabulka 3.2.6. Muži – plazma

Muži – plazma (ze souboru 101 hodnot byla odstraněna 2 odlehlá pozorování)						
počet hodnot:		99				
střední hodnota:		25,92				
standardní odchylka:		11,94				
referenční meze:		2,52 - 49,33				
dolní mez	dolní mez dolní meze	horní mez dolní meze	horní mez	dolní mez horní meze	horní mez horní meze	chyba konfidenčního intervalu
2,52	-0,85	5,88	49,33	45,96	52,69	14%

4. Závěr

Wilcoxonovým a znaménkovým testem pro dva závislé výběry byl prokázán významný rozdíl mezi hodnotami osteokalcinu získanými analýzou séra a analýzou plazmy. Analýza rozptylu potvrdila hypotézu o rozdílných hodnotách koncentrace osteokalcinu u jednotlivých skupin pacientů – žen v menopauze, žen ve fertilním věku a mužů. Pro každou skupinu pacientů byly určeny referenční meze pro koncentraci osteokalcinu v plazmě i v séru.

Literatura

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MODELING OF HUMAN RESOURCESMANAGEMENT PROCESS USING iGrafix PROGRAM ON THE EXAMPLE OF SMALL CATTERING FACILITY

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Streszczenie W systemach zarządzania jakością mapowanie procesów sprowadza się do pokazania przebiegu sekwencji czynności. Do analizy procesów i ich doskonalenia wykorzystywane są znane jako narzędzia jakości (*Magnificentseven*). Narzędzia te oparte są na prostych narzędziach matematycznych i statystyce matematycznej. Wymagają więc danych pochodzących z procesu. Program iGrafix posiada wbudowane narzędzia, które pozwalają przewidywać skutki jakie spowodują zmiany w przebiegu procesu. W artykule przedstawiono wyniki modelowania procesu zarządzania zasobami ludzkimi w małym lokalu gastronomicznym wykorzystując do modelowania program iGrafix.

Abstract: The quality management systems, process mapping reduce to showing the course of the workflow. To analyze processes and their improvement are used tools known as "*MagnificentSeven*". These tools are based on simple mathematical tools and mathematical statistics. Therefore required data from the process. IGrafix program has built-in tools that allow to predict the effects that result in changes in the process. The paper presents the results of modeling of human resources management process in a small restaurant using the modeling software "iGrafix".

INTRODUCTION

Quality management systems include eight principles of which are: customer orientation, process approach, making decisions based on facts, continuous improvement. The primary objective of any organization is to maximize profit. This can be achieved by providing product that meets customer satisfying legal requirements and the limitation of actions not bringing added value.

In the case of a catering facility, these objectives are achieved by:

- ✓ Providing consumer products with organoleptic qualities expected;
- ✓ Providing the consumer with products that meet legal requirements;
- ✓ Limitation of actions not bringing added value

In the case of a catering added value do not bring excessive perishable commodity stocks and the unused staff time.

PURPOSE AND SCOPE OF RESEARCH AND RESULTS OBTAINED

The study was conducted in one of the Silesian restaurant focused on creating a unique atmosphere in the premises and serving the discerning customer special quality dishes. The study was conducted during the period of three months. The aim of the study was to develop a model to optimize the selection of staff employed in each day of functioning of a catering assuming:

- ✓ minimize the costs of employing staff on individual days and
- ✓ meet customer expectations.

The study was conducted in three stages which allowed for:

- ✓ evaluation of customer preferences,
- ✓ assessment of the external factors influencing the cardinality of customers visiting premises,
- ✓ create a model that allows for proper selection of personnel on each day of functioning of premises.

The reason for the loss of a client is in most cases his displeasure, which most often results from a discrepancy between what the customer expects and what experiences (Figure 1). Assessment of the client's preferences were based on a questionnaire to assess the gap between customer expectation and satisfaction. The study also takes into account the margin of tolerance, ie the customer. Deviation from the expected quality of service that the customer will not cause negative reactions.

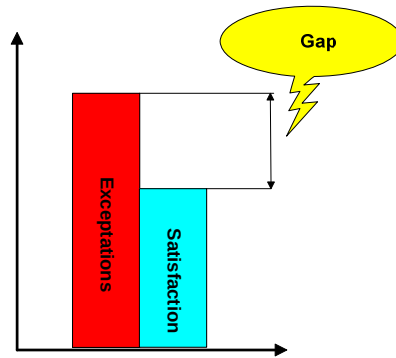


Figure 1 gap between customer expectations and their satisfaction.

Studies have shown that customers expect tested restaurant:

1. have a nice atmosphere
2. good kitchen
3. compliant service
4. short time of service.

Expectations routed in accordance with the results obtained from surveys. Customer survey also included a margin of tolerance specified for the waiting time for serving dishes. The average acceptable waiting time was 37 minutes.

The study also included an assessment of the factors affecting the number of customers visiting a place. The evaluation of these factors was carried out using a control card I-mR.

In the period under reported the number of visitors to the premises. It has been found that there are some days where attendance varies significantly. The results helped plot the control chart I-mR.

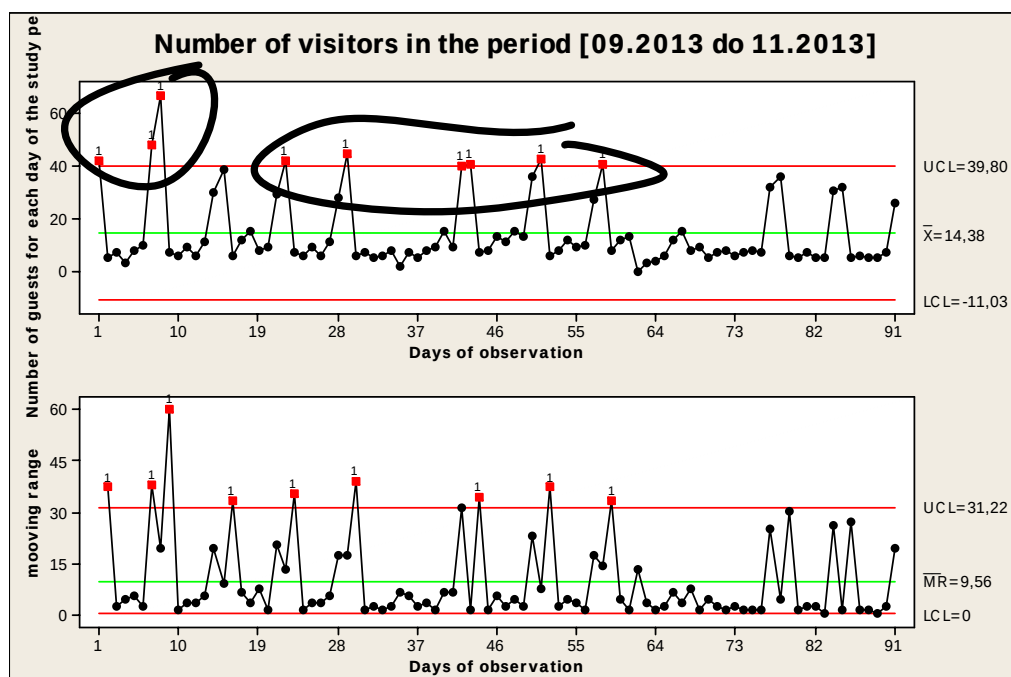


Figure 2 I-mR card for the amount of visitors to the premises during the period.

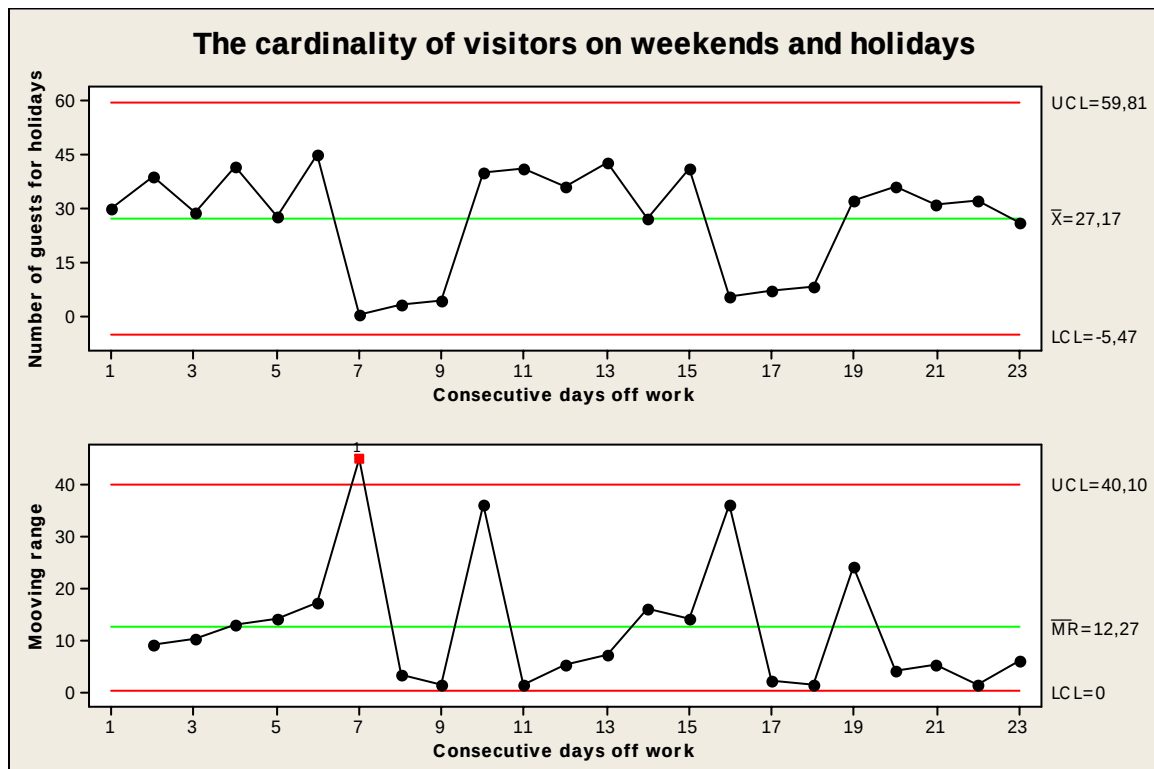


Figure 3 I-mR card for the amount of visitors to the premises during periods indicating the influence of special factors.

The data in Table 1 and in control cards I-mR indicate that the factors that point to are:

1. long weekends / holidays related to trips
2. weather
3. events held in the city
4. days statute prohibiting trade
5. transmissions of important national scale sporting events.

To simulate the customer service process map of the processes has been created taking into account sub-processes occurring at customer service (Figure 4). The effectiveness of customer service and satisfaction determines sub-process of direct customer service and sub-process realized in the kitchen. The rate of service determines the number of staff employed.

The program iGrafix lets you track simulation step by step. The simulation results are presented in the report. The report refers to organizational units, transaction duration, cost and resource utilization. For objects can be assigned attributes, which can be assigned values.

CONCLUSIONS

The study showed that the process of customer service in the restaurant is influenced by factors specific - reasonably foreseeable. The number of customers in a restaurant is determined by such factors as the accumulation of days off from work, sports, weather and public

prohibiting trade. Number of staff employed on such days can be estimated by simulating the process runs with the help of the program iGrafix. Analysis of the number of customers visiting a place to specify the process or transaction type and the limit value of the attribute. Transaction attributes can be assigned any transaction occurring within the process. They can also be assigned to organizational units (processes) - eg. costs of the task.

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Tablica 1 Number of gusts in the restaurant during the period of investigations.

Data	01.09	07.09	08.09	14.09	15.09	21.09	22.09	28.09	29.09		
Number of guests	42	48	67	30	39	29	42	28	45		
Data	12.10	13.10	20.10	21.10	27.10	28.10					
Number of guests	45	41	36	43	27	41					
Data	01.11	02.11	03.11	09.11	10.11	11.11	16.11	17.11	23.11	24.11	30.11
Number of guests	0	3	4	5	7	8	32	36	31	32	26

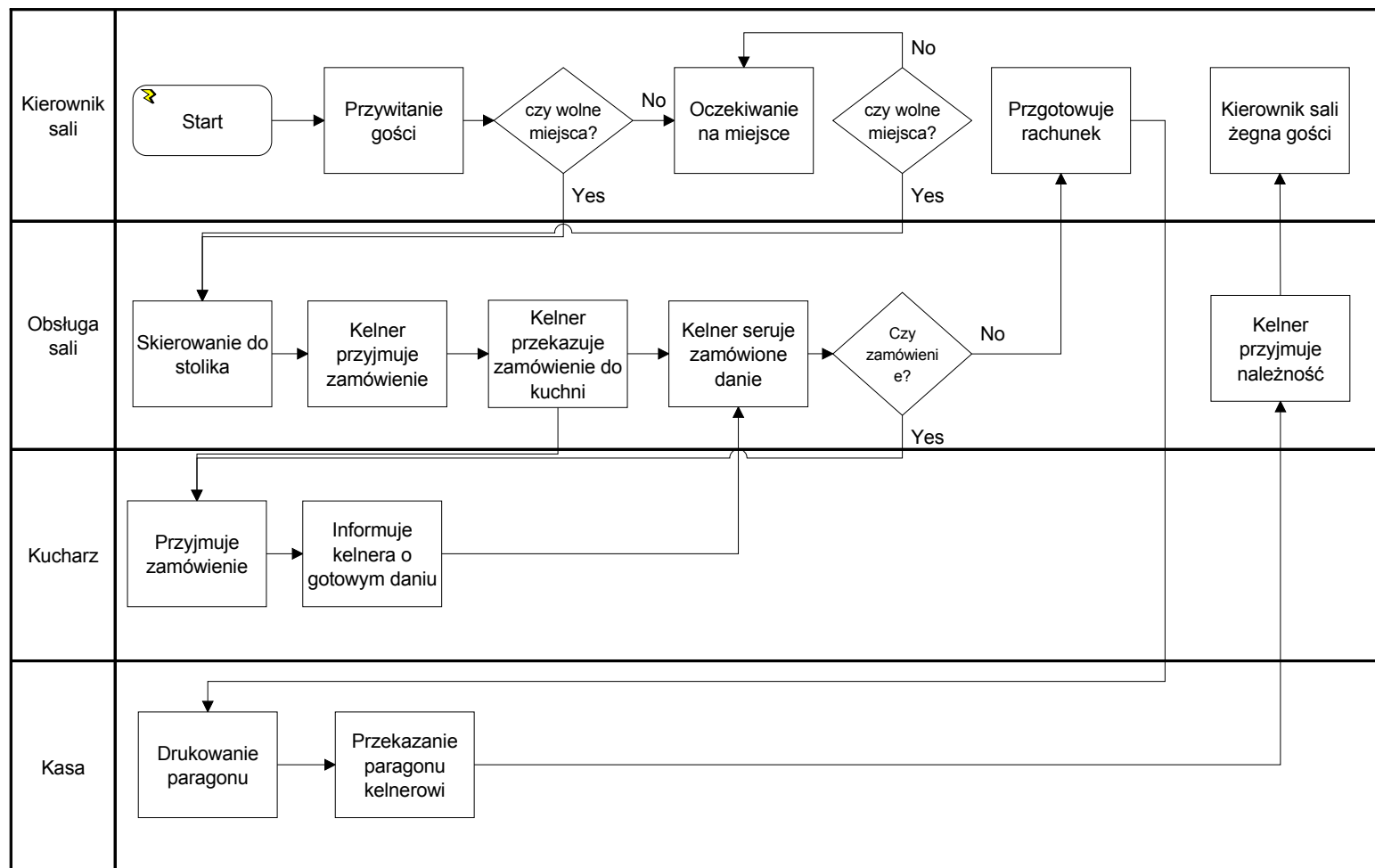


Figure 4 Map of the processes in the restaurant.

GUIDELINES FOR THE FORMAL SEMANTICS OF OBJECT-ORIENTED DESCRIPTION OF DIGITAL SYSTEMS FOR SIMULATION

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Abstrakt: .

Abstract: Models of connection nets created by object modeling technique are widely used in digital system design. In the paper we confront these models with a problem of digital systems simulation. A proposed solution is a components-based model of connection net (CMC), which implements a simulation function. For this purpose we map an example of a modular digital system by the CMC implemented in C#. Using a concept of ‘components chains’ to underlying the CMC by components we implement a simulation function. The final effect is the code of CMC worked out by hand, what pointed out the components chain as ‘mechanism’ supporting the programmer work.

1. Introduction

A digital system consists of ports, logical gates, modules, which are connected together by logical signals. Characteristic dynamic properties of digital system components define a delay and a signal level. An answer to a problem of switching function verification gives hardware description languages (HDL's). In front of HDL's complex definitions, as the VHDL, using object modeling technique we can define models of the logical connections net. Components-based models of connections net (in short: Component Model of Connections - CMC), being products of OMT, are created mapping hardware portions by objects implemented in an object-oriented language. CMC's are used in example in fast prototyping. However the simulation function is necessary for CMC to support the full cycle of digital systems design. Problems of hardware description and simulation have a series of publications, especially about the VHDL. Alike many publications report a wide use of OMT in the area of hardware description languages. In some opposition to these publications we propose to revise a definition of CMC giving the priority to the simulation function. The problem we formulate as follow: the ‘head’ of a CMC definition is a set of classes, which map a specific hardware, but if an implementation of the simulation function can underlying different sets of classes, defined for specific hardware.

In the paper, assuming modularity of digital systems, we define a simply implementation of CMC in the object oriented language. Having a sample of code we use a ‘components chain’ as a mechanism, which allows to underlying the sample by code of the simulation function. A draft of a modular digital system is given in section 2. To simplify a discussion one set an assumption that some typical, reference components-based model of the modular digital system exists. A concept of this reference components-based model gives the section 3. An example of the CMC is discussed in the section 4.

2. Modularization of Digital System Model

Modularization is a typical method of syntheses the hardware for complex switching function. Main goal is a unification of modules by input – output protocols. Another goal is to define scalable modules from unified modules. Usually system build from unified modules has a structure as on figure 1.

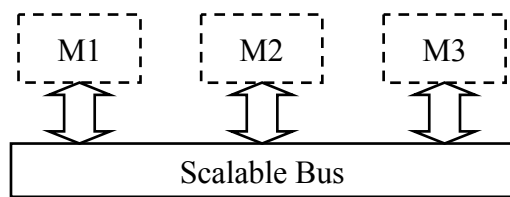


Figure 1. Modular scalable system

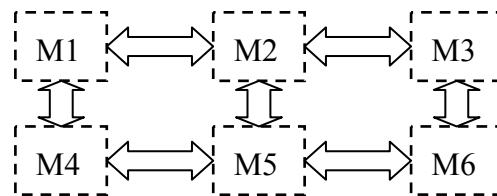


Figure 2. Matrix scalable system

Key aspects of modular, scalable digital system are following:

- a module has inputs and outputs,
- outputs of the module dynamically respond on inputs stimulations,
- a unified module has a behavioral or architectural specification,
- architectural specification can be nested, the module consists of modules

The listed key aspects caused that the VHDL description of hardware is elaborated and compiled to a net list in order to execute simulation. Mapping hardware by the CMC, which is implemented in an object oriented language, there is a problem to use the same way as VHDL for simulation.

3. Mapping The Modular Digital System To Components-Based Model

Mapping the modular digital system to components-based model, based on the class theory, are explored in digital systems design and prototyping e.g.: [1][3]. The full definition of CMC for specific hardware is complex, but on the level of modules we can create simply definitions of CMC. Assume the system contains of 1 to N modules. Definitions of CMCs for ‘system of 1 module’ and for ‘system of 2 modules’ have simply instantiations in code of the object-oriented language. Based on these definitions we can deduce a definition of the CMC for ‘system of N modules’. This thesis allows to discuss the problem of ‘underlying code’ on examples expecting the solution is true for system of N modules. We define, using the C# code, CMCs, which map digital systems given on figure 3 and figure 4. Second example includes a one way logical connection between modules. CMC maps only a structure and ports of the digital system. These definitions are abstract and have the same name ‘digitalsystem’, because in this work are distinguished by ‘namespace’. At this point we should remark that a set of classes e.g.: for modular digital systems has the semantic. We focus on problem of ‘underlying code’ of a simulation function so the implementation of CMC is used.

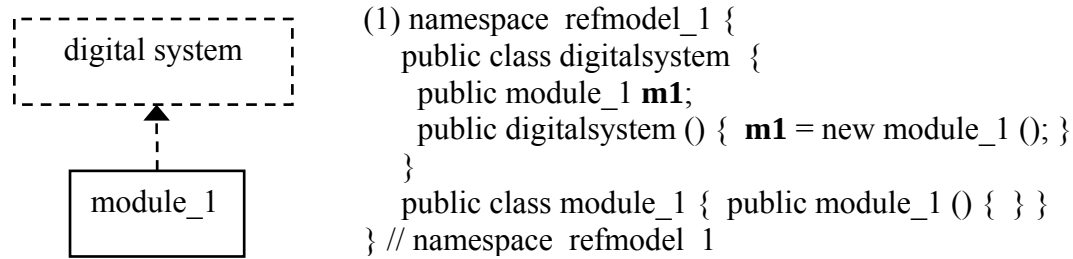


Figure 3. One module digital system model and the code mapping

Analyzing the digital system model on figure 3, which includes the modules, it is built a class definition. The class includes objects of types „module” and „connections”. Mapping the model we obtain the class ‘digitalsystem’, in C# code, as (1). Second example, on figure 4, is the digital system, which contains two modules and one connection. Mapping the model we obtain the class ‘digitalsystem’, in C# code, as (2)

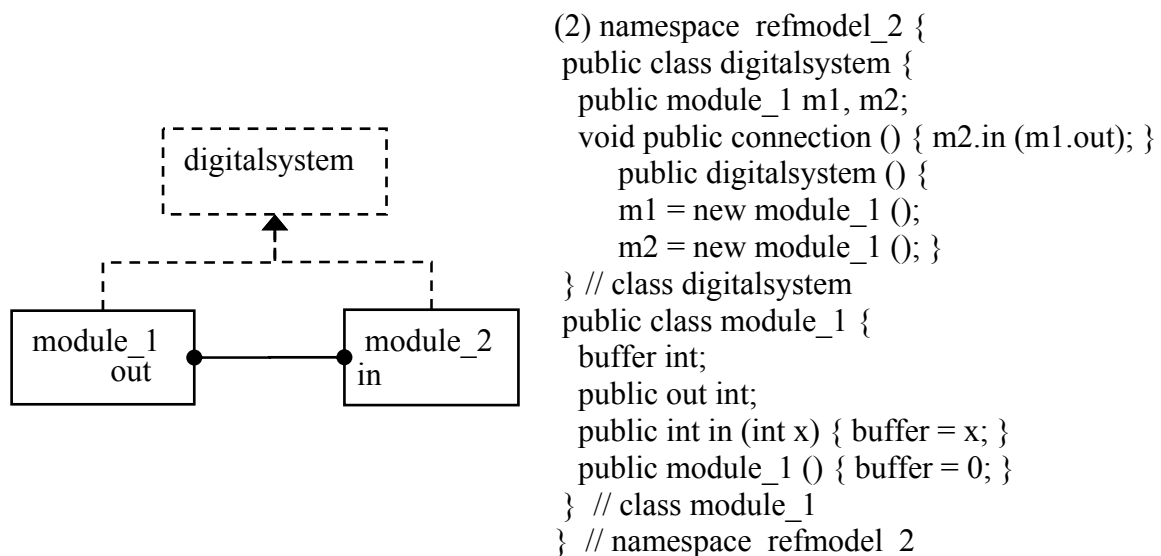


Figure 4. Digital system model and the code mapping

4. Use Of The Components Chain In A Simulation Function Implementation

4.1. The event-driven simulation

Let a digital system is a reactive system with inputs, and outputs. Behavior of the system defines the transactions (3) where N is a number of transactions, and i is an index of the transaction.

$$(3) \quad \{ \{x_1, t_1\}, \dots, \{x_i, t_i\}, \dots, \{x_N, t_N\} \}$$

The order of transactions defines the condition (4)

$$(4) \quad t_1 < t_i < t_N$$

(At this point we do consider that any transactions fulfill the condition: $t_1 \leq t_i \leq t_N$). A set of transactions we interpret as “a discreet time” of the system. If there are transactions on the input of a system, and the system is an inertial delay, the output defines transaction (5).

$$(5) \quad \{ \{y_1, t_1+dt\}, \dots, \{y_i, t_i+dt\}, \dots, \{y_N, t_N+dt\} \}$$

On input, output transactions defines condition (6), where dt is the value of an inertial delay, and $x_i, y_i \in \{0,1\}$.

$$(6) \quad t_i < t_{i+1}+dt$$

Above condition gives ‘short’ definition of an event – driven simulation algorithm for the use in the publication. The digital system simulator is a program, which processes input transactions. Considering a ‘short’ implementation the ‘simulator’ the code of the program contains the object of a digital system type, input and output data structures, and the loop. The code gives section (7)

```
(7) void simulator (int N) {
    int tg; // a global time of simulation
    int dt; // a minimum time of output transactions
    int n; int i;
    digitalsystem uc = new digitalsystem ();
    tg = 0; dt = -1; n = N;
    for (i=0; i<N; ++i) { uc.updateouts (int dt); uc.eventsoninputs ();
        dt = uc.getsampleoftime (); tg = tg + dt; // progress of a global time } //for
    }
```

The code of ‘main’ is not complete. Data structures of input, output transactions are leaved. A ‘short’ event – driven algorithm is implemented as the loop, which process methods of the ‘uc’ object, which maps the modular digital system.

4.2. Components chain in CMC of a one module digital system

Let the definition of CMC for a modular digital system express the code (1). When we assume the CMC has a simulation function then the code (1) has to be ‘underlying’ to fulfill the assumption. Assume equations (3)(4)(5)(6) define a simulation function and (7) gives a ‘short’ code of a simulator. The program implements a simulation function by a specific pattern, which consist of the following code:

```
(8) uc.updateouts (int dt);
    uc.eventsoninputs ();
    dt = uc.getsampleoftime ();
    tg = tg + dt; // progress of a global time
```

Where the ‘uc’ object is an instantiation of the definition of CMC, which has to be underlying. The code (9) of namespace CMC_1 is the code (1) of a CMC definition, which underlying the pattern (8). Bolded name ‘m1’ point out the underlying code. The CMC_1 consists of one module named ‘module_1’. A definition of ‘module_1’ gives the code (10). The definition is more sophisticated containing full definitions of all components (updateouts, eventsoninputs, getsampleoftime) of the pattern (8). These components implements equations (3)(4)(5)(6).

```
(9) namespace CMC_1 {
    public class digitalsystem {
        public module_1 m1;
        public void updateouts (int dt) { m1.updateouts (dt); }
        void public eventsoninputs () { m1. eventsoninputs (); }
        int getsampleoftime () { return m1.getsampleoftime (); }
        public digitalsystem { m1 = new module_1 (); }
    } // digitalsystem
```

```
(10) public class module_1 {
    public int x, y; private int delay, buffer, value_1;
    public void updateouts(int dt) { if ( delay == -1 ) { y = x; // logical signal
        } else { delay = delay - dt; // progress of local time
```

```

        if ( delay == 0 ) y = buffer; delay = -1; // output after delay
    } } // updateouts

    public void eventsoninputs () { if ( x != buffer ) { delay = value_1; }
        buffer = x; } // eventsoninputs

    public int getsampleovertime () { return delay; }

    public module_1 () { x = 0; y = 0; // start values
        delay = -1; buffer = 0; // state value
        value_1 = 100; // the parameter of inertial delay } // module_1
    } // class module_1
} // namespace CMC_1

```

The simulator of CMC_1 consist of code (7), (9) and (10). The simulation function is implemented by components chains, which start on lowest level (10) and are executed on highest level (7). An assumption that chains of components are immutable implies the code, which has to underlying CMC in order to implement a simulation function. Next subsection using the CMC definition explores a problem of code synthesis to fulfill the assumption.

5 Conclusion

Mapping modular digital systems by components-based models (CMC) is not a new problem, but has many solutions. One aspect of these solution is components-based models are implemented in object-oriented language. Confronting these models with a problem of the digital systems simulation we propose a solution by ‘components chains’. The solution allows a components-based model (CMC) underlying by components, which implement the simulation function. In the paper we define a CMC for a modular digital system. Using an implementation of the CMC in C# we define underlying components, ordered into chains, which implement the simulation function. Although, the final effect is the code of CMC worked out by hand, we pointed out the components chain as ‘mechanism’ supporting the programmer work

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PROBLEM OF PLANNING MATERIAL NEEDS IN THE INDUSTRY AND CONSTRUCTION

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Abstrakt:

Obecnie wykorzystuje się różne metody i techniki planowania potrzeb materiałowych dostosowane do aktualnych warunków danej firmy. Przy wyborze metody planowania uwzględnia się na ogół różne czynniki, a w szczególności typ produkcji (masowa, seryjna lub jednostkowa) oraz związana z tym długość cyklu produkcyjnego [1].

Abstract:

At present various methods and techniques of planning material needs adapted for apt conditions of business data are being exploited. At the choice of method the planning in general various factors, in particular a type production are being taken into account (mass, serial or individual) and length associated with it of a production cycle [1].

Exchanged factors indeed affect the accuracy and the technique of planning and the accuracy of arrangements of the plan of the supply. Material needs (P_m) appoint three basic components: planned material consumption - P_z , the set safety stock - Z_B , the real wrestling for the start of the period embraced with plan - Z_p .

Between these elements the following relation is occurring:

$$P_m = P_z + Z_B - Z_P \quad (1)$$

It is possible to take this relation back both to planning demand for the determined material assortment, as well as the group of materials or the whole of material needs of the plant. Planning the material consumption is a base of planning material needs. Depending on character of material consumption (base materials whether support) various methods and techniques of planning are being exploited. Consuming the base materials essential for the planned production to do is determined based on production programs and the structural structure of products and consumption standards [3].

Planned consuming base materials determines the relation:

$$P_z = S \cdot N_z \quad (2)$$

P_z – planned material consumption in the given period,

S – throughput of casts in the given period,

N_z – material consumption per unit of product,

In case of installation processes understood as the join of building blocks into the product a material list constitutes the ground for estimating the material consumption. It is the description of complexity selected by developing his structural structure, and then adding elements repeating itself up and single putting them in description [3]. The material list is appearing in two varieties: analytical, where materials are being sorted according to sub-assemblies in which they are appearing, and synthetic which all materials being included in a ready cast are included in.

For understood production processes as processes, in which the change of physicochemical properties, shapes, dimensions is taking place and the like planning the material consumption is appointed from the model:

$$P_z = S \cdot N_t \quad (3)$$

P_z – planned material consumption in the given period,

S – throughput of casts in the given period,

N_t – technical norm of the material consumption.

Consumption standard, being an important factor influencing the forming directly oneself sizes of material consumption, should be based on technical premises. On that account for calculating individual consumption standards appropriate formulae taking

into account of property materials and technological processes are applicable. Every norm is based on the same principle: an answering size justified losses and technological waste needs to add the sum extras to amount material a product contains which.

A safety stock is the second element plan of material needs. He determines the amount given kind material essential to ensure continuity production in period between two next supplies. In case of regulating it is supply of specific material accepted as the norm for the given period. The need of keeping the specific level supply results from the instability of provoked activity conditions of enterprise with the changeability demand for materials, unevenness and the nonrhythmicity of supplies. From here many enterprises are keeping the certain size of supply as the buffer or the safety stock in the case of the appearance of disruptions in completion of supplies or mistakes made at forecasting the demand.

In the first case on the basis of data from the past an average demand for material is settling accounts in period between two supplies as well as the maximum demand for products is being estimated [3].

The level safety stock must be in addition so big as to cover the difference between the average and maximum demand. This way so it is possible to describe level of safety stock as the time function new delivery:

$$Z_B = (P_{\max} - P_0) \cdot T \quad (4)$$

Z_B – level of safety stock,

$P_{\max}$ – maximum demand for the time unit,

P_0 – expected demand for the time unit,

T – delivery time.

For them the shorter, all the smaller delivery time of materials can be level safety stock. In the second case he is betting that level of safety stock depends on size of mistake, forecasting and plausibility of his address [4.5]. In the course of given cycle add the probability that magnitude of demand will exceed given level supplies, is settling with normal distribution. In practice industrial is applying increasing simplified ways

of estimating level of safety stock about sure about percent size delivery, e.g. for strategic materials 10 , of 20%, remaining materials by 5%.

With source and of formation there is a report of state materials produced based on the conducted control state of materials for determining real supplies. Being characteristic different ways and methods planning it isn't possible to forget MRP about the method of planning material needs based on forecast demand or orders customers called the method. Fundamentals this method are leaning against forecasting to which exclusively a demand for finished products is reporting (independent demand). However material needs (dependent demand) are being calculated directly based on structure of product. A division of needs into needs is an important component proceedings gross and net. Gross needs are needs of products in materials and elements resulting from efficient production plan and different standards; they determine the kind and the amount of materials and elements needed for course of a production process. Needs net correspond to materials indeed subjected to components for processing in given period. Establishing material needs gross and net constitutes the sure synchronization in setting lengths of time the demand for given assortment, with simultaneous agreeing on dates of demand and dates placing an order. Knowing cycle of order processing from external suppliers or cycle in house order processings (length of a production cycle) it is possible very much precisely to determine date of ordering materials.

This method allows for very precise establishing the moment of appearance the demand for given element, but also lets describe the magnitude of this demand. A minimization of supplies is fundamentals of this method [3].

Efficient managing the supply requires taking strategic decisions, allowing to adapt abilities of plant to a given production cycle in developing market economy. Making plans for the material demand in the industry is based on three basic kinds of demand: primitive, secondary and supplementing. For planning material needs of enterprise a conventional method is usually used. He lets for very precise establishing the moment appearance demand to given material and magnitudes the demand what allows for minimization of supplies essential for batch the accomplishment determined of products.

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APPLICATION OF THE RELIABILITY THEORY ELEMENTS FOR ANALYSING THE FAILURE FREQUENCY OF TECHNICAL FACILITIES

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Abstract: The article presents a possibility of using the reliability theory elements for evaluating the failure frequency of particular elements contained in a mechanised support used in hard coal mines. The reliability of particular elements translates directly into a failure-free period of work of the whole mechanised support. Using selected reliability measures, it has been shown which elements of a mechanised support fail most frequently. On the basis of such an analysis one can establish measures to be taken so as to prolong a failure-free period of work of an examined technical facility.

Introduction

Currently all enterprises are striving to minimise manufacturing costs while maintaining an adequate level of the product quality. A similar situation is observed in the case of hard coal mines, which have to improve their effectiveness. This can be done among others by increasing the failure-free time of work of mining machinery and equipment. People responsible for the condition of equipment should analyse the technical condition of its particular elements and carry out frequent inspections and overhauls, which should be included in relevant procedures. Another important thing is to observe physical parameters and analyse crucial assemblies and subassemblies of mining machinery and equipment by means of reliability measures.

Selected reliability measures for technical facility analysis

Below have been presented selected measures of reliability which enable evaluating the reliability of technical facilities.

The reliability of a technical facility in a form of exponential distribution

$$R(t) = e^{-\lambda t} \quad (1)$$

where:

- λ – intensity of damage,
- probability that a technical facility is serviceable at moment (t)

$$R_g(t) = 1 - F(t) + \int_0^t [1 - F(t-x)] \cdot \mu(x) dx \quad (2)$$

where:

$F(t)$ – serviceability time between failures cumulative distribution function,

$\mu(t)$ – function of technical facility regeneration [2,3,4].

The second group of reliability measures includes the ones which are related to the technical condition of a technical facility as well as the organisational and economic activity of service teams in the plant. Measures in this group are as follows [4]:

- mean time between failures – MTBF

$$MTBF = \frac{\text{Czas działania (operacyjny)}}{\text{Ilość uszkodzeń w jednostce czasu}} \quad (3)$$

- mean time to repair – MTTR (taking into consideration the time of reaction before taking repair measures)

$$MTTR = \frac{\text{Czas przestoju}}{\text{Ilość uszkodzeń w jednostce czasu}} \quad (4)$$

- mean downtime. It is a mean time from the moment repair activities are undertaken, together with the time needed to provide appropriate resources necessary to remove a failure [4].

The above described measures of reliability have provided a basis for creating two models which can be directly applied for technical facilities in industrial plants [3, 4].

- The model of technical facility availability, which takes into account a mean time between failures and a mean downtime.

$$A = \frac{MTBF}{MTBF + MTTR} \quad (5)$$

In practice this model shows a relationship between reliability and maintenance works related to the operation of machines and equipment [2, 4].

- The technical facility effectiveness model is described by the following dependence [3, 4]:

$$E = \frac{MTBF}{MTBF + MTTR} \quad (6)$$

This model has been used to present a dependence illustrating a mean time between failures and a mean time to repair. The effectiveness model reflects a relationship between the reliability of technical facilities and maintenance teams' effective and fast reaction to the failure. The model allows evaluating the efficiency of maintenance teams, responsible for removing failures in an enterprise [3, 4].

In the further research part will be presented reliability measures (indexes) belonging to the second group i.e. related to the technical condition of a technical facility as well as the ones taking into consideration the organisational and economic activity of relevant maintenance teams.

Readiness of a technical facility is related to reliability. It is one of reliability measures. According to PN-ISO/IEC2382-14:2001 standard, the definition of readiness is as follows: „This is a feature of a facility which allows it to fulfil the required functions in particular conditions, at a pre-set time or time interval, assuming that external sources (in this case: facility power supply) are ensured”.

„Moreover, an additional comment has been made: Readiness is defined here as internal readiness, assuming that the external sources – other than sources supplying the service workshop – do not influence the readiness of the facility. However, internal readiness

requires external sources to be ensured”. External sources are understood as consumables, fuel, etc. [2].

Readiness is a feature attributed to repairable facilities. We distinguish the following types of readiness [1]:

1. Mechanical readiness, also defined in subject literature as technical readiness – this is readiness for mechanical, preventive or service repair [1].
2. Physical readiness is a ratio of the number of machine’s work hours plus the number of downtime hours to the overall time.
3. Functional readiness is a ratio of the number of machine’s work hours to the sum of the number of work hours and the number of downtime hours [1].

Major indexes by means of which readiness can be characterised include operational readiness, which can be considered in relation to the construction and the system, expressed by the following formula [1, 3, 4]:

$$Go = \frac{MTBF}{MTBF + MDT} \quad (7)$$

where:

MTBF – mean time between failures,

MDT – mean downtime.

And in relation to serviceability and unserviceability according to the following formula:

$$GQ = \frac{T_o}{T_o + MDT} \quad (8)$$

where:

T_o– mean time of the system’s proper work,

MDT – mean downtime.

An example of applying the measures of reliability for evaluating the failure frequency of particular elements in a mechanised support.

Table 1 presents the mean time of the whole mechanised support, which reaches MTBF = 30480 minutes. For particular systems the value of MTBF is as follows: for the electrical system – 274320, the longest time, whereas for organisational failures it reaches 42203 minutes. The duration of failures for all the system and organisational failures is comparable, ranging from 38 to 44 and, in consequence, readiness reaches high values.

Table 1

MTBF, MDT, MUT, G and reparability for a mechanised support

Failure	MTBF	MDT	MUT	G	Reparability (N)
hydraulic	182880	38	182843	0.9998	0.0002
electrical	274320	41	274279	0.9999	0.0001
organisational	42203	44	42159	0.9990	0.0010
whole mechanised support	30480	38	30443	0.9990	0.0012

In the chart (Fig. 1) the probability of a mechanised support’s failure-free work after approximately a month of its operation is 61%. The highest probability characterises the mechanical system – 95% and the hydraulic system – 90%. The probability of a mechanised support’s failure-free work due to organisational reasons reaches 72%.

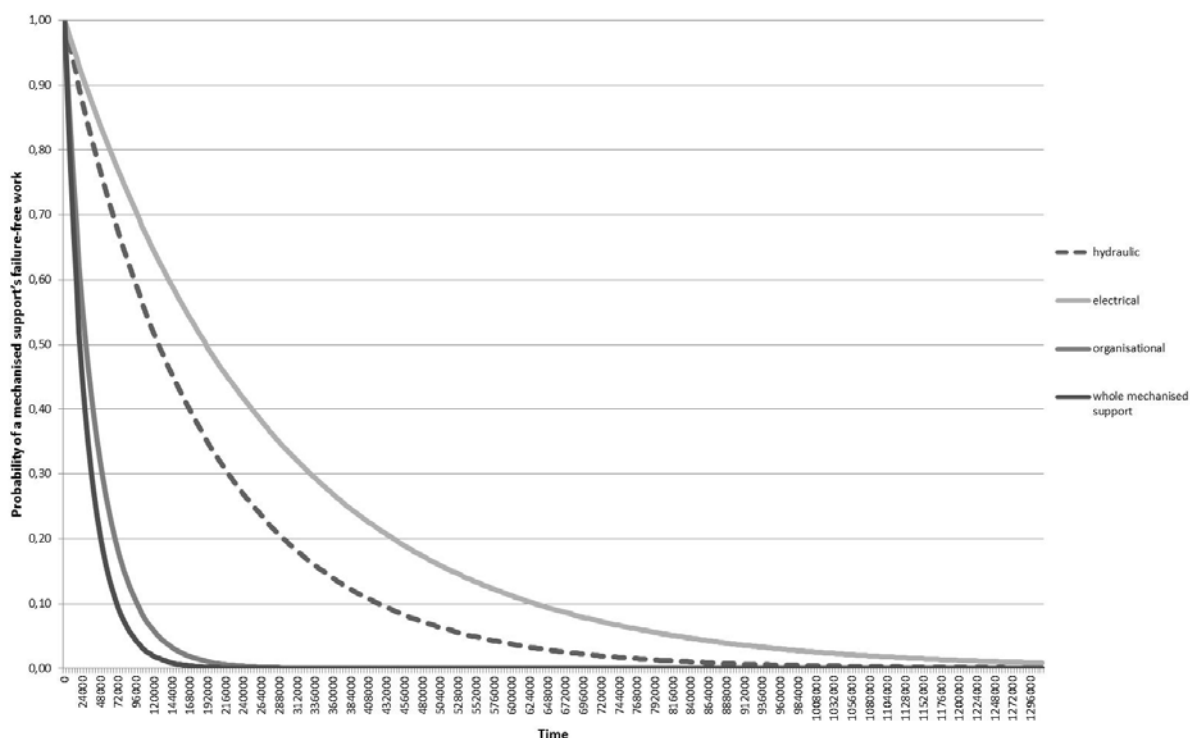


Fig. 1. The function of a mechanised support's reliability

Summary

The mean time between failures (MTBF) for a mechanised support is 30480 minutes; it is the shortest for the electrical system – MTBF = 27432 minutes, and for failures due to organisational reasons the value of MTBF reaches 42203 minutes. The duration of failures and downtimes due to organisational reasons is comparable – from 38 to 44 minutes and, in consequence, readiness reaches high values. Monitoring of particular systems in mining machines and equipment enables better control over the crucial elements of technical facilities, which translates into an increased time of their failure-free work and reliability. Evaluation of the reliability and readiness of particular elements in mining machines and equipment will influence the planning of spare parts purchase and the scheduling of downtimes for maintenance works and overhauls.

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LEPAGEAN EQUIVALENTS VERSUS MULTISYMPLECTIC FORMS

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Abstract: This paper is devoted to geometric formulation of Hamiltonian systems based upon the Lepagean $(n + 1)$ -forms. The relations to multisymplectic theory are studied.

Abstrakt: Článek je věnován geometrické formulaci Hamiltonovy teorie založené na použití Lepageových $(n + 1)$ -forem. Souvislosti s multisymplektickou teorií jsou studovány.

1 Lepagean forms and Hamiltonian systems

A surjective submersion $\pi : Y \rightarrow X$ is called a *fibred manifold* with a *base* X , $\dim X = n$ and a *total space* Y , $\dim Y = m + n$. For every point $x \in X$, the submanifold $\pi^{-1}(x) \in Y$ is called a *fibre* over x ($\dim \pi^{-1}(x) = m$).

To every fibred chart (V, ψ) , $\psi = (x^i, y^\sigma)$, $1 \leq i \leq n$, $1 \leq \sigma \leq m$, on π there exist an *associated fibred chart* on J^1Y , denoted by (V_1, ψ_1) , $\psi_1 = (x^i, y^\sigma, y_j^\sigma)$.

We recall Saunders ([8]) for more details on calculus of fibred manifold and its associated structures.

Every form $\eta \in \Lambda^q(J^sY)$ admits a unique (canonical) decomposition into a sum of q -forms on $J^{s+1}Y$ as follows:

$$\pi_{s+1,s}^* \eta = h(\eta) + \sum_{k=1}^q p_k(\eta),$$

where $h(\eta)$ is a horizontal form, called the *horizontal part* of η , and $p_k(\eta)$, $1 \leq k \leq q$, is a k -contact form, called the *k -contact part* of η . ([3])

A $(n+1)$ -form E on $J^s Y$, $s \leq 1$, is called a *dynamical form* if it is 1-contact and $\pi_{s,0}$ -horizontal. This means that E is a dynamical form iff in every fibred chart

$$E = E_\sigma \omega^\sigma \wedge \omega_0,$$

where E_σ are functions on $V_s \subset J^s Y$. A section γ of π is called a *path* of E if $E \circ J^s \gamma = 0$. In fibred coordinates this equation represents a *system of m partial differential equations of order s* .

Definition. (Krupková [4]) Let $s \geq 0$. A *closed $(n+1)$ -form α* on $J^s Y$ will be called *Lepagean* if $p_1 \alpha$ is a dynamical form (i.e., $p_1 \alpha = E$).

We say that two Lepagean $(n+1)$ -forms α_1 and α_2 are *equivalent* if (up to a possible projection)

$$p_1 \alpha_1 = p_1 \alpha_2. \quad (1)$$

The equivalence class of α will be denoted by $[\alpha]$. Note that, the forms α_1 and α_2 are possibly of different orders.

By a *Hamiltonian system of order s* we shall mean a Lepagean $(n+1)$ -form α on $J^s Y$. A section δ of the fibred manifold π_s is called a *Hamilton extremal* of α if

$$\delta^* i_\xi \alpha = 0, \quad (2)$$

for every π_s -vertical vector field ξ on $J^s Y$.

Note that Hamilton equations are not uniquely determined by an Euler–Lagrange form (respectively, by a Lagrangian) but depend upon the form $\pi_{s+1,s}^* \alpha - E$, i.e., the part of α which is at least 2-contact. Consequently, one has many different “Hamilton theories” associated to a given variational problem.

On the other hand, we can see that two different Lepagean n -forms ρ_1 and ρ_2 (possibly of different orders) give rise to the same Hamiltonian system whenever $d\rho_1 = d\rho_2$, i.e., locally, $\rho_2 = \rho_1 + d\eta$. In this sense we can understand a Hamiltonian system to be the equivalence class for (generally locally defined) Lepagean n -forms, differing by closed n -forms.

Definition. [5] A section δ of the fibred manifold $\pi_s : J^s Y \rightarrow X$ is called a *Dedecker section* if $\delta^* \mu = 0$ for every at least 2-contact form μ on $J^s Y$.

A Dedecker’s section which is a Hamilton extremal of $\pi_{s+1,s}^* \alpha$ is called *Dedecker–Hamilton extremal* of λ . Hamilton equations, considered as equations for Dedecker’s sections, are called *Dedecker–Hamilton equations*.

We shall study in detail the case of Lepagean $(n+1)$ -forms defined on $J^1 Y$. These Hamiltonian systems are the most simple ones from the mathematical point of view, and, moreover, from the physical point of view they represent Hamiltonian counterparts of all the most interesting Lagrangian systems in field theory.

For details of first order systems you can see [6], [7] and for higher order generalizations we refer to [5].

Let α be a Lepagean $(n+1)$ -form on $J^1 Y$. Using the canonical decomposition of α into the sum of i -contact components, $1 \leq i \leq n+1$, we write

$$\pi_{2,1}^* \alpha = E + F + G,$$

where $E = p_1 \alpha$, $F = p_2 \alpha$, and G is at least 3-contact. E satisfies the Anderson–Duchamp–Krupka conditions, i.e.,

$$\frac{\partial E_\sigma}{\partial y^\nu} - \frac{\partial E_\nu}{\partial y^\sigma} + d_j \frac{\partial E_\nu}{\partial y_j^\sigma} - d_j d_k \frac{\partial E_\nu}{\partial y_{jk}^\sigma} = 0, \quad (3)$$

$$\begin{aligned}\frac{\partial E_\sigma}{\partial y_j^\nu} + \frac{\partial E_\nu}{\partial y_j^\sigma} - 2d_k \frac{\partial E_\nu}{\partial y_{jk}^\sigma} &= 0, \\ \frac{\partial E_\sigma}{\partial y_{jk}^\nu} - \frac{\partial E_\nu}{\partial y_{jk}^\sigma} &= 0,\end{aligned}$$

and F takes the form

$$\begin{aligned}F &= \left(\frac{1}{4} \left(\frac{\partial E_\sigma}{\partial y_i^\nu} - \frac{\partial E_\nu}{\partial y_i^\sigma} \right) - d_j f_{\sigma\nu}^{i,j} \right) \omega^\sigma \wedge \omega^\nu \wedge \omega_i \\ &+ \left(\frac{\partial E_\sigma}{\partial y_{ij}^\nu} - 2f_{\nu\sigma}^{j,i} \right) \omega^\sigma \wedge \omega_j^\nu \wedge \omega_i + f_{\sigma\nu}^{jk,i} \omega_j^\sigma \wedge \omega_k^\nu \wedge \omega_i,\end{aligned}\tag{4}$$

where

$$f_{\sigma\nu}^{j,k} - f_{\nu\sigma}^{k,j} - d_i f_{\sigma\nu}^{jk,i} = 0$$

and $f_{\sigma\nu}^{j,k}, f_{\sigma\nu}^{jk,i}$ are arbitrary functions satisfying the antisymmetry relations

$$f_{\sigma\nu}^{j,k} = -f_{\sigma\nu}^{k,j}, f_{\sigma\nu}^{ki,j} = -f_{\sigma\nu}^{ji,k}, f_{\sigma\nu}^{ki,j} = -f_{\nu\sigma}^{ik,j}.$$

Denote

$$D_\alpha^1 = \{i_\xi \alpha \mid \text{where } \xi \text{ runs over all } \pi_1\text{-vertical vector fields on } J^1 Y\}\tag{5}$$

We call the ideal of differential forms on $J^1 Y$ generated by the system of n -forms D_α^1 the Hamiltonian ideal related with α . Note that now, Hamilton equations (2) mean that Hamilton extremals identify with integral sections of the Hamiltonian ideal.

Denote

$$\hat{\alpha} = E + F$$

and call $\hat{\alpha}$ the *principal part of* α . Note that $\hat{\alpha}$ is an $(n+1)$ -form on $J^2 Y$, generally not closed.

Definition. [4] Let $V\pi_1$ be a set of all π_1 -vertical vector fields on $J^1 Y$. A Hamiltonian system α on $J^1 Y$ is called *regular* if $D_\alpha^1 = \text{rank} V\pi_1$, and the system of local generators of D_α^1 contains all the n -forms $\omega^\sigma \wedge \omega_i$, $1 \leq \sigma \leq m$, $1 \leq i \leq n$.

Theorem. [4] *Let α be a first-order Hamiltonian system. If α is regular then it holds:*

(1) *Every Dedecker-Hamilton extremal $\hat{\delta}$ of α projects onto an extremal δ of $E = p_1 \alpha$ (i.e., $\delta = \pi_{2,1} \hat{\delta}$).*

(2) *The map J^1 is a bijection of the set of extremals of $E = p_1 \alpha$ onto the set of $\pi_{2,1}$ -projections of Dedecker-Hamilton extremals of E .*

Theorem. [4] *Let α be a first order Hamiltonian system. Suppose that $f_{\sigma\nu}^{jk,i} = 0$. The following conditions are equivalent:*

(1) *It holds*

$$\det \left(\frac{\partial E_\sigma}{\partial y_{ij}^\nu} - 2f_{\nu\sigma}^{i,j} \right) \neq 0.\tag{6}$$

where in the indicaten $(mn \times mn)$ -matrix, (σ, i) labels rows and (ν, j) labels columns.

(2) α is regular.

Definition. [4] A Hamiltonian system α will be called *strongly regular* if Hamilton extremals of λ are in bijective correspondence with extremals of $E = p_1 \alpha$.

Note that every strongly regular Hamiltonian system is regular. On the other hand, there exist regular Hamiltonian systems which are not strongly regular.

2 Relations to multisymplectic theory.

Recall that an $(n+1)$ -form Ω on a manifold M is called *multisymplectic* if it is *closed*, and the map $\xi \rightarrow i_\xi \Omega$, mapping vector fields on M to n -forms, is *injective* [1].

A *closed* $(n+1)$ -form α on $J^s Y$ will be called *Lepagean* if $p_1 \alpha$ is a dynamical form (i.e., $p_1 \alpha = E_\sigma \omega^\sigma \wedge \omega_0$) [4].

Variationality. The $p_1 \alpha$ of Lepagean $(n+1)$ -form α is locally variational (i.e., we can always find the corresponding Lagrangian). For given multisymplectic form we need not find the corresponding Lagrangian.

Regularity. In the multisymplectic theory, the regularity condition is property of the form. In the Lepagean case, the regularity condition follows from the Hamiltonian equations.

Forms on Y : All closed $(n+1)$ -forms on Y are *uniquely* characterized by the following formula

$$\begin{aligned} \alpha &= E_\sigma \omega^\sigma \wedge \omega_0 + \sum_{k=1}^n \frac{1}{k!(k+1)!} \frac{\partial^k E_\sigma}{\partial y_{j_1}^{\nu_1} \dots \partial y_{j_k}^{\nu_k}} \omega^\sigma \wedge \omega^{\nu_1} \wedge \dots \wedge \omega^{\nu_k} \wedge \omega_{j_1 \dots j_k} \\ &= B_\sigma dy^\sigma \wedge \omega_0 + \sum_{k=1}^n \frac{1}{(k+1)!} B_{\sigma \nu_1 \dots \nu_k}^{j_1 \dots j_k} dy^\sigma \wedge dy^{\nu_1} \wedge \dots \wedge dy^{\nu_k} \wedge \omega_{j_1 \dots j_k}, \end{aligned}$$

where

$$\begin{aligned} E_\sigma &= B_\sigma + B_{\sigma \nu_1}^{j_1} y_{j_1}^{\nu_1} + \dots + B_{\sigma \nu_1 \dots \nu_n}^{j_1 \dots j_n} y_{j_1}^{\nu_1} \dots y_{j_n}^{\nu_n}, \\ B_{\sigma \nu_1 \dots \nu_p \dots \nu_q \dots \nu_k}^{j_1 \dots j_p \dots j_q \dots j_k} &= B_{\sigma \nu_1 \dots \nu_q \dots \nu_p \dots \nu_k}^{j_1 \dots j_q \dots j_p \dots j_k}, \quad B_{\sigma \nu_1 \dots \nu_p \dots \nu_k}^{j_1 \dots j_p \dots j_k} = B_{\nu_p \nu_1 \dots \sigma \dots \nu_k}^{j_1 \dots j_p \dots j_k}. \end{aligned}$$

The form $E = E_\sigma \omega^\sigma \wedge \omega_0$ is variational (i.e., this form satisfies the formulas (3))

Example: ([2]) On $R^n \times R^m \rightarrow R^n$ consider a form α as follows:

$$\alpha = B_\sigma dy^\sigma \wedge \omega_0 + \frac{1}{2} B_{\sigma \nu}^1 dy^\sigma \wedge dy^\nu \wedge \omega_1, \quad d\alpha = 0, \quad \det(B_{\sigma \nu}^1) \neq 0.$$

The corresponding dynamical form is obviously $E = (B_\sigma + B_{\sigma \nu}^1 y_1^\nu) \omega^\sigma \wedge \omega_0$, where $B_\sigma, B_{\sigma \nu}^1$ are functions independent of the variables y_j^ρ , and satisfying the variationality conditions (3)

$$B_{\sigma \nu}^1 = -B_{\nu \sigma}^1, \quad \frac{\partial B_\sigma}{\partial y^\nu} - \frac{\partial B_\nu}{\partial y^\sigma} + \frac{\partial B_{\sigma \nu}^1}{\partial x^1} = 0, \quad \frac{\partial B_{\sigma \rho}^1}{\partial y^\nu} + \frac{\partial B_{\rho \nu}^1}{\partial y^\sigma} + \frac{\partial B_{\nu \sigma}^1}{\partial y^\rho} = 0.$$

Since, by assumption, the rank of the $(m \times (m+1))$ -matrix $(B_\sigma, B_{\sigma \nu}^1)$ is maximal (equal to m), the form α is *regular zero order Hamiltonian system*. However, computing

$$\begin{aligned} i_\zeta \alpha &= \zeta^\sigma (B_\sigma \omega_0 + B_{\sigma \nu}^1 dy^\nu \wedge \omega_1) - \zeta^i \left(B_\sigma dy^\sigma \wedge \omega_i - \frac{1}{2} B_{\sigma \nu}^1 dy^\sigma \wedge dy^\nu \wedge \omega_{1i} \right) \\ &= \zeta^\sigma B_\sigma \omega_0 - \left(\zeta^\nu B_{\sigma \nu}^1 + \zeta^1 B_\sigma \right) dy^\sigma \wedge \omega_1 - \sum_{i=2}^n \zeta^i B_\sigma dy^\sigma \wedge \omega_i \\ &\quad + \frac{1}{2} \zeta^i B_{\sigma \nu}^1 dy^\sigma \wedge dy^\nu \wedge \omega_{1i} = 0 \end{aligned}$$

for a vector field $\zeta = \zeta^i \frac{\partial}{\partial x^i} + \zeta^\sigma \frac{\partial}{\partial y^\sigma}$, and taking into account that $\omega_{11} = 0$, we obtain

$$\zeta^\sigma = -M^{\sigma \nu} B_\nu \zeta^1, \quad 1 \leq \sigma, \nu \leq m, \quad \zeta^2 = \dots = \zeta^n = 0,$$

where $(M^{\sigma\nu})$ is the inverse matrix to $(B_{\sigma\nu}^1)$. Hence, the map $\zeta \rightarrow i_\zeta \alpha$ has a one-dimensional kernel spanned by the everywhere non-zero vector field $\Xi = \frac{\partial}{\partial x^1} - M^{\sigma\nu} B_\nu \frac{\partial}{\partial y^\sigma}$, meaning that the form α is *not multisymplectic*.

As a consequence of the above we get that multisymplectic forms do not coincide with regular Hamiltonian systems.

Forms on J^1Y : The situation on J^1Y is different from situation on Y . In general, forms on J^1Y are not $\pi_{1,0}$ -horizontal (i.e., $p_1\alpha = E_\sigma\omega^\sigma \wedge \omega_0 + E_\sigma^i\omega_i^\sigma \wedge \omega_0$). This fact means that there exist multisymplectic forms which are not Lepagean $(n+1)$ -form.

One can characterize the closed $\pi_{1,0}$ -horizontal forms α (i.e., $p_1\alpha = E_\sigma\omega^\sigma \wedge \omega_0$) by the formula

$$\begin{aligned} \alpha = & E_\sigma\omega^\sigma\omega_0 + \left(\frac{1}{4} \left(\frac{\partial E_\sigma}{\partial y_i^\nu} - \frac{\partial E_\nu}{\partial y_i^\sigma} \right) - d_j f_{\sigma\nu}^{i,j} \right) \omega^\sigma \wedge \omega^\nu \wedge \omega_i \\ & + \left(\frac{\partial E_\sigma}{\partial y_{ij}^\nu} - 2f_{\nu\sigma}^{j,i} \right) \omega^\sigma \wedge \omega_j^\nu \wedge \omega_i + f_{\sigma\nu}^{jk,i} \omega_j^\sigma \wedge \omega_k^\nu \wedge \omega_i + G, \end{aligned} \quad (7)$$

where $f_{\sigma\nu}^{j,k} - f_{\nu\sigma}^{k,j} - d_i f_{\sigma\nu}^{jk,i} = 0$ and $f_{\sigma\nu}^{j,k}, f_{\sigma\nu}^{jk,i}$ are arbitrary functions satisfying the antisymmetry relations $f_{\sigma\nu}^{j,k} = -f_{\sigma\nu}^{k,j}$, $f_{\sigma\nu}^{ki,j} = -f_{\sigma\nu}^{ji,k}$, $f_{\sigma\nu}^{ik,j} = -f_{\nu\sigma}^{jk,i}$. Comparing the regularity condition of the Lepagean $(n+1)$ -forms (6) we can see that the regular Lepageans $(n+1)$ -forms do not coincide with multisymplectic forms.

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MATRIX CALCULUS AND SPREADSHEET IN PLANNING OF OPERATING COSTS

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Abstract: The article is a discussion on the concept of the matrix calculus application in combination with a spreadsheet for planning of operating costs. Application of the matrix calculus in cost planning has been discussed in the first part of the paper. The approach proposed expands the concepts presented previously in the literature of the subject. Further sections of the article address a sample set of matrices and array formulas enabling the solution proposed to be used in a spreadsheet.

Introduction

Planning of operating costs is an important problem from the perspective of an enterprise's operations. Particularly in large corporations, having a highly sophisticated organisational structure, complex production process and diversified range of products or services, taking the most significant dependencies into consideration in the course of the planning process may prove challenging. An approach one can potentially adopt in this respect is to refer to the existing dependencies in corresponding matrices and applying the matrix calculus when planning costs. The solution proposed further in the article expands the concepts presented in the literature, devoted to using the matrix calculus in cost management [1, 2]. Developing cost plans involves a necessity to process considerable amounts of data. It is required to use suitable IT tools to support such a process. Some popular tools commonly used to support information handling under the processes performed in various areas of an enterprise are spreadsheets [3, 4]. A sample plain matrix set, along with appropriate formulas enabling a cost plan to be created using the MS Excel spreadsheet by application of the matrix calculus being discussed, has been provided in the final part of the article.

Cost planning using matrix calculus

For the sake of cost planning, in accordance with the MSRP method proposed [5], an enterprise can be brought down to a series of connected components corresponding to products, semi-finished products, materials, operations performed, labour etc. For every single component, one can establish its flow expressed by means of appropriate natural units, such as pieces, kilo grams, number of operations, man-hours or the applicable currency. One may observe specific connections occurring between the components, assuming the form of consumption standards envisaged (Fig. 1). The determine standard volumes of flow of secondary components assuming a unit flow volume of the primary component.

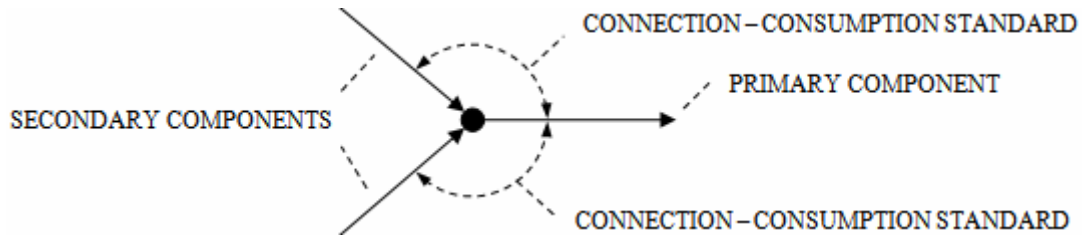


Fig. 1 Connections between components

All components may be divided into 2 groups: quantitative components and cost components (Fig. 3).

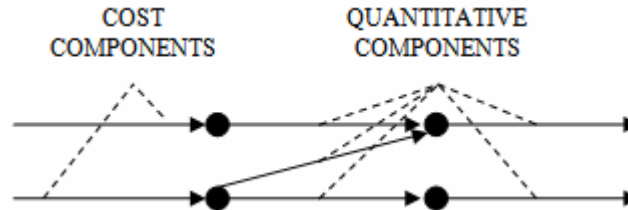


Fig. 2 Breakdown of components into groups

Quantitative components are those whose flow volume can be expressed in natural units other than currency. The components grouped under this category may serve as primary as well as secondary components while establishing connections. Cost components are references to costs which are to be planned. The natural unit in this respect is the applicable currency. While establishing connections, these components may only occur as the secondary ones. Connections between components may be represented by means of two matrices. A dependence between individual quantitative components has been shown in matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \quad (1)$$

where:

n – number of quantitative components

a_{ij} ; $i, j = 1, 2, \dots, n$; – flow volume of the i^{th} component on the assumed unit flow of the j^{th} component

A dependence between quantitative components and cost components has been shown in matrix

$$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix} \quad (2)$$

where:

n – number of quantitative components

m – number of cost components

b_{ij} – $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$ – flow volume of the i^{th} component on the assumed unit flow of the j^{th} component

Having assumed specific dependencies between individual components, in order to establish a cost plan one must necessarily determine external flows for the quantitative components. External flows are those which determine all other flows of components. While planning them, one should entail various targets, these including:

1. sales, if necessary, broken down into various sales areas
2. shifts in stock levels of products, semi-finished products, work in progress etc.
3. fixed consumption of individual components (number of operations or permanent electricity consumption irrespective of the production volume) in a breakdown into individual organisational units of the enterprise.

The plan may be represented using two matrices:

$$C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1k} \\ c_{21} & c_{22} & \dots & c_{2k} \\ \dots & \dots & \dots & \dots \\ c_{n1} & c_{n2} & \dots & c_{nk} \end{bmatrix} \quad (3)$$

where:

k – number of targets for external flows,

n – number of quantitative components,

c_{ij} – $i = 1, 2, \dots, n$; $j = 1, 2, \dots, k$ – planned external flow of the i^{th} component towards the target of j

Individual targets of external flows may be assigned to the aforementioned three groups of targets using the following matrix

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ \dots & \dots & \dots \\ d_{k1} & d_{k2} & d_{k3} \end{bmatrix} \quad (4)$$

where:

k – number of targets for external flows,

$d_{ij} = 1$, when target “ i ” belongs to a group of targets “ j ”, $d_{ij} = 0$, when target “ i ” does not belong to a group of targets “ j ”, $i = 1, 2, \dots, k$; $j = 1, 2, 3$

By multiplying matrix C by matrix D, one obtains a matrix of external flows entailing the division into target groups.

$$E = C \cdot D \quad (5)$$

Based on matrices E and A, one can develop the matrix of overall flows for the quantitative components.

$$F = (I - A)^{-1} \cdot E \quad (6)$$

In three columns of matrix F, there are quantitative flows corresponding to three groups of external flows.

Based on matrices F and B, one can develop the matrix of overall flows for the cost components.

$$G = B \cdot F \quad (7)$$

One may also claim that:

$$G = B \cdot (I - A)^{-1} \cdot C \cdot D \quad (8)$$

In three columns of matrix G, there are flows corresponding to three groups of external flows. By that means, one can obtain a cost plan based on a breakdown into costs resulting from the volume of sales and shifts in stock levels, and entailing the permanent nature of internal consumption.

Spreadsheet-based cost plan

The concept proposed may be implemented in a spreadsheet by means of array formulas based on such functions as IF(), MMULT() or MINVERSE(). A sample set of matrices in a spreadsheet with the formulas applied has been provided in Figures 3 and 4.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Matrix B									Matrix A							
2		1	2	3	4	5	6	7			1	2	3	4	5	6	7
3	1	0,8	0	0	0	0	0	0		1	0	1,1	0	0	0	0	1,3
4	2	0	0	1,2	0	0	0	0		2	0	0	0	0	0	1	0
5	3	0	7	0	0	0	1	0		3	0	0	0	0	0	14	0
6	4	0	0	0	45	20	0	0		4	0	0,6	0	0	0	0	0
7	5	0	0	0	20	0	0	0		5	0	0	0	0	0	0	0,3
8										6	0	0	0	0	0	0	2
9										7	0	0	0	0	0	0	0
10	Matrix C																
11	C	1	2	3	4	5											
12	1	0	0	0	0	20				Matrix D							
13	2	0	0	0	0	0				D	1	2	3				
14	3	0	0	-10	0	0				1	1	0	0				
15	4	0	0	0	0	15				2	1	0	0				
16	5	0	0	0	0	0				3	0	1	0				
17	6	30	0	0	25	0				4	0	1	0				
18	7	80	40	0	0	0				5	0	0	1				

Fig. 3 Sample set of matrices A, B, C, D in a spreadsheet

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
19	Matrix E					Matrix F					Matrix G			
20		1	2	3			1	2	3			1	2	3
21	1	0	0	20		1	453,0	27,5	20,0		1	362,4	22,0	16
22	2	0	0	0		2	270,0	25,0	0,0		2	4536,0	408,0	0
23	3	0	-10	0		3	3780,0	340,0	0,0		3	2160,0	200,0	0
24	4	0	0	15		4	162,0	15,0	15,0		4	8010,0	675,0	675
25	5	0	0	0		5	36,0	0,0	0,0		5	3240,0	300,0	300
26	6	30	25	0		6	270,0	25,0	0,0					
27	7	120	0	0		7	120,0	0,0	0,0					
28														
29														
F01	{=MMULT(B12:F18;K14:M18)}													
F02	{=MMULT(MINVERSE(IF(K2:Q2=J3:J9;1-K3:Q9;-K3:Q9));B21:D27)}													
F03	{=MMULT(B3:H7;G21:I27)}													

Fig. 4 Sample set and collation of array formulas for matrices E, F, G

With reference to the above set of matrices, the time required to perform all calculations for several exemplary equipment setups were conducted. For each setup, times were examined for 100, 500 and 1,000 quantitative components. In each case, the

number of cost components and external flow targets came to 50. The results obtained have been illustrated in Figure 5.

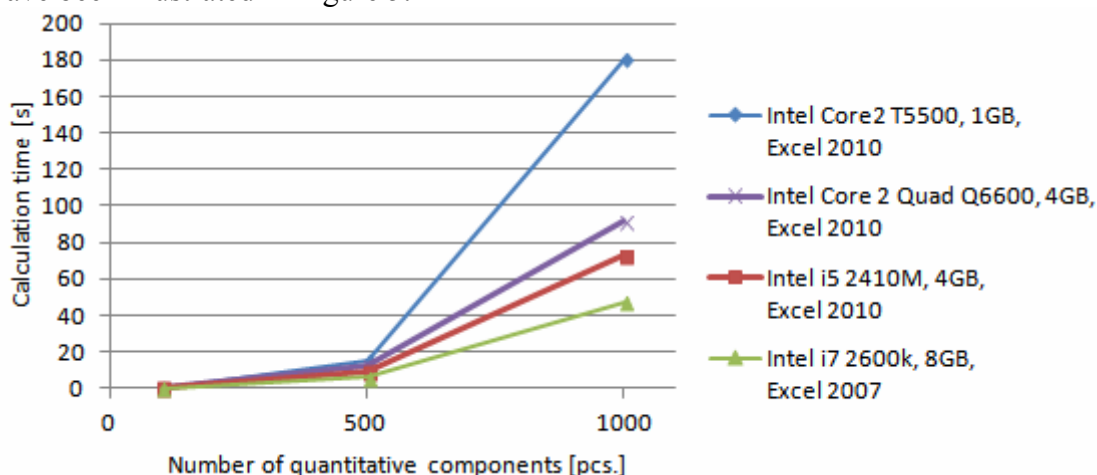


Fig. 5 Time of cost calculations depending on the number of quantitative components

Conclusions

The concept proposed and described above proves that a matrix calculus can be successfully used for cost planning entailing both the sales volume and shifts in stock levels as well as fixed external consumption. A spreadsheet can be successfully used for data processing in this respect. The calculation times obtained for the set of matrices and array formulas proposed are considerable enough to cause that data editing must be handled in the manual calculations mode, however, particularly for latest processors, they are short enough to ensure appropriate functionality of the tool used.

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MAJORIZACE POSLOUPNOSTÍ

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Abstrakt: Některé posloupnosti přirozených čísel lze uspořádat pomocí tzv. relace majorizace. V příspěvku se nebudeme zabývat obecnou teorií této relace, ale představíme konkrétní posloupnosti, mezi nimiž vztah majorizace platí.

Abstract: Certain sequences of nonnegative integers can be ordered using a majorization relation. In this contribution, we do not discuss the general theory of this relation but we present several particular sequences which fulfill mentioned relationship.

V celém příspěvku budeme pracovat s maticemi nad polem komplexních či reálných čísel, a to pouze se čtvercovými maticemi, resp. se čtvercovými blokovými maticemi, jejichž bloky na diagonále jsou čtvercové. Symbolem E budeme značit jednotkovou matici příslušného řádu. Charakteristický polynom matice A budeme uvažovat ve tvaru $\det(\lambda E - A)$

Uveďme nyní definice základních pojmů.

Definice 1: Nechť A je komplexní čtvercová matice řádu n . *Jádrem* $\text{Ker } A$ *matice* A rozumíme množinu vektorů v , pro které $Av^T = o^T$. *Nulitou* nul A *matice* A rozumíme dimenzi jejího jádra.

Jádro matice tedy tvoří nulový vektor a všechny vlastní vektory matice A příslušné vlastnímu číslu 0. Nulita matice je rovna rozdílu jejího řádu a hodnoty.¹

Definice 2: Je-li matice A singulární, potom existuje přirozené číslo t , pro které

$$0 \neq \text{Ker } A \subset \text{Ker } A^2 \subset \text{Ker } A^3 \subset \dots \subset \text{Ker } A^t = \text{Ker } A^{t+1} = \dots$$

Nejmenší přirozené číslo této vlastnosti nazveme *index matice* A .

¹ Singulární matice A má vlastní číslo 0 a nul $A > 0$.

Definice 3: Necht' t je index singulární matice A . *Zobecněné jádro* $\text{GKer } A$ matice A řádu n je definováno vztahem

$$\text{GKer } A = \text{Ker } A^t.$$

Je zřejmé, že $\text{Ker } A \subseteq \text{GKer } A = \text{Ker } A^n$, kde n je řád matice A .

Definice 4: Necht' A je singulární matice, t její index a necht'

$$\begin{aligned} \eta_1 &= \text{nul } A = \dim \text{Ker } A, \\ \eta_2 &= \text{nul } A^2 - \text{nul } A = \dim \text{Ker } A^2 - \dim \text{Ker } A, \\ &\dots\dots\dots \\ \eta_t &= \text{nul } A^t - \text{nul } A^{t-1} = \dim \text{Ker } A^t - \dim \text{Ker } A^{t-1}. \end{aligned}$$

Posloupnost $\eta(A) = (\eta_1, \eta_2, \dots, \eta_t)$ se nazývá *výšková charakteristika matice* A .

Lze dokázat, že $\sum_{i=1}^t \eta_i = \dim \text{GKer } A$ je násobnost vlastního čísla 0 matice A .

Definice 5: Čtvercovou matici, kterou lze pomocí simultánních permutací řádků a sloupců, tj. pomocí transformace $P^{-1}AP (= P^TAP)$, kde P je permutační matice, převést na tvar

$$\begin{pmatrix} K & L \\ O & M \end{pmatrix}, \quad \text{resp. na tvar} \quad \begin{pmatrix} K & O \\ L & M \end{pmatrix},$$

v němž K a M jsou čtvercové matice řádu alespoň jedna a O je nulová matice, nazýváme *reducibilní (rozložitelnou)*. V opačném případě nazýváme matici *ireducibilní (nerozložitelnou)*. Matice řádu 1 (včetně nulové) považujeme za ireducibilní, nulové matice řádu alespoň dva za reducibilní.

Definice 6: Reálnou matici, jejíž všechny prvky jsou nezáporná čísla, nazýváme *nezápornou*.

Definice 7: *Z-maticí* rozumíme matici A , pro níž existuje nezáporná matice B taková, že $A = kE - B$. *Z-matici* $A = kE - B$ nazveme *M-maticí*, jestliže $k \geq \varrho(B)$, kde $\varrho(B)$ značí tzv. *spektrální poloměr* matice B , tj. největší z absolutních hodnot vlastních čísel matice B .

Z-matice je tedy matice, jejíž prvky neležící na diagonále jsou nekladné, o prvcích na diagonále nepředpokládáme nic. Je známo,² že pro čtvercovou nezápornou matici $B = (b_{ij})$ je $\varrho(B) \geq b_{ii}$ pro všechna i . Proto je *M-matice* čtvercovou maticí s nezápornými prvky na diagonále a nekladnými prvky na ostatních místech.

Pojem *Z-matice*, resp. *M-matice* byl motivován následující úvahou. Číslo k je s -násobným vlastním číslem matice B právě tehdy, když číslo 0 je s -násobným vlastním číslem matice $kE - B$. *M-matice* $kE - B$ je singulární právě tehdy, když $k = \varrho(B)$. Studujeme-li vlastní číslo 0 singulární *M-matice* $\varrho(B)E - B$, studujeme tedy v podstatě vlastní číslo $\varrho(B)$ matice B .

² Viz [1].

Důležitými pro náš další výklad budou pojmy teorie grafů.

Definice 8: Grafem $G(A)$ matice $A = (a_{ij})$ řádu n nazýváme orientovaný graf s n vrcholy $1, 2, \dots, n$, ve kterém existuje orientovaná hrana z vrcholu i do vrcholu j právě tehdy, když $a_{ij} \neq 0$. Redukovaným grafem $R(A)$ blokové matice A tvaru

$$\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1p} \\ A_{21} & A_{22} & \cdots & A_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ A_{p1} & A_{p2} & \cdots & A_{pp} \end{pmatrix}$$

rozumíme orientovaný graf s p vrcholy $1, 2, \dots, p$, v němž existuje hrana z i -tého vrcholu do j -tého právě tehdy, když $A_{ij} \neq O$.

Definice 9: Necht' A je čtvercová bloková matice, jejíž bloky A_{ii} na diagonále jsou čtvercové. Vrchol i redukovaného grafu $R(A)$ se nazývá *singulární*, jestliže je A_{ii} singulární matice. Je-li navíc 0 jednoduchým vlastním číslem matice A_{ii} , potom se i nazývá *jednoduchý singulární vrchol*.

Definice 10: Cestou nazýváme posloupnost $(i_1, i_2, \dots, i_k)$ navzájem různých vrcholů grafu, ve které jsou každé dva po sobě jdoucí vrcholy spojeny orientovanou hranou, tj. hrany jdou z i_1 do i_2 , z i_2 do i_3 atd. až z i_{k-1} do i_k . Cestou rovněž rozumíme posloupnost obsahující jediný vrchol. Délku cesty definujeme jako počet jejích vrcholů. Cyklem nazýváme posloupnost vrcholů $(u_1, u_2, \dots, u_m, u_1)$, ve které první a poslední vrchol splývají, ostatní vrcholy jsou navzájem různé a v grafu existují hrany $(u_1, u_2), (u_2, u_3), \dots, (u_m, u_1)$. Smyčkou nazýváme hranu (u, u) .

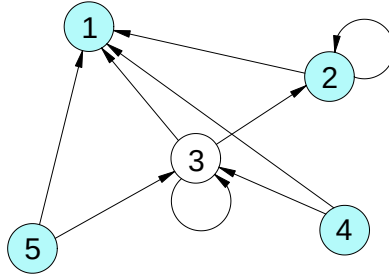
Definice 11: Uvažujme singulární vrchol i redukovaného grafu $R(A)$ a všechny cesty v $R(A)$ v něm končící. Z těchto cest vyberme takovou cestu, která obsahuje největší počet singulárních vrcholů. Úroveň singulárního vrcholu i redukovaného grafu $R(A)$ potom rozumíme počet singulárních vrcholů na této cestě.

Definice 12: Největší z úrovní všech singulárních vrcholů v $R(A)$ blokové matice A označme m . Posloupnost $\lambda(A) = (\lambda_1, \lambda_2, \dots, \lambda_m)$, kde λ_k je počet singulárních vrcholů grafu $R(A)$ úrovně k , nazýváme *úrovňovou charakteristikou* $\lambda(A)$ matice A .

Upevněme pochopení nově zavedených pojmů na konkrétní blokové matici

$$A = \left(\begin{array}{c|c|c|c|c|c} 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & -1 & 3 & 0 & 0 & 0 \\ \hline 0 & 2 & -6 & 0 & 0 & 0 \\ \hline 1 & 1 & 2 & -1 & 0 & 0 \\ \hline -1 & 0 & 0 & 2 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 \end{array} \right).$$

Singulárními vrcholy, které budeme při zakreslování grafů zvýrazňovat podbarvením, jsou vrcholy 1, 2, 4 a 5; redukovaný graf $R(A)$ matice A vypadá takto:



Z cest končících ve vrcholu 1 mají nejvíce singulárních vrcholů cesty $(5, 3, 2, 1)$ a $(4, 3, 2, 1)$, a to tři. Úroveň vrcholu 1 je proto tři. Ve vrcholu 2 končí dvě cesty $(4, 3, 2)$ a $(5, 3, 2)$, obě obsahují dva singulární vrcholy, úroveň vrcholu 2 je tedy dva. V singulárních vrcholech 4 a 5 končí pouze cesty jednovrcholové, úroveň obou vrcholů je proto jedna. Úrovňová charakteristika $\lambda(A)$ matice A je tedy $(2, 1, 1)$.

Je známo, že každou čtvercovou matici lze pomocí simultánních permutací řádků a sloupců převést na tzv. *Frobeniův normální tvar*, což je bloková matice tvaru

$$\begin{pmatrix} A_{11} & O & \cdots & O \\ A_{21} & A_{22} & \cdots & O \\ \vdots & \vdots & \ddots & \vdots \\ A_{q1} & A_{q2} & \cdots & A_{qq} \end{pmatrix},$$

kde matice A_{ii} jsou čtvercové a ireducibilní. Až na nepodstatné změny v uspořádání prvků (které nemají na naše studium vliv) existuje tento tvar pro danou matici jediný. Uvědomme si, že po provedení simultánních permutací řádků a sloupců se diagonální prvky pouze přemístí v rámci diagonály, nediagonální prvky zůstanou nediagonálními. Frobeniův normální tvar M -matice a rovněž každý jeho blok na diagonále tedy zůstává M -maticí. Je známo (z Perronovy-Frobeniovy teorie), že 0 je jednoduchým vlastním číslem singulární ireducibilní M -matice, a tedy i každého singulárního bloku, který leží na diagonále Frobeniova normálního tvaru matice A . Na následujících stranách budeme vždy předpokládat, že matice A je ve Frobeniově normálním tvaru, čímž zajistíme jednoznačné přiřazení úrovňové charakteristiky.

Zavedli jsme dvě posloupnosti, výškovou $\eta(A)$ a úrovňovou charakteristiku $\lambda(A)$ matice A , a to terminologiemi různých matematických disciplín – teorie matic a teorie grafů. Je možné, aby mezi těmito posloupnostmi byl nějaký vztah? Odpověď je překvapivě kladná, uvedené charakteristiky jsou v některých speciálních případech dokonce shodné.

Věta 13: *Nechť A je M -matice a t její index. Potom tvrzení (i) a (ii) jsou navzájem ekvivalentní a rovněž tvrzení (iii) a (iv) jsou navzájem ekvivalentní:*

(i) $\eta(A) = (t)$,	(iii) $\eta(A) = (1, 1, \dots, 1)$,
(ii) $\lambda(A) = (t)$,	(iv) $\lambda(A) = (1, 1, \dots, 1)$,

kde počet jedniček v tvrzeních (iii) a (iv) je t .

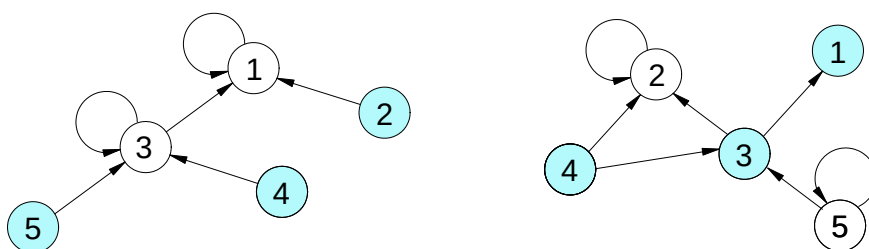
Případ příslušný tvrzením (i) a (ii) odpovídá redukovanému grafu $R(A)$, ve kterém každá cesta obsahuje maximálně jeden singulární vrchol. Druhá speciální

situace (tvrzení (iii) a (iv)) nastává, existuje-li cesta grafu $R(A)$ obsahující všechny jeho singulární vrcholy. V obou případech platí vztah $\eta(A) = \lambda(A)$.

Demonstrujme oba speciální případy na příkladech. Nechť

$$B = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -2 & 0 & 4 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -4 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}.$$

Matice B i C jsou M -maticemi, jsou ve Frobeniově normálním tvaru, bloky na diagonále jsou řádu 1. Přísluší jim následující grafy $R(B) = G(B)$ a $R(C) = G(C)$:



Všechny singulární vrcholy 2, 4, 5 grafu $R(B)$ (obr. vlevo) mají úroveň jedna, proto $\lambda(B) = (3)$. Dle Věty 13 by mělo platit $\eta(B) = (3)$. Protože $\text{nul } B = \text{nul } B^2 = 3$, je i dle výpočtu pomocí rozdílu nulit skutečně $\eta(B) = (3)$, a tedy $\lambda(B) = \eta(B)$. Všechny singulární vrcholy 1, 3, 4 grafu $R(C)$ (obr. vpravo) jsou na jediné cestě a mají po řadě úrovně tři, dva a jedna, proto $\lambda(C) = (1, 1, 1)$. Jelikož je $\text{nul } C = 1$, $\text{nul } C^2 = 2$, $\text{nul } C^3 = \text{nul } C^4 = 3$, je $\eta(C) = (1, 1, 1)$, a proto opět $\lambda(C) = \eta(C)$.

Obecně však rovnost charakteristik neplatí. Je tedy zcela přirozené se ptát, kdy posloupnosti splývají, resp. jaký je mezi nimi vztah. Již přibližně 25 let je známo 35 podmínek ekvivalentních rovnosti úrovně a výškové charakteristiky. Vzhledem k jejich množství a poměrné složitosti a také vzhledem k omezenému rozsahu tohoto příspěvku odkazujeme čtenáře přímo do originálních článků.³

I v případě, že jsou výšková a úrovněvá charakteristika odlišné, však mezi nimi platí určitá relace. Definujme nyní tento vztah.

Definice 14: Uvažujme posloupnosti $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_t)$, $\beta = (\beta_1, \beta_2, \dots, \beta_t)$ nezáporných celých čísel o stejném počtu prvků. Říkáme, že *posloupnost β majorizuje posloupnost α* a píšeme $\alpha \preceq \beta$, jestliže

$$\alpha_1 + \dots + \alpha_k \leq \beta_1 + \dots + \beta_k$$

pro všechna $k = 1, \dots, t-1$ a

$$\alpha_1 + \dots + \alpha_t = \beta_1 + \dots + \beta_t.$$

³ Uvedené otázky si položil již Hans Schneider ve své disertaci [2] z roku 1952. Odpovědi našli Hans Schneider a Daniel Hershkowitz zhruba po třiceti sedmi letech; publikovány byly především v článcích [3], [4] a [5].

Popišme obecný vztah mezi výškovou a úroňovou charakteristikou M -matice.⁴

Věta 15: *Pro každou M -matici A platí $\lambda(A) \preceq \eta(A)$.*

Tento vztah lze zpřesnit, neboť platí i v případě, že libovolně permutujeme pořadí prvků posloupnosti $\lambda(A)$ (symbol $\hat{\lambda}(A)$ nechť dále značí posloupnost získanou z posloupnosti $\lambda(A)$ přemístěním jejích prvků do nerostoucí posloupnosti). Navíc relace zůstane zachována, rozšíříme-li M -matice na obecnější třídu matic a také má-li výšková charakteristika méně prvků než úroňová – v tomto případě doplníme za poslední člen charakteristiky $\eta(A)$ takový počet nul, aby obě posloupnosti měly stejný počet prvků; tuto modifikovanou posloupnost budeme značit $(\eta(A), 0)$.

Věta 16: *Nechť A je blokově trojúhelníková matice, jejíž redukovaný graf má všechny singulární vrcholy jednoduché. Potom platí⁵*

$$\hat{\lambda}(A) \preceq (\eta(A), 0).$$

Pokusme se nalézt další posloupnosti, které by bylo možno porovnat (alespoň pro některé třídy matic) s výškovou a úroňovou charakteristikou pomocí majorizace. Jedna z takových posloupností je zavedena pomocí tzv. *pokrytí grafu cestami*.

Definice 17: *Pokrytím $\mathcal{P}$ grafu G cestami nazýváme množinu cest grafu G , které jsou disjunktní, tj. žádné dvě z nich nemají společný vrchol, a každý vrchol grafu G náleží právě jedné z těchto cest.*

Definice 18: Symbol $p_k(G)$, $k = 1, 2, \dots$, nechť značí maximální počet vrcholů bez smyček, které mohou být zahrnuty v k (či méně) disjunktních cestách grafu G . Tato množina vrcholů se nazývá k -cesta. Položme ještě $p_0(G) = 0$. Dále nechť t je nejmenší počet disjunktních cest grafu G , které jsou nutné k pokrytí všech vrcholů grafu G , které nemají smyčky. Zřejmě je $p_{k-1}(G) < p_k(G)$ pro $1 < k \leq t$ a $p_{k-1}(G) = p_k(G)$ pro $k > t$. Pro $k = 1, 2, \dots, t$ tedy lze definovat přirozená čísla

$$\pi_k(G) = p_k(G) - p_{k-1}(G).$$

Charakteristikou cest $\pi(G)$ budeme nazývat posloupnost

$$(\pi_1(G), \pi_2(G), \dots, \pi_t(G)).$$

V dále uvedených majorizacích však nebude figurovat přímo posloupnost $\pi(G)$, ale posloupnost k ní tzv. *duální*.

Definice 19: Nechť $\alpha = (\alpha_1, \dots, \alpha_t)$ je nerostoucí posloupnost přirozených čísel. Uvažujme tzv. *Ferrersův diagram posloupnosti α* vytvořený z t sloupců teček, z nichž k -tý sloupec zleva má právě α_k teček. První (spodní) tečky všech sloupců jsou umístěny na stejném, posledním řádku, tečky druhé na předposledním řádku atd. Posloupnost α^* nazveme *duální k posloupnosti α* , jestliže její členy značí počty teček v jednotlivých řádcích (čteno odspodu) Ferrersova diagramu posloupnosti α .

⁴ Viz [3], str. 158.

⁵ Viz [6], str. 116.

Věta 20: Pro téměř trojúhelníkovou matici A , tj. matici, kterou lze simultánními permutacemi řádků a sloupců převést na dolní (nebo horní) trojúhelníkovou matici, platí vztah⁶

$$\pi(G(A))^* \preceq \eta(A).$$

Dosud jsme představili dvě posloupnosti, které majorizuje výšková charakteristika $\eta(A)$. Jednak charakteristiku $\hat{\lambda}(A)$, je-li A blokově trojúhelníková matice se čtvercovými bloky na diagonále, jejíž redukovaný graf má jednoduché singulární vrcholy, jednak posloupnost $\pi(G(A))^*$, je-li matice A téměř trojúhelníková. Uvědomíme-li si, že Frobeniův normální tvar téměř trojúhelníkové matice je matice trojúhelníková, která je speciálním případem blokově trojúhelníkové matice se čtvercovými bloky (řádu 1) na diagonále a s odpovídajícím grafem, jehož singulární vrcholy jsou jednoduché, napadne nás přirozeně otázka, zda lze (alespoň) pro tuto třídu matic nalézt vztah majorizace mezi posloupnostmi $\hat{\lambda}(A)$ a $\pi(G(A))^*$. Relace majorizace mezi nimi skutečně platí. Dokonce mezi ně můžeme vsunout posloupnost další, která jednu z nich majorizuje a zároveň je majorizována posloupností druhou. Definujme nyní tuto novou posloupnost.

Definice 21: Nechť G je graf bez cyklů (který však může mít smyčky). Množina vrcholů grafu G , které nemají smyčky, se nazývá k -systém, jestliže žádná její $(k+1)$ -prvková podmnožina neleží na stejné cestě. Libovolný z k -systémů s maximálním počtem prvků označme symbolem Ω_k . Symbol $d_k(G)$ nechť značí počet prvků množiny Ω_k a nechť $d_0(G) = 0$. Nechť t značí největší počet vrcholů bez smyček grafu G ležící na téže cestě. Zřejmě $d_{k-1}(G) < d_k(G)$ pro $1 < k \leq t$ a $d_{k-1}(G) = d_k(G)$ pro $k > t$. Pro $k = 1, 2, \dots, t$ označme

$$\delta_k(G) = d_k(G) - d_{k-1}(G).$$

Charakteristikou systémů $\delta(G)$ budeme nazývat posloupnost přirozených čísel

$$(\delta_1(G), \delta_2(G), \dots, \delta_t(G)).$$

Vztah této posloupnosti k posloupnosti $\pi(G)^*$ vyjadřuje následující věta.⁷

Věta 22: Pro každý graf G bez cyklů, který však může obsahovat smyčky, platí relace

$$\delta(G) \preceq \pi(G)^*.$$

Na pozici majoranty posloupnost δ naopak figuruje v následující větě, jejíž tvrzení je snadno představitelné. Stačí uvážit, že žádné dva vrcholy bez smyček mající stejnou úroveň neleží na téže cestě grafu $G(A)$.

Věta 23: Každá téměř trojúhelníková matice A splňuje relaci

$$\hat{\lambda}(A) \preceq \delta(G(A)).$$

⁶ Viz [7], str. 180.

⁷ Viz [7], str. 177.

Jelikož graf téměř trojúhelníkové matice nemá cykly, je triviálním důsledkem dvou právě vyslovených tvrzení hledaný vztah mezi posloupnostmi $\hat{\lambda}(A)$ a $\pi(G(A))^*$.

Věta 24: *Pro každou téměř trojúhelníkovou matici A je*

$$\hat{\lambda}(A) \preceq \pi(G(A))^*.$$

Shrňme výsledky týkající se majorizace posloupností příslušných k téměř trojúhelníkovým maticím do přehledného sledu pěti navzájem se majorizujících posloupností.

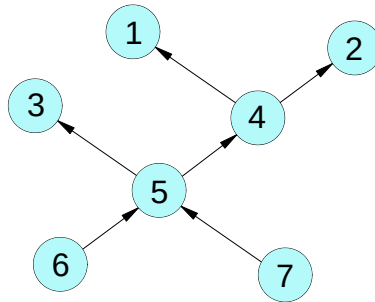
Věta 25: *Každá téměř trojúhelníková matice A splňuje vztahy*

$$\lambda(A) \preceq \hat{\lambda}(A) \preceq \delta(G(A)) \preceq \pi(G(A))^* \preceq \eta(A).$$

Ukažme existenci majorizací mezi všemi pěti posloupnostmi na konkrétním příkladu (téměř) trojúhelníkové matice D .⁸ Nechť

$$D = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & 0 \end{pmatrix}.$$

Graf $G(D)$ matice D je na následujícím obrázku:



Hledejme postupně posloupnosti příslušné matici D , resp. grafu $G(D)$. Úrovnňová charakteristika $\lambda(D) = (2, 1, 2, 2)$, tudíž $\hat{\lambda}(D) = (2, 2, 2, 1)$.

Vrcholy $\{1, 2, 3\}$ bez smyček tvoří 1-systém, který neobsahuje dva vrcholy bez smyček ležící na téže cestě a má z množin této vlastnosti největší počet prvků, a to tři. Tedy $\Omega_1 = \{1, 2, 3\}$ a $d_1(G(D)) = 3$. Množinou splňující podmínky, že žádná trojice jejích vrcholů bez smyček neleží na téže cestě a je co největší, je množina $\Omega_2 = \{1, 2, 3, 6, 7\}$, proto $d_2(G(D)) = 5$. Postupně určíme, že $\Omega_3 = \{1, 2, 3, 4, 6, 7\}$ (nebo také $\Omega_3 = \{1, 2, 3, 5, 6, 7\}$) a $\Omega_4 = \{1, 2, 3, 4, 5, 6, 7\}$. Odtud $d_3(G(D)) = 6$ a $d_4(G(D)) = 7$. Dostáváme posloupnost $\delta(G(D)) = (3, 2, 1, 1)$.

⁸ Zadání matice D je inspirováno obecnějším příkladem 5.8 z článku [8], str. 185.

Pouhým pohledem na graf $G(D)$ zjistíme, že $p_1(G(D)) = 4$, $p_2(G(D)) = 5$, $p_3(G(D)) = 6$ a $p_4(G(D)) = 7$. Dostáváme nejprve $\pi(G(D)) = (4, 1, 1, 1)$ a z Ferrer-sova diagramu dále $\pi(G(D))^* = (4, 1, 1, 1)$.

Vidíme, že nulita matice D je 4. Vypočítáme, že s každou vyšší mocninou matice D se její nulita zvýší o jednu, přičemž se tento postup zastaví u matice D^4 . Proto má $\eta(D)$ čtyři prvky a $\eta(D) = (4, 1, 1, 1)$.

V našem konkrétním příkladě má tedy onen sled pěti posloupností tvar

$$(2, 1, 2, 2) \preceq (2, 2, 2, 1) \preceq (3, 2, 1, 1) \preceq (4, 1, 1, 1) \preceq (4, 1, 1, 1),$$

přičemž s výjimkou posledního vztahu se jedná o majorizace, které nejsou rovnostmi.

Vzpomeňme ještě na třídu M -matic, konkrétně na dva speciální případy, v nichž $\lambda(A) = \eta(A)$ (viz Věta 13). Právě uvedená řada majorizací, v nichž pouze graf $G(A)$ nahradíme redukováným grafem $R(A)$, zůstává v platnosti, tj. pro M -matici A , pro kterou buď $\eta(A) = (t)$, nebo $\eta(A) = (1, 1, \dots, 1)$, je

$$\lambda(A) = \hat{\lambda}(A) = \delta(R(A)) = \pi(R(A))^* = \eta(A).$$

Na závěr poznamenejme, že výšková charakteristika je totožná s tzv. *Weyrovou charakteristikou matice příslušnou vlastnímu číslu 0* (viz např. [9]) a že je duální posloupností k tzv. *Segreově charakteristice příslušné vlastnímu číslu 0*, což je nerostící posloupnost řádů Jordanových buněk příslušných vlastnímu číslu 0.

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ASYMPTOTICKÁ STABILITA LINEÁRNÍCH DIFERENČNÍCH ROVNIC

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Abstrakt: Článek uvádí přehled kritérií, které umožňují rozhodnout o asymptotické stabilitě lineární diferenční rovnice s konstantními koeficienty. U speciálního případu jsou uvedeny nutné a postačující podmínky pro asymptotickou stabilitu v různých formách, a sice v závislosti na koeficientech rovnice a jejím řádu.

Abstract: The paper gives a survey of asymptotic stability criteria for linear difference equations with real constant coefficients. There are introduced several forms of necessary and sufficient conditions for some special few term difference equations.

1 Úvod

Uvažujme lineární diferenční rovnici m -tého řádu

$$y(n) + a_1 y(n-1) + a_2 y(n-2) + \cdots + a_m y(n-m) = 0, \quad n = 0, 1, 2, \dots, \quad (1)$$

kde $m \in \mathbb{Z}^+$ a $a_1, a_2, \dots, a_m$ jsou reálné koeficienty.

Řešení diferenční rovnice (1) nazýváme *asymptoticky stabilní*, jestliže pro každé řešení (1) platí $\lim_{n \rightarrow \infty} y(n) = 0$. Dále platí známý vztah mezi asymptotickou stabilitou lineární diferenční rovnice s konstantními koeficienty a dislokací kořenů jejího charakteristického polynomu: Rovnice (1) je asymptoticky stabilní právě tehdy, když všechny kořeny charakteristického polynomu

$$p(\lambda) = \lambda^m + a_1 \lambda^{m-1} + a_2 \lambda^{m-2} + \cdots + a_{m-1} \lambda + a_m \quad (2)$$

leží uvnitř jednotkového kruhu v komplexní rovině.

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Této základní charakteristiky asymptotické stability rovnice (1) se zhusta využívá při její analýze. V této souvislosti uvedeme Schurovo-Cohnovo kritérium, které udává nutné a postačující podmínky, aby všechny kořeny polynomu (2) ležely uvnitř jednotkového kruhu.

Věta 1.1 (Schurovo-Cohnovo kritérium). *Všechny kořeny polynomu $p(\lambda)$ (viz (2)) leží uvnitř jednotkového kruhu právě tehdy, když jsou zároveň splněny všechny tři následující podmínky:*

1. $p(1) > 0$;
2. $(-1)^m p(-1) > 0$;
3. matice B_{m-1}^+ a B_{m-1}^- , které jsou dány vztahem

$$B_{m-1}^{\pm} = \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ a_1 & 1 & \ddots & & 0 \\ a_2 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & 0 \\ a_{m-2} & \cdots & a_2 & a_1 & 1 \end{pmatrix} \pm \begin{pmatrix} 0 & 0 & \cdots & 0 & a_m \\ 0 & & & a_m & a_{m-1} \\ \vdots & & & & a_{m-2} \\ 0 & a_k & & & \vdots \\ a_m & a_{m-1} & a_{m-2} & \cdots & a_2 \end{pmatrix},$$

mají kladné determinanty a mají kladné také determinanty všech centrálních submatic (positive innerwise).

Poznámka 1.2. *Ilustrujme konstrukci centrálních submatic na následujícím příkladě:*

Nechť $M = \Delta_n$ je matice $n \times n$. Matici Δ_{n-2} $((n-2) \times (n-2))$ vytvoříme z matice Δ_n tak, že odebereme její první a poslední řádek a první a poslední sloupec. Tímto způsobem získáme množinu matic $\{\Delta_1, \Delta_3, \dots, \Delta_{n-2}\}$ pro n liché a množinu matic $\{\Delta_2, \Delta_4, \dots, \Delta_{n-2}\}$ pro n sudé. Pro ilustraci

$$M_5 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} \end{bmatrix},$$

$$M_6 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & m_{36} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} & m_{46} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} & m_{56} \\ m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{bmatrix}.$$

Centrální submatice matice M_5 jsou Δ_1 a Δ_3 ; centrální submatice matice M_6 jsou Δ_2 a Δ_4 . Více informací o dané problematice lze nalézt např. v [5].

Hlubší analýzu o vlastnostech polynomů, potažmo umístění kořenů polynomů v komplexní rovině, lze nalézt v [9].

2 Kritéria asymptotické stability

V této kapitole budeme ilustrovat dvě formy kritérií asymptotické stability pro lineární diferenční rovnice s několika málo nenulovými členy. Typickým případem je rovnice

$$y(n+1) + \alpha y(n) + \gamma y(n-k+1) + \beta y(n-k) = 0, \quad n = 0, 1, 2, \dots, \quad (3)$$

jejíž charakteristický polynom má tvar

$$\tilde{p}(\lambda) = \lambda^{k+1} + \alpha \lambda^k + \gamma \lambda + \beta.$$

Rovnice tohoto typu jsou obvykle označovány jako zpožděné diferenční rovnice. Rovnice (3) byla z hlediska asymptotické stability analyzována v čistě zpožděném případě $\gamma = 0$ v [8], v případě $\alpha = 0$ v [4] a v obecném tvaru $\alpha\gamma \neq 0$ v [1].

Nyní uvedeme dva druhy podmínek na diferenční rovnici, kterou získáme z (1) volbou $m = k+2$, $a_2 = \alpha$, $a_k = \beta$ a $a_1 = a_3 = a_4 = \dots = a_{k-1} = 0$, tj.

$$y(n+2) + \alpha y(n) + \beta y(n-k) = 0, \quad n = 0, 1, 2, \dots \quad (4)$$

Poznamenejme, že analýzu rovnice (4) pro k sudé lze snadno převést na případ (3) s $\gamma = 0$. Pak stačí substituovat λ^2 výrazem λ a $k/2$ hodnotou k . Tím obdržíme charakteristický polynom

$$\tilde{p}(\lambda) = \lambda^{k+1} + \alpha \lambda^k + \beta,$$

jehož analýza je provedena v [8]. Stačí tedy vyšetřit (4) pro k liché.

Ren odvodil následující podmínky pro asymptotickou stabilitu rovnice (4) (viz [10]).

Věta 2.1. *Nechť α, β jsou nenulové reálné parametry a $k \in \mathbb{Z}^+$ je liché. Pak (4) je asymptoticky stabilní právě tehdy, když platí buďto*

$$-1 < \alpha < 0, \quad |\beta| < 1 + \alpha,$$

nebo

$$0 < \alpha < 1, \quad |\beta| < (\alpha^2 + 2\alpha \cos 2\phi + 1)^{1/2}, \quad (5)$$

kde ϕ je řešením nelineární rovnice

$$\sin(kx)/\sin((k+2)x) = -1/\alpha \quad (6)$$

na intervalu $((k+1)\pi/(2k+4), \pi/2)$.

Alternativou k tomuto tvrzení je kritérium odvozené v [2]:

Věta 2.2. *Nechť α, β jsou nenulové reálné parametry a $k \in \mathbb{Z}^+$ je liché.*

(i) *Nechť $\alpha < 0$. Pak (4) je asymptoticky stabilní právě tehdy, když*

$$|\alpha| + |\beta| < 1.$$

(ii) Necht' $\alpha > 0$. Pak (4) je asymptoticky stabilní právě tehdy, když platí buď

$$|\alpha| + |\beta| \leq 1,$$

anebo

$$\beta^2 < 1 - \alpha < |\beta|, \quad k < 2 \arcsin \frac{1 - \alpha^2 - \beta^2}{2|\alpha\beta|} / \arccos \frac{\alpha^2 - \beta^2 + 1}{2|\alpha|}.$$

Uvedená tvrzení mají svůj přínos především pro rovnice vysokých řádů. Klasické Schurovo–Cohnovo kritérium např. pro rovnici 21. řádu pro potvrzení asymptotické stability vyžaduje výpočet dvaceti determinantů. Forma věty 2.1 vyžaduje v případě $0 < \alpha < 1$ vyřešit nelineární rovnici. Výsledek uvedený ve větě 2.2 pro dané α a β uvádí explicitně interval přípustných k , pro které je rovnice (4) asymptoticky stabilní. Ilustrujme nyní použití výše uvedených kritérií na tomto příkladě

Příklad 2.3. Rozhodněte o asymptotické stabilitě diferenční rovnice

$$y(n+2) + 0.52y(n) + 0.5y(n-k) = 0, \quad n = 0, 1, 2, \dots \quad (7)$$

pro $k = 7$. Podle věty 2.1 dostaneme řešením rovnice (6) $\phi \approx 1.466556$ a dosazením ověříme platnost (5) ve tvaru $0.5 < 0.502911$. Tedy pro $k = 7$ je rovnice (7) asymptoticky stabilní.

Použití věty 2.2 (bod (ii)) dává po vyčíslení $k < 12.075107$. Z tohoto výsledku vyplývá, že diferenční rovnice (7) je asymptoticky stabilní pro $k = 1, 3, 5, 7, 9, 11$, uvažujeme-li pouze lichá k .

Analýza asymptotické stability lineárních rovnic s několika málo členy je poměrně aktuálním tématem, o čemž svědčí řada publikací, které se danou problematikou zabývají (viz např. [1, 2, 3, 6, 7]). Kvalitativní zkoumání rovnic s několika členy je využitelné také např. u vyšetřování stability numerických schémat pro diferenciální rovnice s posunutým argumentem.

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THE PRELIMINARY NUMERICAL ANALYSIS OF AIRFLOW THROUGH GOAFS ZONE

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Abstract: *The basic meaning for the security of persons working in underground mining heading their ventilation of headings. Providing the fresh air into the active mining heading is a fundamental task of ventilation service in the mine. Very significant impact on the efficiency of the ventilation process has the physical parameters of supplied airflow, such as its amount, speed and pressure. These parameters can be determined based on the "in situ" tests or modeling tests. Carrying out the tests in underground conditions is very expensive and not always available due to exploitation. As an alternative, in such cases the modeling tests can be used, which give more possibilities of analysis the impact of differentials factors on the studied parameters. In the paper there is presented results of modeling of airflow in mining heading, obtained basing on the numerical simulations with use of finite volume method in ANSYS Fluent software. In the paper results of numerical analysis connected with airflow through the goafs zone in coal mine were presented. To find the solution of the mathematical model, the k-ε turbulence model was used.*

1. Introduction

The fundamental meaning for the safety of persons working in underground mine headings has proper ventilation of headings.

Operating longwalls can be ventilated in different ways: on „U”, „Y”, „Z” and in the „H” way, which requires maintenance longwall gates. Stream of fresh air flowing through the longwall has tendency to head to caving zone along its entire length, however its greatest amount gets to gobs at crossing of longwall with bottom gate. Migration of air to the gobs occurs during supplied air and also carried away air from wall by longwall gate roads.

Test of airflow through rockfall goaves “*in situ*” conditions is practically impossible, so it becomes essential to search another, alternative methods enabling analysis of this phenomenon.

Such possibilities create modeling studies of flows, based on the numerical simulations. In recently years, these methods are more often used for solving problems associated to process of ventilation of mining headings [2, 3, 5, 6].

In this paper results of simulations of the airflow through longwall and goaf zone in coal mine are presented.

2. Mathematical model of airflow

To perform the model tests, ANSYS Fluent software was used, which uses the Finite Volume Method. This method is used to solve problems, in which the principle of continuity of the field variable in the considered area has not to be fulfilled [4].

Problem connected with the airflow through mining headings in the Ansys Fluent software are solved based on the equation of mass conservation and on the equation of momentum conservation, which take the following form [1]:

a) *The Mass Conservation Equation*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_m \quad (1)$$

where $\vec{v}$ is velocity (m/s), ρ is density (kg/m³), t is time (s) and S_m is the mass added to the continuous phase from the dispersed second phase, kg/s.

b) *The Momentum Conservation Equations*

$$\frac{\partial}{\partial t} (\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p + \nabla \cdot (\bar{\tau}) + \rho \vec{g} + \vec{F} \quad (2)$$

where p is static pressure (Pa), $\bar{\tau}$ is the stress tensor (Pa), $\vec{g}$ is the gravitational body force (m/s²) and $\vec{F}$ is the external body forces (N).

Porous media are modeled by the addition of a momentum source term to Eq. (1). The source term is composed of two parts: a viscous loss term and an inertial loss term:

$$S_i = - \left(\sum_{j=1}^3 D_{ij} \mu v_j + \sum_{j=1}^3 C_{ij} \frac{1}{2} \rho |v| v_j \right) \quad (3)$$

where S_i is the source term for Navier – Stokes equation, μ (kg/(m·s)) is kinetic viscosity of the gas, D and C are prescribed matrices.

Ignoring convective acceleration and diffusion, the porous media model then reduces to Darcy's Law:

$$\nabla p = - \frac{\mu}{k} \vec{v} \quad (4)$$

where ∇p is pressure drop (Nm⁻²), k is the permeability (m²).

3. Discrete model of airflow

In order to perform an analysis, the geometrical models of operating longwalls ventilated in a “U” way from borders and in a “Y” way with rebloving of top gate were developed (fig. 1a, fig. 1b). So developed geometric models were subjected to discretization.

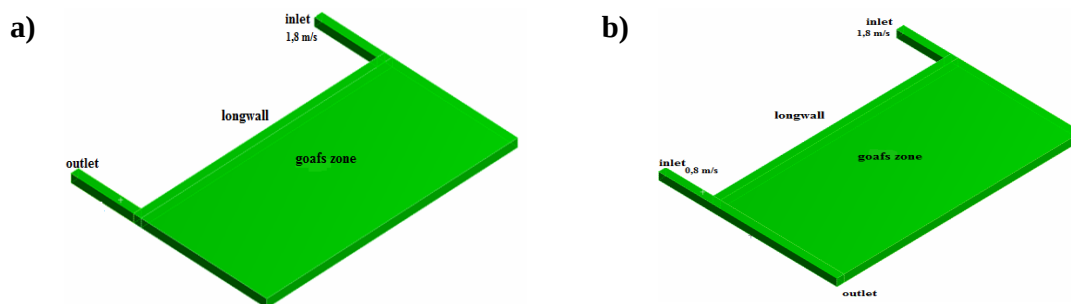


Fig.1. The geometrical models of the longwalls.

It was assumed, that length of longwall gates equals to 25 m, and length of wall to 100 m. Length of top gate during ventilation with „Y” way amounts to 50 m. Length of the operating longwall for analyzed ways of ventilating equals to 50 m. Height and width of wall and longwall gates equal to 3 m.

As an inlet boundary condition there was assumed constant velocity field of air stream, constant values of kinetic energy of turbulence, and its rate of dissipation, which was determined assuming 5-percentage of turbulence intensity at the inlet.

In the inlet cross-section, located in a bottom gate, for longwalls ventilated with “Y” way from borders and with „Y” way with reblowing of top gate, uniform distribution of the velocity of value 1.8 m/s was applied. In a case of ventilation with „Y” way with reblowing of top gate it was assumed, that the velocity of air at the inlet section of top gate equals to 0.8 m/s.

For analyzed model outlet boundary condition was defined as pressure - outlet, whereas walls were defined as impermeable, which surface roughness corresponded to the height of 0.2 m.

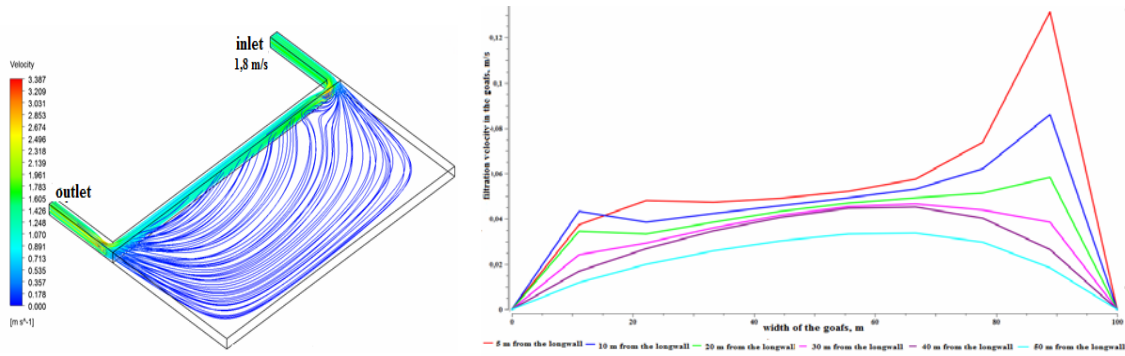
It was assumed, that porosity of goafs equals to 50%, and coefficient of their permeability is constant and equals to 1 per m^2 .

So developed model, was subjected to the numerical analysis.

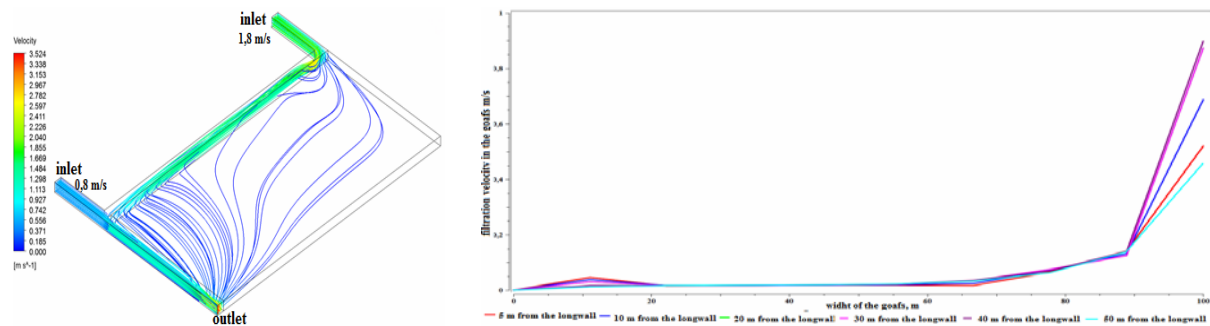
4. Results of numerical analysis

Based on performed calculations, distributions of velocity and pressure fields of air stream flowing through longwall gates and operating walls were determined.

In Figures 2a and 3a, there is presented trajectories of airflow particles through operating longwall and longwall gates and goafs for analyzed ways of ventilating walls. In Figures 2b and 3b, there are presented velocity characteristics of air stream flowing through the goafs.



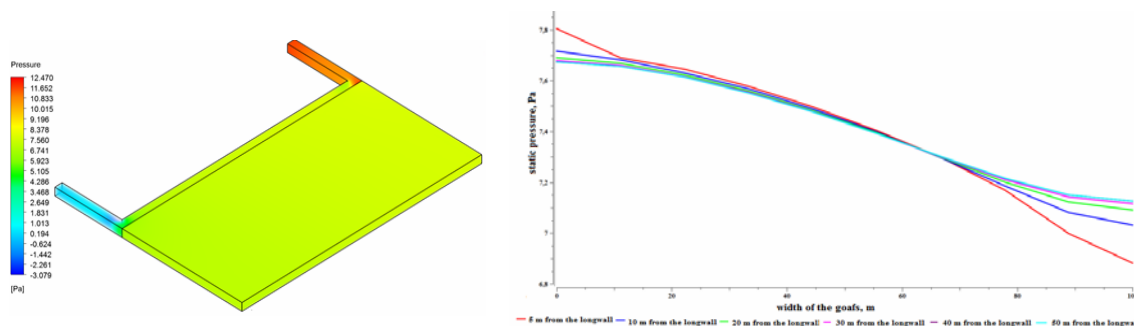
a) b)
Fig. 2. The path of air particles at the longwall method of ventilation on the „U” (a) and the characteristics of air velocity in goaf zone (b).



a) b)
Fig. 3. The path of air particles at the longwall method of ventilation on the „Y” (a) and the characteristics of air velocity in goaf zone (b).

Analyzing obtained distributions of velocity of air in gobbs of tested walls, one can conclude, that the greatest values of filtration occurring at the corners of walls for “Y” way of ventilating with reblowing of top gate. In a case of ventilation longwall with „U” way, velocity of filtration in the corners is smaller, whereas at the central part of gobbs is greater than during the ventilation with “Y” way. The greater distance from the longwall face, the smaller velocity of air flowing through goafs.

Distributions of static pressure in wall and longwall gates and gobbs are presented in Figures 5a and 6a, whereas in Figures 5b and 6b the characteristics of pressure drop in goafs.



a) b)
Fig. 5. Static pressure distribution at the longwall method of ventilation on the "U" (a) and the characteristics of the static pressure in goafs zone (b).

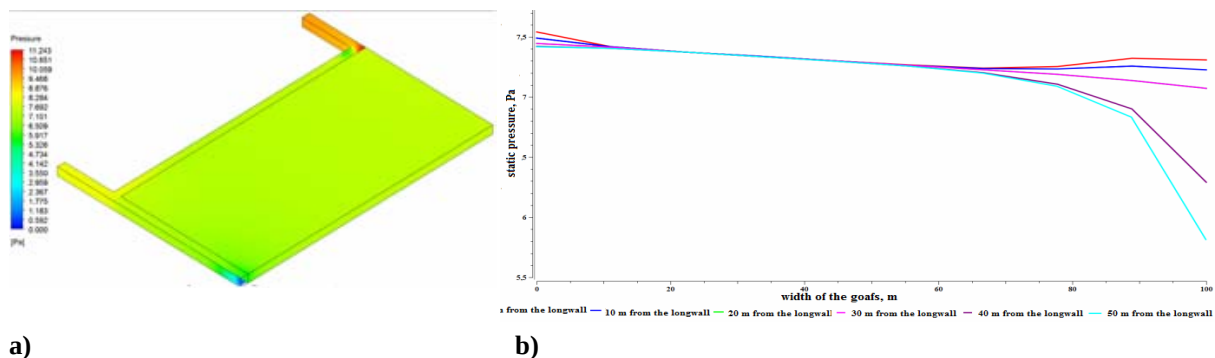


Fig. 6. Static pressure distribution at the longwall method of ventilation on the "Y" (a) and the characteristics of the static pressure in goafs zone (b).

Analyzing obtained distributions of static pressure, one can conclude, that in the region of upper and lower corner of longwalls, linear drops are not occurred. Beyond the upper and lower corner, for analyzed ways of ventilating the walls, pressure drop is linear. In the region of corner of longwalls, change of direction of airflow impact on pressure drop (after flow into longwall the air stream changes direction of flow by 90°; the same phenomenon occurs when air flows out from a longwall).

4. Conclusion

In the paper results of numerical analysis connected with airflow through the goafs zone in coal mine were presented.

Used for numerical analysis of airflow through rockfall goaves models enabled to determine distributions and characteristics of velocity of air in gobs, and also distributions of static pressure and their characteristics. From performed analysis clearly one can see, that together with increasing distance from the wall, the velocity of filtration in rockfall goaves decreases. A similar phenomenon applies to static pressure drop in rockfall goaves, the greatest distance from longwall face, the lower value of pressure in gobs.

It should be emphasized that developed model and applied software give a lot of possibilities for more complicated analysis of problems in a scope of ventilation process of mining headings.

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EFFECTIVE MEDIUM APPROXIMATION OF ANISOTROPIC NANOSTRUCTURE

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Abstract: Optical function of heterogeneous material composed by anisotropic metallic nanoparticles in (generally) anisotropic host medium is modelled using effective medium approximation (EMA). The classical Bragg-Pippard model is extended about the interaction of dipole moments simulated by discrete dipole approximation (DDA). Theoretical results are illustrated for in-plane distributed spheroidal gold nanoparticles.

1 Introduction

Implementation of ferromagnetic components into nanostructures made out of noble metals has been studied to optimize either plasmonic or magneto-optic response relating to possible applications – see e.g. [1]-[4]. As it was demonstrated in the pioneering work [5], an observable change of polarization state of scattered light can be reached even without presence of ferromagnetic material. The physical reason follows from the polarizability enhancement in plasmonic nanostructures at relatively low magnetic fields (tenths of Tesla). In our previous work [6] we analysed this idea for more heterogeneous system of gold nanodots deposited on oxidized Si substrate, where the dot diameter varies from 4 to 22 nm. It was established that measured Kerr rotation and ellipticity exhibit pronounced enhancement at the same frequency, where the permittivity resonance occurs as a consequence of localised surface plasmon resonance.

As the faithful determination of optical function is essential in the mentioned research, we supplied previously used effective medium approximation model by an interaction of nanoparticle dipole moments. To this purpose, we modified the well-known discrete dipole approximation (DDA) formula [7] for regularized in-plane distribution of nanoparticles. In this paper, we present main theoretical results with several examples.

2 Theory

2.1 Averaged fields model

We will consider a system of nanoparticles with relative permittivity tensor $\hat{\epsilon}_p$ randomly distributed in a host medium characterized by relative permittivity $\hat{\epsilon}_h$. In the frame of prescribed geometrical parameters we seek for effective permittivity $\hat{\epsilon}_{ef}$ by given total volume fraction f of nanoparticles under following assumptions: (i) constitutive parameters do not depend on magnetic field; (ii) the percolation limit of local fields is preserved with regard to the low fill-factor; (iii) nanoparticles have spheroidal form with parallel symmetry axes. Above conditions allow to prefer the Bragg-Pippard model of effective relative permittivity based on average field approximation.

In the standard model (see e.g. [8]) the polarization in heterogeneous system is expressed as the sum of dipole moments of nanoparticles $\mathbf{P}$ related to the total electrical field $\mathbf{E}_0$:

$$(\hat{\epsilon}_{ef} - \hat{\epsilon}_h)\mathbf{E}_0 = \mathbf{P} . \quad (1)$$

Averaged fields model is based on the assumption that the balance relation for electromagnetic fields present in heterogeneous system can be written as

$$\mathbf{E}_0 = f\langle\mathbf{E}_p\rangle + (1 - f)\langle\mathbf{E}_h\rangle, \quad (2)$$

where $\langle\mathbf{E}_p\rangle$, $\langle\mathbf{E}_h\rangle$ are averaged fields in nanoparticles and host medium, respectively, which fulfill the relations

$$\langle\hat{\alpha}\rangle\langle\mathbf{E}_h\rangle = (\hat{\epsilon}_p - \hat{\epsilon}_h)\langle\mathbf{E}_p\rangle, \quad \langle\mathbf{P}\rangle = f(\hat{\epsilon}_p - \hat{\epsilon}_h)\langle\mathbf{E}_p\rangle. \quad (3)$$

Rearranging the equation (2) with the help of previous relations we obtain required formula for effective permittivity tensor

$$\hat{\epsilon}_{ef} = \hat{\epsilon}_h + f(\hat{\epsilon}_p - \hat{\epsilon}_h) \left[f\mathbf{I} + (1 - f)\langle\hat{\alpha}\rangle^{-1}(\hat{\epsilon}_p - \hat{\epsilon}_h) \right]^{-1}. \quad (4)$$

Here we write $\mathbf{I}$ for identity matrix and introduce the averaged polarizability tensor

$$\langle\hat{\alpha}\rangle = (\hat{\epsilon}_p - \hat{\epsilon}_h) [\hat{\epsilon}_h + \mathbf{N}(\hat{\epsilon}_p - \hat{\epsilon}_h)]^{-1}, \quad (5)$$

where $\mathbf{N}$ is the diagonal matrix of depolarization factors of nanoparticles, N_i , indexed correspondingly to Cartesian coordinates x, y, z . As we consider spheroidal nanoparticles with the symmetry axis parallel to the z -axis, $N_x = N_y = N$, $N_z = 1 - 2N$. In this case, the value N can be calculated analytically [9].

2.2 Radiative corrections

If the distance between neighbouring nanoparticles is not sufficiently large, the electric dipole act one to the other. In such case, an additional external polarization $\mathbf{P}_{ext}$ of particles must be taken into account. The presented relations are derived by certain rearrangement of the DDA model referred in [7].

Let $\mathbf{p}^{(k)} = \hat{\alpha}^{(k)}\mathbf{E}_p^{(k)}$ be the dipole moment of the k -th nanoparticle having the volume $v^{(k)}$ located at a point with radius vector $\mathbf{r}_k$. For a twice of particles, we denote $\mathbf{r}_{jk} = \mathbf{r}_j - \mathbf{r}_k$, $r_{jk} = \|\mathbf{r}_{jk}\|$, $\mathbf{r}_{jk,0} = \mathbf{r}_{jk}/r_{jk}$. Considering the small elements

(with characteristic dimension $d \ll$ wavelength λ), the dipole moment of the j -th particle caused by the k -th dipole is expressed as

$$\mathbf{P}^{(jk)} = \frac{v^{(k)}}{r_{jk}^3} e^{i\kappa r_{jk}} \left\{ \left[3(1 - i\kappa r_{jk}) - \kappa^2 r_{jk}^2 \right] (\mathbf{r}_{jk,0} \cdot \mathbf{p}^{(k)}) \mathbf{r}_{jk,0} - (1 - i\kappa r_{jk} - \kappa^2 r_{jk}^2) \mathbf{p}^{(k)} \right\}, \quad (6)$$

where $\kappa = 2\pi/\lambda$ is the wavenumber. Thus, we introduce the total polarization effect as the sum of all the dipole moments,

$$\mathbf{P}_{ext} = \sum_j \sum_{k \neq j} \mathbf{P}^{(jk)}. \quad (7)$$

It is obvious that an use of this formula demands large data amount in practice, when the great number of randomly distributed particles constitutes modeled system. Since the magnitude of dipole moment $\mathbf{P}^{(jk)}$ decreases by third power of distance in (6), it suffices to consider the nearest particles only. Moreover, we proposed to simplify described problem in this way that an equivalent regular distribution of particles is assumed thereby the important characteristics (fill-factor, particle dimensions) are preserved.

With regard to following application we study the planar case only, when the plane fill-factor F and dot diameter d are given. We aim to estimate averaged dipole moment $\langle \mathbf{P}_{ext} \rangle$ toward the nearest K particles placed in vertices of regular polygon around the arbitrary element located in the polygon center. The particle in the k -th vertex is characterized by unit radius vector $\mathbf{r}_0^{(k)} = (\cos \frac{2k\pi}{K}, \sin \frac{2k\pi}{K}, 0)$. All the elements have the same dipole moment $\mathbf{p} = (p_1, p_2, p_3) = \langle \hat{\alpha} \rangle \langle \mathbf{E}_p \rangle$. Thus, the projection of the last one into vector $\mathbf{r}_0^{(k)}$ gives

$$(\mathbf{r}_0^{(k)} \cdot \mathbf{p}) \mathbf{r}_0^{(k)} = \frac{1}{2}(p_1, p_2, 0) + \frac{1}{2}(p_1, -p_2, 0) \cos \frac{4k\pi}{K} + \frac{1}{2}(p_2, p_1, 0) \sin \frac{4k\pi}{K}.$$

Using well-known summation rule for goniometrical functions we obtain

$$\sum_{k=0}^{K-1} (\mathbf{r}_0^{(k)} \cdot \mathbf{p}) \mathbf{r}_0^{(k)} = \frac{1}{2} K \mathbf{p},$$

therefore, the total averaged dipole coupling leads by (6) to this result:

$$\langle \mathbf{P}_{ext} \rangle = \underbrace{\frac{1}{2} K \frac{v}{r^3} e^{i\kappa r} (1 - i\kappa r + \kappa^2 r^2)}_{G(K)} \langle \hat{\alpha} \rangle \langle \mathbf{E}_p \rangle. \quad (8)$$

If we add this correction term to the averaged polarization $\langle \mathbf{P} \rangle$ in (3), the effective permittivity tensor (4) changes into the final form

$$\hat{\epsilon}_{ef} = \hat{\epsilon}_h + f(\hat{\epsilon}_p - \hat{\epsilon}_h + G \langle \hat{\alpha} \rangle) \left[f \mathbf{I} + (1 - f) \langle \hat{\alpha} \rangle^{-1} (\hat{\epsilon}_p - \hat{\epsilon}_h) \right]^{-1}. \quad (9)$$

3 Application

3.1 Formulation of problem

An external magnetic field causes electrical polarization in noble metal dots. Together, localised surface plasmons (LSP's) excited in metal nanoparticles enhance this effect that can be observed and measured as magneto-plasmonic response [5]. Resulting resonance of permittivity leads to induced anisotropy of gold inclusions. In order to illustrate effect, we will model effective permittivity in the system of identical oblate spheroidal gold nanoparticles with air as the host medium. The dots with diameter d and height h are randomly deposited on the substrate (oxidized Si) with plane fill-factor F ; and, they have axes parallel to the z axis of co-ordinate system (Fig. 1).

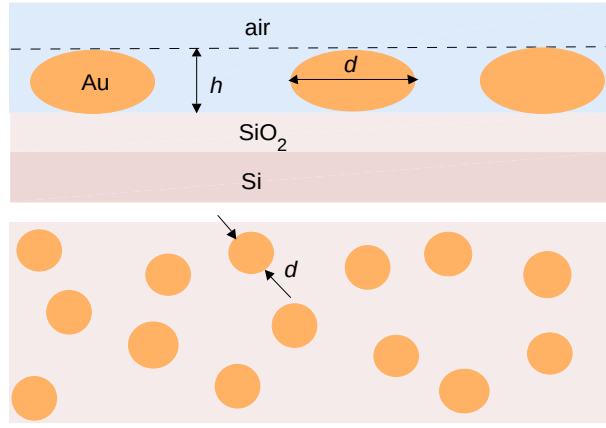


Fig. 1. In-plane distribution of spheroidal nanoparticles: vertical crossection (upper part) and planar scheme (lower part)

An averaging of distance between particle centers enables to apply above derived relation. There are two basic types of uniform in-plane ordering – the rectangular or triagonal one, therefore, the nearest dipoles to the reference particle are located in vertices of a square or hexagon (Fig. 2).

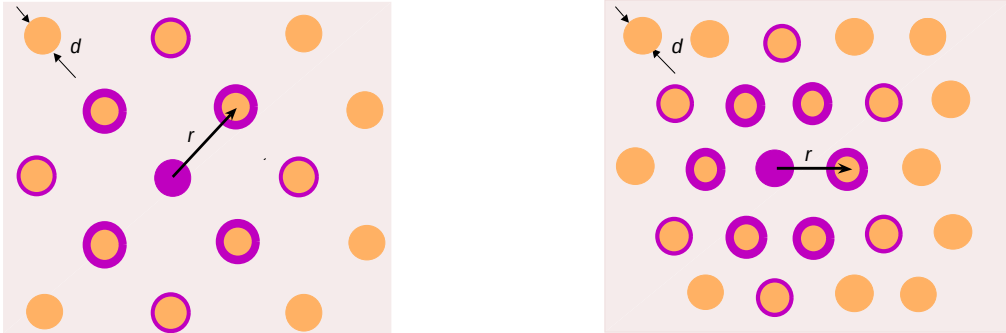


Fig. 2. Uniform square and hexagonal distribution of nanoparticles

Not very complicated calculation leads to following expressions for the averaged dipole distance r :

$$r_{sq} = \frac{d}{2} \sqrt{\frac{\pi}{F}}, \quad r_{hex} = \sqrt{\frac{2}{\sqrt{3}}} \cdot \frac{d}{2} \sqrt{\frac{\pi}{F}}. \quad (10)$$

Note that the volume factor $f = 2F/3$ for in-plane distribution of equal spheroids.

3.2 Numerical results

As the polar magneto-optical configuration is assumed, the relative permittivity of gold is the second rank tensor as a function of wavelength λ :

$$\hat{\epsilon}_p(\lambda) = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ -\epsilon_{xy} & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{xx} \end{bmatrix}. \quad (11)$$

The components ϵ_{xx} , ϵ_{xy} at the wavelengths λ from 400 to 800 nm are derived from Drude-Lorenz model using the data referred in [10] (see also [6] for detailed explanation) by magnetic flux density $B = 0.4$ Tesla. The air ($\epsilon_h = 1$) is assumed as the host medium, therefore we set $\hat{\epsilon}_h = \epsilon_h \mathbf{I}$.

The basic input information about geometrical parameters contains the dot diameter $d = 10$ nm and its height $h = 4$ nm by the plane fill-factor $F = 0.3$. In the computational models we tested three variants of averaged medium:

1. without radiative correction (Eq. 4);
2. correction by square ordering related to the $K = 4$ nearest dipoles – see the dots with thick dark border in the left Fig. 2 (Eq. 9);
3. correction by triangular ordering related to two distance levels of the nearest elements ($6 + 6$ dots with dark border in the right part of the Fig. 2); if $r = r_{hex}$ by (10) for the first level, then $r = r_{hex}\sqrt{3}$ for the second one, as can be easy shown (combined use of Eq. 9).

Resulting spectral dependence of effective permittivity in heterogeneous layer is presented in the Figs. 3 and 4. We observe resonance peaks close to the wavelength 550 nm corresponding to localized surface plasmon excitation that predicts generation of an electric dipole in the dots.

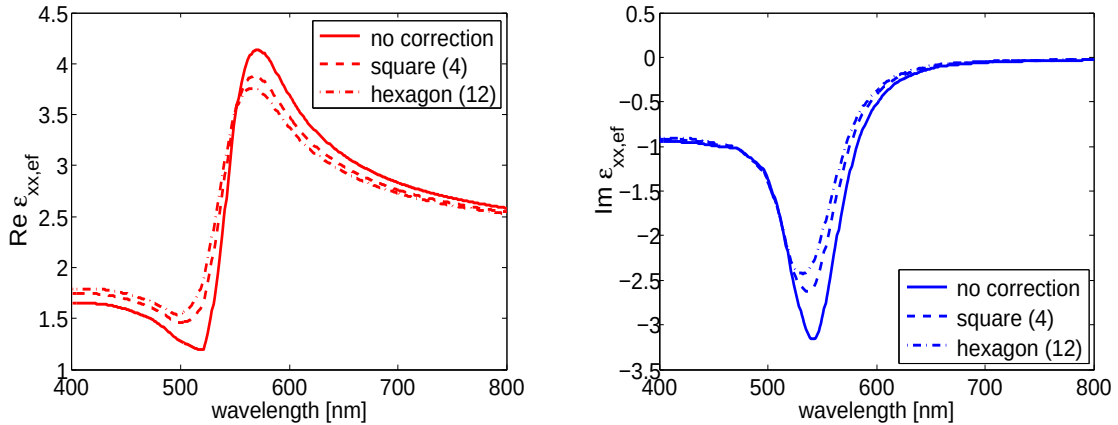


Fig. 3. Diagonal element of effective relative permittivity tensor

Together, discussed influence of radiative correction is clearly demonstrated that decreases magnitude of the effective permittivity tensor elements. As follows from obtained correction formula (8) this effect is produced by the nearest surrounding dipoles.

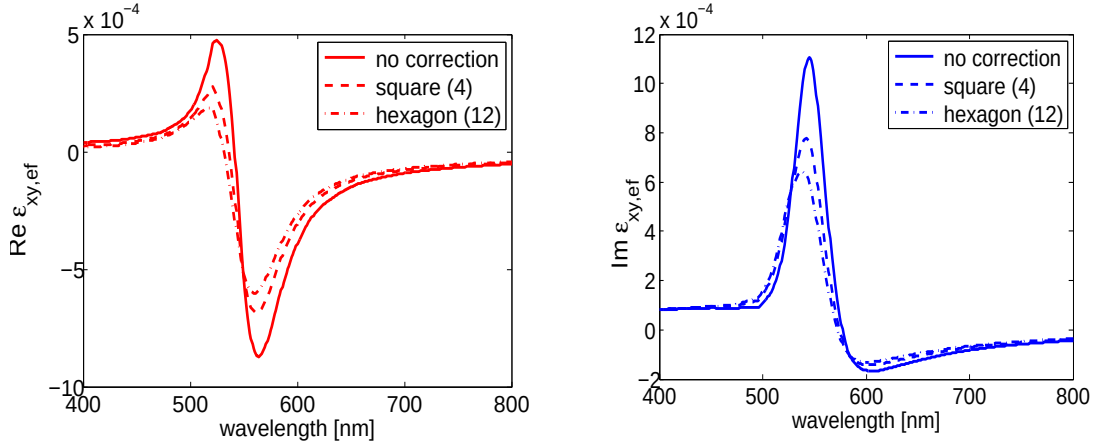


Fig. 4. Off-diagonal element of effective relative permittivity tensor

4 Concluding remarks

The improved theoretical model of anisotropic effective medium has been derived including the radiative corrections applied to dipole moments. Although the results showed that an influence of the nearest elements predominates, the presented model can be used more extensively, if the nanoparticle ordering is uniform.

Acknowledgement

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LINEAR ALGEBRA AND DATA ENCRYPTION¹

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Abstract: The simple application of linear algebra - data encryption is presented. It is a demonstration of the concrete application of the matrix theory in GeoGebra.

Abstrakt: Prezентujeme jednoduchou aplikaci lineární algebry - šifrování dat. Jedná se o demonstraci konkrétního použití operace násobení matic prostřednictvím programu GeoGebra.

1. Encryption and linear algebra

The whole history of mankind stretches need to hide information from any unauthorized person. How to ensure that the message reads only and only its recipient? It was necessary in some manner agreed to conceal the message, encrypt. The messenger, or any other person who somehow got the message, in such a case should not be able to read the message.

The whole issue of encryption has undergone rapid development. There were always new and sophisticated methods of data encryption. However, the history of encryption (cryptography) is not the aim of this paper. Our goal is to make with students a simple applet that allows encrypt given alphabetical sequence according to a pre-agreed key. The whole procedure of encryption will be implemented through an appropriate given numerical matrix. Suitability of that matrix will be discussed later in the context of a decryption script.

We also make a procedure that decrypts the encrypted data back.

Applets were created in GeoGebra version 4.4.36.0.

2. Encryption

For simplicity, we limit ourselves to coding only 26 lowercase letters of the English alphabet, i.e. a - z, plus space as a special character. Any other character will be replaced in the encryption process by this space character.

¹ Author appreciates support of her department.

The principle of coding:
to a given text string we assign a numerical sequence → we will choose four numbers of this sequence → each quadruple is multiplied by the given suitable regular matrix of the fourth order and we construct new sequence → the new sequence will be converted back to the text string.

Decoding is carried out completely analogously, only the matrix inverse modulo 27 to the suitable regular matrix of the fourth order is used.

At first, we define the required objects in GeoGebra.

Construction

1.		In Algebra window click on the right mouse button, choose Auxiliary Objects .
2.		In the Input field enter empty text string, OriginalText = "" , hide this object.
3.		In the Input field enter empty text string CodedText = "" , hide this object.
4.		In the Input field enter empty text string DecodedText = "" , hide this object.
5.		Create Input Box, Caption= OriginalText , Linked Object= OriginalText = "" .
6.		Create Input Box, Caption= CodedText , Linked Object= CodedText = "" .
7.		Create Input Box, Caption= DecodedText , Linked Object= DecodedText = "" .
8.		Create matrix of the fourth order, $B = \{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$.
9.		Create Button, Caption= Encrypt .
10.		Create Button, Caption= Decrypt .

We prepare a script for a button **Encrypt**. We click on the button **Encrypt** using right mouse button and we choose **ObjectProperties**, then **Scripting**, **On Update** and **GeoGebra Script**.

Applet - encryption

```
OriginalList={};
CodedList={};
```

We introduce auxiliary objects that are set to empty numerical lists.

```
SetValue[OriginalList,TextToUnicode[OriginalText]];
```

We convert typed text to a numerical list, the letters a - z are assigned numerical values in Unicode, ie. 97–122.

```
lengthA=Length[OriginalText];
```

We measure the length of the original text string.

```
SetValue[OriginalList,Join[Sequence[  
If[(Element[OriginalList,j]≥97)^(122≥Element[OriginalList,j]),  
Element[OriginalList,j],123],j,1,lengthA],  
Sequence[123,i,1,Mod[4-Mod[lengthA,4],4]]]];
```

We set the new value of the numerical list `OriginalList`, so that all the values from the original list that are not in the interval 97–122, we change to 123 and at the same time we add to the list as many numbers 123, that the length of the numerical string `OriginalList` was divisible by four, because we decided at the beginning that the coding will be done using fourth order matrix.

```
lengthB=Length[OriginalList];
```

We measure the length of the numerical list once again.

```
SetValue[CodedList,Join[Sequence[Element[  
{Take[OriginalList,4*j+1,4*j+4]}*B,1],j,0,(lengthB-3)/4]]];
```

We create vectors formed gradually successive quadruples from the numerical list. By the each vector we multiply the given matrix `B` and the results we join into a coded numerical list.

```
SetValue[CodedList,Sequence[  
97+Mod[Element[CodedList,j]-Mod[97,27],27],j,1,lengthB]];
```

We rescale the coded numerical list. We go through the each number of coded numerical list and their value modulo 27 using a calibration by the numbers 97 and $\text{Mod}[97,27] = 16$ we move into the interval 97–123.

```
SetValue[CodedList,Sequence[If[Element[  
CodedList,j]==123,32,Element[CodedList,j]],j,1,lengthB]];
```

All values 123 in the list will be changed to 32, which is a numerical value representing the space symbol in Unicode.

```
SetValue[CodedText,UnicodeToText[CodedList]];
```

The coded numerical list will be converted into a coded text string.

3. Decryption

Completely analogously we proceed to create a script to decrypt the encrypted text. Multiplication will be implemented using the inverse matrix of the matrix `B`

modulo 27.

For encryption, one can use an arbitrary matrix of the fourth order. But not such encrypted message can be decrypted uniquely. Therefore, it is necessary to use for the encryption suitable matrix, i.e. matrix for which one can find the inverse matrix modulo 27.

The algorithm to find the suitable encryption matrix and the inverse matrix modulo 27:

1. we choose `detB`, determinant of the matrix `B`,
2. the number `detB` we gradually multiply modulo 27 by the integers from the interval 0–26,
3. if there is at least one number 1 between modules (remainders after division), the matrix is suitable for encryption, i.e. there exists inverse matrix modulo 27,
4. the integer that when multiplied by the `detB` modulo 27 gives number 1, we proclaim the determinant of the inverse matrix modulo 27, and we denote by `detinvB`,
5. the inverse matrix `invB` then will be obtained by multiplying the number `detinvB` with the adjoint matrix to the matrix `B`.

The elements of the matrix `invB` are integers. Multiplying the matrix `B` and `invB` we get a diagonal matrix where the diagonal consists of the numbers which modulo 27 gives the remainder equal to one.

Analogously to the previous case, we click on the button **Decrypt** using right mouse button and choose **ObjectProperties**, then **Scripting**, **On Click** and finally **GeoGebra Script**. The script is completely analogous, as in the case of the button **Encrypt**, only at the beginning we first count inverse matrix modulo 27 to the matrix `B`; instead of `OriginalText` and `OriginalList` we have `CodedText` and `CodedList`; and instead of `CodedText` and `CodedList` we have `DecodedText` and `DecodedList`. Next, we use for the multiplication the inverse matrix and for rescaling of the resulting numerical list we use appropriate integers for calibration.

Applet - decryption

```
DetB=Determinant[B];
detinvB=0;
ListModulo=Sequence[Mod[j * DetB,27],j,0,27-1];
If[IsDefined[IndexOf[1,ListModulo]],SetValue[
detinvB,IndexOf[1,ListModulo]-1]];
Adjung=Invert[B]*DetB;
invB=round(Adjung*detinvB);

CodedList={};
DecodedList={};
```

```

SetValue[CodedList,TextToUnicode[CodedText]];

lengthB=Length[CodedText];

SetValue[CodedList,Join[Sequence[If[(Element[
CodedList,j]≥97)^(122≥Element[CodedList,j]),Element[
CodedList,j],123],j,1,lengthB],Sequence[123,i,1,
Mod[4-Mod[lengthB,4],4]]]];

lengthC=Length[CodedList];

SetValue[DecodedList,Join[Sequence[Element[
{Take[CodedList,4*j+1,4*j+4}]* invB,1],j,0,(lengthC-3)/4]]];

SetValue[DecodedList,Sequence[97+Mod[Element[
DecodedList,j]-97,27],j,1,lengthC]];

SetValue[DecodedList,Sequence[If[
Element[DecodedList,j]==123,32,Element[DecodedList,j]],
j,1,lengthC]];

SetValue[DecodedText,UnicodeToText[DecodedList]];

```

Remark: Is it possible to copy individual lines of script from a PDF file. To make the script work, it is necessary to cancel the break of the brimming lines of script. The line always ends up with a semicolon.

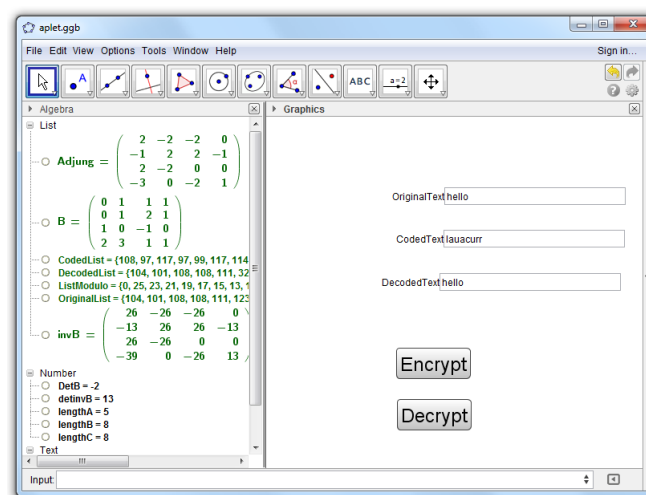


Fig. 1: Applet

Task: Try to decode the following text

„hewqdvshrmokkh“ with respect to the matrix $B = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & -1 & 0 \\ 2 & 3 & 1 & 1 \end{pmatrix}$.

Reference

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HAMILTON EQUATIONS, FIELD THEORY¹

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Abstract: The Hamilton equations in the field theory are studied. The nonholonomic version of Hamilton equations is presented.

Abstrakt: Studujeme problematiku Hamiltonových rovnic v teorii pole a prezentujeme jejich možné neholonomní zobecnění.

1. Introduction

The nonholonomic systems in field theory are a natural generalization of the case of mechanics. Such systems are modelled on fibred manifolds. In contrast of mechanics, the base of fibred manifold is in general n -dimensional. That means we have n independent variables and differential equations representing the motion of a nonholonomic system are partial differential equations. Therefore, the situation in the case of field theory is more complicated and it is difficult to establish the appropriate theory of nonholonomic systems. We present a possible generalization. Details and proofs can be found in [13].

The key object is so-called constraint form. There is a lot of possibilities how to introduce this constraint form. There is a natural requirement to respect constraint form in mechanics. The field version of constraint form must be a direct generalization of let us say classical constraint form.

It was shown that most of such possibilities is unusable because most of such constraint forms generate trivial canonical distribution. Canonical distribution represents the space of possible virtual displacements of a nonholonomic system and in many cases the appropriate motion equations were almost unusable.

For the nonholonomic systems in mechanics exists wide variety of literature and scientific papers, we mention at least some of them, [2–7, 9–12] and references therein. For the field theoryt here is only a few paper dealing with this subject [1, 8, 9, 13].

¹Author appreciates support of his department.

2. Lagrangian systems in field theory

We focus only on Lagrangian systems, i.e. there exists a Lagrangian. But it is possible to study more general case with usage of a concept of Lepage forms [4].

Let us very briefly recall the concept of fibred manifolds in field theory case. Within the text we will consider a fibred manifold $\pi : Y \rightarrow X$ with $\dim X = n$, $\dim Y = m + n$, and its first and second jet prolongations $\pi_1 : J^1Y \rightarrow X$ and $\pi_2 : J^2Y \rightarrow X$.

Local fibred coordinates on Y are denoted by (x^i, y^σ) , where $1 \leq i \leq n$, $1 \leq \sigma \leq m$. The associated coordinates on J^1Y are denoted by $(x^i, y^\sigma, y_j^\sigma)$ and on J^2Y by $(x^i, y^\sigma, y_j^\sigma, y_{jk}^\sigma)$, where $1 \leq j \leq k \leq n$. We skip due to lack of space the whole theory of differential forms and vector fields on fibred manifolds. We remind only important differential form called contact form which has in coordinates expression $\omega^\sigma = dy^\sigma - y_j^\sigma dx^j$.

Let λ be a Lagrangian on J^1Y , θ_λ and E_λ its Poincaré-Cartan and Euler-Lagrange form, respectively. Recall that $E_\lambda = p_1 d\theta_\lambda$. In fibred coordinates where $\lambda = L\omega_0$ we have

$$\theta_\lambda = L\omega_0 + \frac{\partial L}{\partial y_j^\sigma} \omega^\sigma \wedge \omega_j, \quad \omega_j = i_{\partial/\partial x^j} \omega_0 \quad (1)$$

and $E_\lambda = E_\sigma dy^\sigma \wedge \omega_0$, $\omega_0 = dx^1 \wedge \cdots \wedge dx^n$, where

$$E_\sigma = \frac{\partial L}{\partial y^\sigma} - d_j \frac{\partial L}{\partial y_j^\sigma}, \quad (2)$$

the operator d_j , $1 \leq j \leq n$, is the j -th formal derivative operator. The functions E_σ can be expressed by

$$E_\sigma = A_\sigma + B_{\sigma\nu}^{ji} y_{ij}^\nu, \quad (3)$$

where

$$B_{\sigma\nu}^{ji} = -\frac{\partial^2 L}{\partial y_i^\nu \partial y_j^\sigma}, \quad A_\sigma = \frac{\partial L}{\partial y^\sigma} - \frac{\partial^2 L}{\partial x^j \partial y_j^\sigma} - \frac{\partial^2 L}{\partial y^\nu \partial y_j^\sigma} y_j^\nu. \quad (4)$$

A section γ of the fibred manifold π is called an *extremal* of λ if it satisfies the *Euler-Lagrange equations*,

$$J^1\gamma^* i_\xi d\theta_\lambda = 0 \quad \text{for every } \pi_1\text{-vertical vector field } \xi \text{ on } J^1Y. \quad (5)$$

(see [12, 13]). Equations

$$\delta^* i_\xi d\theta_\lambda = 0 \quad \text{for every } \pi_1\text{-vertical vector field } \xi \text{ on } J^1Y, \quad (6)$$

are called *Hamilton* or *Hamilton-De Donder equations* of λ and their solutions, local sections of π_1 , are called *Hamilton extremals*.

Very important is a regularity of a given Lagrangian system. A Lagrangian λ is called *regular* if every solution of the Hamilton equations is holonomic. If

$$\det(B_{\sigma\nu}^{jk}) = \det\left(\frac{\partial^2 L}{\partial y_k^\nu \partial y_j^\sigma}\right) \neq 0 \quad (7)$$

then λ is regular. In this case the Euler-Lagrange equations and Hamilton equations are equivalent. That means in a neighbourhood of every point in J^1Y there exists a

coordinate transformation $(x^i, y^\sigma, y_j^\sigma) \rightarrow (x^i, y^\sigma, p_\sigma^j)$, called *Legendre transformation*, such that the $(n+1)$ -form $d\theta_\lambda$ takes the *canonical form*

$$d\theta_\lambda = -dH \wedge \omega_0 + dp_\sigma^j \wedge dy^\sigma \wedge \omega_j, \quad (8)$$

where

$$p_\sigma = \frac{\partial L}{\partial y_j^\sigma}, \quad H = -L + p_\sigma^j y_j^\sigma. \quad (9)$$

In Legendre coordinates Hamilton equations (6) take the form

$$\frac{\partial(p_\sigma^j \circ \delta)}{\partial x^j} = -\frac{\partial H}{\partial y^\sigma}, \quad \frac{\partial(q^\sigma \circ \delta)}{\partial x^j} = \frac{\partial H}{\partial p_\sigma^j}. \quad (10)$$

3. Nonholonomic constraints

By a *nonholonomic constraint* on J^1Y we shall mean a fibered submanifold Q of $\pi_{1,0}$, $\text{codim } Q = \kappa$, where $1 \leq \kappa \leq mn - 1$. Let

$$f^\alpha(x^i, y^\sigma, y_j^\sigma) = 0, \quad 1 \leq \alpha \leq \kappa, \quad (11)$$

be equations of the constraint Q . By definition,

$$\text{rank} \left(\frac{\partial f^\alpha}{\partial y_j^\sigma} \right) = \kappa. \quad (12)$$

We will assume that $\kappa = kn$, where k is an integer, $1 \leq k \leq m - 1$, and that in a neighborhood of every point in J^1Y , equations (11) can be expressed in a *normal form*

$$f_j^i = 0, \quad \text{where} \quad f_j^i = y_j^{m-k+i} - g_j^i(x^k, y^\sigma, y_l^a), \quad 1 \leq i \leq k, \quad 1 \leq j \leq n. \quad (13)$$

Note that by definition of the constraint submanifold,

$$\text{rank} \left(\frac{\partial f_j^i}{\partial y_l^\sigma} \right) = \max = kn, \quad \text{where } (ij) \text{ label rows and } (\sigma l) \text{ label columns}, \quad (14)$$

(i.e. (14) is a $(kn \times mn)$ -matrix). Moreover, by (13), for every fixed $p = 1, 2, \dots, n$,

$$\text{rank} \left(\frac{\partial f_j^i}{\partial y_p^\sigma} \right) = k, \quad \text{where } (ij) \text{ label rows and } \sigma \text{ label columns}, \quad (15)$$

and

$$\text{rank} \left(\frac{\partial f_j^i}{\partial y_l^\sigma} \right) = k, \quad \text{where } (ijl) \text{ label rows and } \sigma \text{ label columns}. \quad (16)$$

Let (V, ψ) be a fibered chart on Y , (V_1, ψ_1) the associated chart on J^1Y . Let $U \subset V_1$ be an open set. On U consider the following 1-forms

$$\varphi_j^{il} = \frac{1}{n} f_j^i dx^l + \frac{\partial f_j^i}{\partial y_l^\sigma} \omega^\sigma, \quad 1 \leq i \leq k, \quad 1 \leq j, l \leq n. \quad (17)$$

The 1-forms φ_j^{il} will be called *local constraint 1-forms*, associated with the constraint Q .

The forms $\bar{\varphi}_j^{il} = \iota^* \varphi_j^{il}$ generate or span codistribution $\mathcal{C}^0$ called *canonical codistribution* of the constraint Q ,

$$\mathcal{C}^0 = \text{span}\{\bar{\varphi}_j^{il}\}, \quad (18)$$

the mapping ι is the canonical embedding of Q into J^1Y .

In analogy with the case of mechanics, the corresponding distribution $\mathcal{C}$ of corank k will be called the *canonical distribution* of the constraint Q . The ideal $\mathcal{I}$ in the exterior algebra of differential forms on Q generated by the canonical codistribution will be called the *constraint ideal*. A pair $(Q, \mathcal{I})$ where Q is a constraint in J^1Y and $\mathcal{I}$ is the corresponding constraint ideal will be called a *constraint structure* on π_1 .

Let us denote

$$\varphi_j^i = \varphi_j^{il} \wedge \omega_l = f_j^i \omega_0 + \frac{\partial f_j^i}{\partial y_l^\sigma} \omega^\sigma \wedge \omega_l. \quad (19)$$

These n -forms are linearly independent at each point $x \in U$; we will call them *canonical constraint n -forms* [13].

4. Constrained Lagrangian systems

Let $(Q, \mathcal{I})$ be a constraint structure on π_1 , and consider a Lagrangian system defined by a Lagrangian λ on J^1Y .

By a *constrained system* associated with λ and $(Q, \mathcal{I})$ we mean the equivalence class $[\alpha_Q]$ whose elements are the at most 2-contact $(n+1)$ -forms

$$\alpha_Q = \iota^* d\theta_\lambda + \bar{F} + \varphi, \quad (20)$$

where $\bar{F}$ is an arbitrary 2-contact form on Q , and $\varphi \in \mathcal{I}$. The at most 2-contact elements of the equivalence class

$$\iota^* d\theta_\lambda \mod \mathcal{I} \quad (21)$$

then will be called *constraint Poincaré-Cartan $(n+1)$ -forms*.

In fibered coordinates we have

$$\alpha_Q = \bar{A}_a \bar{\omega}^a \wedge \omega_0 + \bar{B}_{ab}^{lj} \bar{\omega}^a \wedge dy_j^b \wedge \bar{\omega}_l + \bar{F}_{ab}^l \bar{\omega}^a \wedge \bar{\omega}^b \wedge \bar{\omega}_l + \varphi, \quad (22)$$

where $\bar{\omega}^a = \iota^* \omega^a$, $\bar{\omega}_l = \iota^* \omega_l$, and

$$\begin{aligned} \bar{A}_a &= \left(A_a + \frac{1}{n} A_{m-k+i} \frac{\partial g_j^i}{\partial y_j^a} + \left(B_{a,m-k+p}^{jl} + B_{m-k+q,m-k+p}^{il} \frac{\partial g_i^q}{\partial y_j^a} \right) \bar{d}_j g_l^p \right) \circ \iota, \\ \bar{B}_{ab}^{lj} &= \left(B_{ab}^{lj} + B_{a,m-k+p}^{li} \frac{\partial g_i^p}{\partial y_j^b} + B_{m-k+p,b}^{ij} \frac{\partial g_i^p}{\partial y_l^a} + B_{m-k+p,m-k+q}^{ri} \frac{\partial g_i^q}{\partial y_j^b} \frac{\partial g_r^p}{\partial y_l^a} \right) \circ \iota. \end{aligned} \quad (23)$$

The operator $\bar{d}_j$ is defined as

$$\bar{d}_j = \frac{\partial}{\partial x^j} + y_j^\sigma \frac{\partial}{\partial y^\sigma}. \quad (24)$$

Let λ be a Lagrangian, $(Q, \mathcal{I})$ a constraint structure on J^1Y , and $[\alpha_Q]$ the corresponding constrained system. A (local) section $\gamma : X \rightarrow Y$ will be called a *constrained extremal* of λ if $J^1\gamma$ is an integral section of $\mathcal{C}$, and

$$J^1\gamma^* i_\xi \alpha_Q = 0 \quad \text{for every } \pi_1\text{-vertical vector field } \xi \in \mathcal{C}. \quad (25)$$

Equations (25) will be called *constrained Euler-Lagrange equations*. Equations

$$\delta^* i_\xi \alpha_Q = 0 \quad \text{for every } \pi_1\text{-vertical vector field } \xi \in \mathcal{C} \quad (26)$$

for integral sections of $\mathcal{C}$ will then be called constrained Hamilton equations of λ . A constrained system $[\alpha_Q]$ is called *regular* if every solution of the constrained Hamilton equations is holonomic. In contrast with Euler-Lagrange equations constrained Hamilton equations depend upon a choice of a representative α_Q in the equivalence class $[\alpha_Q]$.

Theorem [13] *Let $[\alpha_Q]$ be the constrained system related with a Lagrangian system $[\alpha]$ and a constraint structure $(Q, \mathcal{I})$ on $J^1 Y$. If the matrix $(\bar{B}_{ab}^{lj})$ is regular, i.e.,*

$$\det(\bar{B}_{ab}^{lj}) \neq 0, \quad (27)$$

then $[\alpha_Q]$ is regular.

The regularity condition (27) for $\lambda = L \omega_0$ takes the form

$$\det \left(\frac{\partial^2 \bar{L}}{\partial y_l^a \partial y_j^b} - \left(\frac{\partial L}{\partial y_i^{m-k+p}} \circ \iota \right) \frac{\partial^2 g_i^p}{\partial y_l^a \partial y_j^b} \right) \neq 0, \quad \bar{L} = L \circ \iota. \quad (28)$$

It is evident that in general the constrained system of a regular Lagrangian system need not be regular.

5. Hamilton equations for nonholonomic Lagrangian systems

Theorem [13] *Consider a Lagrangian λ and a constraint structure $(Q, \mathcal{I})$ on $J^1 Y$. Let $[\alpha_Q]$ be the related constrained system. Let $x \in Q$ be a point. Suppose that in a neighbourhood of x ,*

$$\frac{\partial \bar{B}_{ab}^{lj}}{\partial y_q^c} = \frac{\partial \bar{B}_{ac}^{lq}}{\partial y_j^b}. \quad (29)$$

Then there exists a neighbourhood $U \subset Q$ of x , and, on U , functions P_a^l , and a n -form η , such that the class $[\alpha_Q]$ has a representative of the form

$$\alpha'_Q = \eta \wedge \omega_0 + dP_a^l \wedge dy^a \wedge \bar{\omega}_l. \quad (30)$$

If, moreover, the constrained system $[\alpha_Q]$ is regular, then $(x^i, y^\sigma, y_l^a) \rightarrow (x^i, y^\sigma, P_a^l)$ is a coordinate transformation on U .

The integrability condition (29) for the $\bar{B}_{ab}^{lj}$'s ensures that it is possible express the functions P_a^l explicitly. We consider a mapping $\chi : [0, 1] \times W \rightarrow W$ defined by $(u, x^i, y^\sigma, y_l^a) \rightarrow (t, y^\sigma, u y_l^a)$, where $W \subset Q$ is an appropriate open set. Then Poincaré Lemma gives us

$$P_a^l = -y_j^b \int_0^1 (\bar{B}_{ab}^{lj} \circ \chi) du = \frac{\partial \bar{L}}{\partial y_l^a} - y_j^b \int_0^1 \left(\left(\frac{\partial L}{\partial y_i^{m-k+p}} \circ \iota \right) \frac{\partial^2 g_i^p}{\partial y_l^a \partial y_j^b} \right) \circ \chi du. \quad (31)$$

We will call the above functions P_a^l the *constraint momenta*, and the corresponding coordinate transformation *constraint Legendre transformation*. The 1-form η in (30) is *determined up to a constraint 1-form*, and *need not be closed*. We will call it a *constraint energy 1-form*.

It can be proved that for the energy 1-form η we have

$$\eta = \eta_i dx^i + \bar{\eta}_a dy^a + \bar{\eta}_{m-k+i} dy^{m-k+i} + \tilde{\eta}_b^i dy_b^i, \quad (32)$$

where η_i is an arbitrary function on U , and

$$\begin{aligned} \bar{\eta}_a &= \bar{A}_a + \frac{\partial P_a^i}{\partial x^i} + y_b^i \left(\frac{\partial P_a^i}{\partial y^b} - \frac{\partial P_b^i}{\partial y^a} \right) + g_i^j \frac{\partial P_a^i}{\partial y^{m-k+j}}, \\ \tilde{\eta}_b^i &= -y_j^a \frac{\partial P_a^j}{\partial y_b^i}, \quad \bar{\eta}_{m-k+i} = -y_j^a \frac{\partial P_a^j}{\partial y^{m-k+i}}, \end{aligned} \quad (33)$$

In Legendre coordinates we can write

$$\eta = \eta_i dx^i + \eta_a dy^a + \eta_l^a dP_a^l \mod \mathcal{I} \quad (34)$$

and the Hamilton equation then take the form

$$\frac{d}{dx^i}(P_a^l \circ \delta) = \eta_a, \quad \frac{d}{dx^l}(y^a \circ \delta) = -\eta_a^l, \quad \frac{d}{dx^l}(y^{m-k+i} \circ \delta) = g_l^i. \quad (35)$$

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MULTIDIMENSIONAL MODELS OF RELATIONSHIP BETWEEN THE CHARACTERISTICS OF LEADERS AND IMPROVEMENT OF THE QUALITY MANAGEMENT

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Abstract: The paper presents results of research on the relationship between quality management and leadership. Relying on the results of the survey were developed based on a multidimensional model using a multiple regression method. Regression models are used to analyze the relationship between several independent variables (explanatory) and the dependent variable (explanatory). Multiple regression analysis allows the prediction of the dependent variable based on knowledge of the independent variables.

1. Introduction

The paper presents results of research on the relationship between quality management and leadership. Relying on the results of the survey were developed based on a multidimensional model using a multiple regression method. Regression models are used to analyze the relationship between several independent variables (explanatory) and the dependent variable (explanatory). Multiple regression analysis allows the prediction of the dependent variable based on knowledge of the independent variables.

There are two attitudes dominating in the studies concerning leadership in an organization. The first one assumes that leadership is based on certain qualities which a leader should have. The second one assumes so-called - a process-like attitude to leadership, according to which leadership is mainly based on the interaction between a leader and employees and does not entirely result from leader's qualities. The attitude based on qualities is in accordance with a concept in which being a leader stands for a set of inborn features which can't be learned. Whereas, the attitude based on the interaction assumes that most managers can learn how to be a leader through gained experience [1, 2, 6, 7, 9].

The leadership qualities decide, to a large extent, if a person is a good and successful leader. The contemporary studies that a good leader should have not only charisma but most of all, should have communication skills as communication with his subordinates, superiors, co-workers, clients, deliverers and stakeholders is a large part of his work. In the conducted studies, the concept of leadership was based on leadership qualities. The qualities, which have been taken into consideration in the research, were chosen by means of the analysis of literature and an expert-like method [3, 4, 5, 8, 11, 12, 13].

2. Description of the research

The questionnaire studies were conducted within the research project of Ministry of Science and Higher Education in the first quarter of 2010. The questionnaires were sent to 2500 companies. 700 of them were industrial companies while 1800 were services ones. 1120 correctly filled -in questionnaires were obtained.

14 variables were taken into account within a scope of the improvement of quality management (D1-D14):

- D1 – having a quality management system being in accordance with standards of PN-EN ISO 9001,
- D2 –attitude to the implementation of standardization in an organization,
- D3 – having trade quality management systems,
- D4 – having environmental management systems,
- D5 – having security management systems,
- D6 – use of team work ,
- D7 – involvement in team work,
- D8 –use of methods and tools of quality management systems (the variable which defines how many methods and tools of quality management is used in a given organization),
- D9 – number of innovative ideas per an employee,
- D10 – rewarding employees for their innovative ideas,
- D11 – pro-innovative attitude of a company,
- D12 – market position of a company in comparison with the best companies in a given branch of business,
- D13 – change of market position in comparison with companies in recent years,
- D14 – financial condition of a company.

In order to analyse the concept of leadership in an organization, the theory of leadership based on leadership qualities was used in this publication. 18 variables, which characterize a successful leader, were used in the studies (P1-P18):

- P1 – physical appearance, such as height or appeal,
- P2 – interpersonal communications skills,
- P3 - vigour,
- P4 – aiming for constant development,
- P5 – planning and action organizing skills,
- P6 – courage,
- P7 – resistance to stress,
- P8 – ability to assess others' work,
- P9 – innovation,
- P10 – diplomatic skills,
- P11 – ability to persuade,
- P12 – aim-oriented attitude,
- P13 – open-minded action taking,
- P14 – self-assessment ability,
- P 15 – truthfulness,
- P16 – being consistent ,
- P 17 – fulfilling promises,
- P 18 – discretion.

3. Multidimensional model of relations

Based on the obtained results it can be next to the account correlation also use the multiple regression analysis. The model should be strongly correlated variables with the dependent variable and how the least correlated [10]. Using multiple regression methods were determined moving backward regression models. Regression models are used to analyze the relationship between several independent variables (explanatory) and the dependent variable (explanatory). Multiple regression analysis allows the prediction of the dependent variable based on knowledge of the independent variables. Backward stepwise regression implies next (step) removal from the model built from all possible variables to those of them which, in a step to the least significant impact on the dependent variable. So proceed until the "best" model. About the extent to which the created model describes the phenomenon studied provide the following factors:

- multiple correlation coefficient R - it measures the extent to which the equation allows the prediction of the dependent variable , and determines the strength of the relationship between the dependent variable and the explanatory variables , taken together ,
- coefficient of determination R², which allows to draw conclusions that part of the variance explained constructed model.

Table 1 shows the variables significant at a significance level of less than $\alpha=0,1$. Empty fields in the table means that the variable does not qualify for the model. Fields in which there are values represent the average change in the dependent variable when you change the independent variable by one unit (the top) and t statistics (the bottom). For each model in the table are also given the constant equation and the value of R (which determines the fit of the model to empirical variables). For example, the model estimated for the attitude of pro-innovation organization has the following form:

$$D11=0,092*P2+0,097*P4+0,061*P5-0,075*P15+1,817 \quad R=0,306$$

It shows that the attitude of innovation-oriented organization depends positively on interpersonal communication skills (P2), striving for continuous development (P4) and the ability to plan and organize activities (P5) and negatively on truthfulness (P15) leader.

Full interpretation of the parameters of this equation is:

- with increasing interpersonal communication skills of one unit pro-innovation attitude organization grows an average of 0,092 units,
- with increasing desire for continuous development of leaders for innovation-oriented attitude of one unit of the organization to grow by an average of 0,097 units,
- with an increase in the ability to plan and organize activities by the leader of one unit of pro-innovation attitude organization grows an average of 0,061 units,
- with the increase of the truthfulness of the leaders of one unit of pro-innovation attitude falls by an average of 0,075 units.

Table 1. Models multidimensional relationship between the characteristics of leaders and improvement of the quality management

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	wyraz wolny	R
D1	0,032 (2,38)							0,039 (1,89)	0,043 (2,17)						-0,358 (-2,0)		0,047 (2,43)		0,355	0,159
D2	0,049 (2,65)																0,04 (1,65)		2,227	0,178
D3		0,024 (1,65)	-0,036 (-2,5)											0,029 (2,16)					0,048	0,144
D4		-0,03 (-1,8)												0,031 (2,04)					0,144	0,135
D5							-0,029 (-1,7)	0,028 (1,73)											0,262	0,131
D6	0,045 (3,51)	-0,033 (-1,7)					-0,437 (-2,2)	0,058 (2,93)		0,036 (1,89)	-0,035 (-2,0)								0,166	0,198
D7	0,077 (1,90)			0,106 (1,70)				0,062 (2,04)		0,14 (2,36)	-0,134 (-2,4)					-0,108 (-1,7)			0,388	0,187
D8																				
D9				0,296 (1,84)												-0,365 (-2,2)			1,073	0,145
D10															0,052 (2,97)	-0,037 (-1,8)		-0,032 (-1,9)	0,411	0,171
D11		0,092 (2,79)		0,097 (2,94)	0,061 (1,70)										-0,075 (-2,6)				1,817	0,306
D12		0,071 (1,79)																	3,420	0,137
D13				-0,087 (-1,9)											-0,081 (-2,1)		0,097 (2,31)		2,412	0,132
D14													0,071 (1,85)				0,073 (1,91)		3,234	0,141

Source. Authors own research.

The resulting coefficient of determination R^2 shows that 9.4% of the variability in attitudes explain the organization of pro-innovation leadership qualities managers. This result is typical for social variables. In the same way you can make the interpretation of the remaining regression equations.

4. Conclusion

The analysis of multiple regression confirms the conclusions wznikajce of correlation analysis. Also in this case a very important leadership qualities in terms of improvement of quality management can be considered as the ability to judge the work of others, skills, self-esteem and fulfillment of promises by the leader. An interesting difference is the physical characteristics of the manager, which according to this method are much more important than suggested by the correlation analysis. This is due to the fact that this variable is poorly correlated with other variables leadership.

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AKO ZAPOJIŤ ŠTUDENTOV DO SYSTEMATICKEJ PRÁCE POČAS SEMESTRA - 2. ČASŤ

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Abstrakt: Počet študentov, ktorí úspešne ukončia predmet Matematika I na SjF STU z roka na rok klesá. Vplyv na tento fakt má nedostatočná príprava z matematiky na strednej školy a taktiež aj to, že študenti nie sú naučení priebežne študovať. Je možné aby sa tento postoj študentov k štúdiu na vysokej škole zmenil? V článku sa popisuje experiment, ktorý Ústav matematiky a fyziky SjF STU zrealizoval v prvom semestri akademických rokov 2012/13 a 2013/14. Tento experiment bol zameraný na zlepšenie práce študentov počas celého semestra a následne aj na lepšie zvládnutie predmetu Matematika I. V príspevku je vyhodnotený experiment akademického roka 2013/14 a porovnaný s výsledkami experimentu z predchádzajúceho akademického roka.

Abstract: Number of students who successfully complete the subject Mathematics I at SjF STU is constantly decreasing. This situation is caused by inadequate preparation at secondary schools and also by the fact that students are not accustomed to systematic study. Is it possible to change this attitude of students to university study? The paper describes the experiment, Institute of mathematics and physics implemented in the first semester of the school years 2012/13 and 2013/14. This experiment was aimed at improving students' work during the whole semester and consequently at better mastering of the subject Mathematics I. In the paper the experiment of the academic year 2013/14 is evaluated and compared with experimental results from the previous academic year.

1. Úvod

Dôsledkom reforiem školstva na Slovensku je aj to, že študenti, ktorí prichádzajú na na vysoké školy technického zamerania majú nedostatočné vedomosti z prírodovedných predmetov, hlavne z matematiky a fyziky. Na druhej strane však matematika a fyzika sú teoretickým základom erudovaného strojného inžiniera. Na Strojníckej fakulte STU v Bratislave sa snažíme už dlhé roky vyrovnáť stredoškolské vedomosti študentov z matematiky, buď pomocou kurzu zo stredoškolskej matematiky pred začiatkom semestra, alebo pomocou Doplnkových cvičení z matematiky I počas semestra. Kurz pred začiatkom semestra je dobrovoľný, Doplnkové cvičenia z Matematiky I sú povinné pre všetkých tých študentov, ktorí nezvládnu jednoduchý vstupný test zo stredoškolskej

matematiky na začiatku semestra. Vyrovnanie vedomostí pomocou absolvovania tohto predmetu sa osvedčilo [1], no aj napriek tomu, výsledky z Matematiky I sú neuspokojivé [2]. Počas semestra väčšina študentov pracuje nepravidelne, nevyužívajú materiály, ktoré sú im dostupné na štúdium, dalo by sa povedať, že ignorujú konzultačné hodiny učiteľov. Menší pohyb nastane pred priebežnými kontrolkami počas semestra a tiež aj pred termínmi skúšok. Otázky na konzultáciách však nie sú zamerané na podstatu nejakého problému, ale skôr na to, aké príklady budú na kontrolke, resp. na skúške. Niektorí študenti sa dokonca radšej naučia príklady z minulých školských rokov naspamäť, ako keby sa mali nad nimi zamyslieť a pochopiť ich.

2. Popis experimentu

Dobré základy vedomostí zo stredoškolskej matematiky a pravidelná práca počas semestra sú istým predpokladom úspešného absolvovania predmetu Matematika I. Ako bolo už v úvode povedané, vedomosti študentov nie sú na rovnakej úrovni a systematická príprava študentov na cvičenia je skôr výnimkou ako pravidlom. Ako keby väčšina z nich si nechcela uvedomiť, že štúdium na vysokej škole je dobrovoľné a nie každý človek musí mať vysokoškolské vzdelanie. Sú rôzne metódy ako donútiť študentov, aby pracovali počas celého semestra a získané vedomosti aj využili potom pri skúške z Matematiky I. Nie každá z nich však prinesie očakávané výsledky a veľakrát učiteľ stratí pri preverovaní vedomostí študentov oveľa viac energie ako je získaný výsledok.

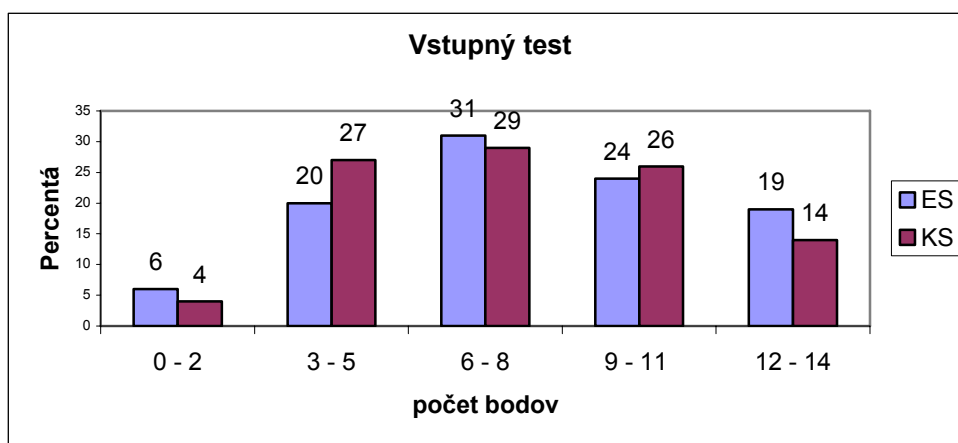
Na Ústave matematiky a fyziky na cvičeniach z Matematiky I sme v akademických rokoch 2012/13 a 2013/14 vyskúšali v jednej z troch paraleliek metódu, ktorou sme chceli naučiť študentov priebežne sa pripravovať na cvičenia. V akademickom roku 2012/13 to bolo v paralelke, kde študenti dosiahli vo vstupnom teste najhoršie výsledky, v akademickom roku 2013/14 to bola paralelka s najlepšími výsledkami. Vyhodnotenie experimentu z roku 2012/13 je v publikácii [3].

V predmete Matematika I pri hodnotení na skúške sa berie do úvahy aj práca študentov počas celého semestra. Na zápočet z Matematiky I je potrebných 15 bodov zo 40 (akademickom roku 2012/13 to bolo 13 bodov). Na začiatku semestra v experimentálnej paralelke každý študent získal 13 bodov. Aby si ich udržal, musel na tom pravidelne počas semestra pracovať. Študenti dostávali na domácu úlohu príklady, ktoré si mali vypracovať a pripraviť na nasledujúce cvičenie. V prvej polovici ďalšieho cvičenia študenti tieto príklady samostatne riešili a vysvetľovali bez použitia poznámok. Boli vyvolávaní sporadicky, nebol v tom žiaden systém. Každý študent počas semestra bol vyvolaný 13-krát. V prípade, že sa nepripravil, stratil bod. Keďže semester trvá 13 týždňov a cvičenia z Matematiky I sú vo výmere 4 hodiny, teda dve cvičenia za týždeň, požiadavka trinástich samostatných vystúpení pred tabuľou počas semestra bola realizovateľná. Tiež, ak bol komplikovanejší príklad, tak riešenia tohto príkladu sa zúčastnili viacerí študenti. Problematika, z ktorej boli domáce úlohy, bola vysvetlená na prednáškach a na predchádzajúcom cvičení v druhej polovici cvičenia sa podrobne riešili podobné príklady ako tie, ktoré boli zadane na domácu úlohu. Študenti mohli využiť aj konzultačné hodiny. Tie boli naplánované tak, že pri riešení domácich úloh medzi cvičeniami študenti mohli vyhľadať pomoc učiteľa ak sa vyskytol nejaký problém. To ich malo donútiť pracovať v predstihu a nenechávať si úlohu na poslednú chvíľu. Tiež mali k dispozícii skriptá s riešenými príkladmi z Matematiky I. Počas semestra študenti experimentálnej paralelky podobne, ako aj zvyšných dvoch, písali dve kontrolky za 27 bodov.

Na skúške je maximálny počet bodov, ktoré môže študent získať 60, z toho 40 bodov za príklady a 20 bodov za teoretický test. Nutné podmienky k získaniu skúšky sú minimálne 25 bodov z príkladov, 10 bodov teoretický test, 35 bodov zo skúšky a 56 bodov spolu za cvičenia a skúšku. V prípade, že študent nesplní niektorú nutnú podmienku okrem prvej, tak ešte absolvuje ústnu skúšku.

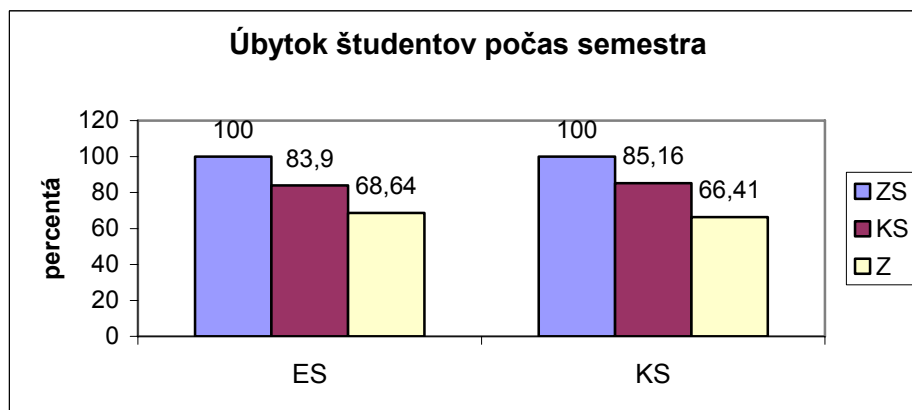
3. Vyhodnotenie experimentu

Na začiatku semestra všetci študenti písali vstupný test zo stredoškolskej matematiky s jednoduchými otázkami za 14 bodov [1]. Jeho vyhodnotenie v percentách je na nasledujúcom grafe.

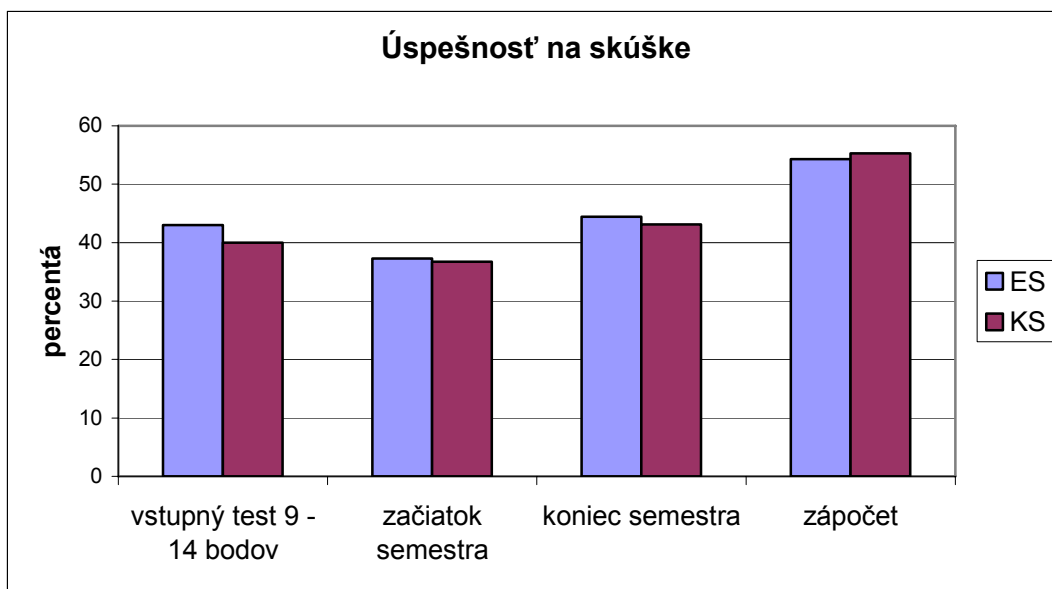


Na otázky, ktoré boli v teste, by mal každý absolvent strednej školy zodpovedať na 100%, prípadne by mohlo byť rozmedzie 12 – 14 bodov. Boli sme však benevolentní a porovnali sme rozmedzie 9 – 14 bodov a 0 – 8 bodov. 9 - 14 bodov získalo 43% študentov experimentálnej skupiny a 40% študentov kontrolnej skupiny. Vo nižšom bodovom hodnotení 0 - 8 bodov je pomer opačný. Tento počet bodov získalo 57% študentov experimentálnej skupiny a 60% študentov kontrolnej skupiny.

Úbytok študentov počas semestra, ukazuje nasledujúci graf. Pripomeňme, že už počas semestra niektorí študenti zistia, že štúdium nezvládnu, a preto ho ukončia, resp. iba sa zapíšu na štúdium a nenastúpia do školy. Preto je aj rozdiel medzi počtom študentov na začiatku semestra (ZS) a konci semestra (KS).

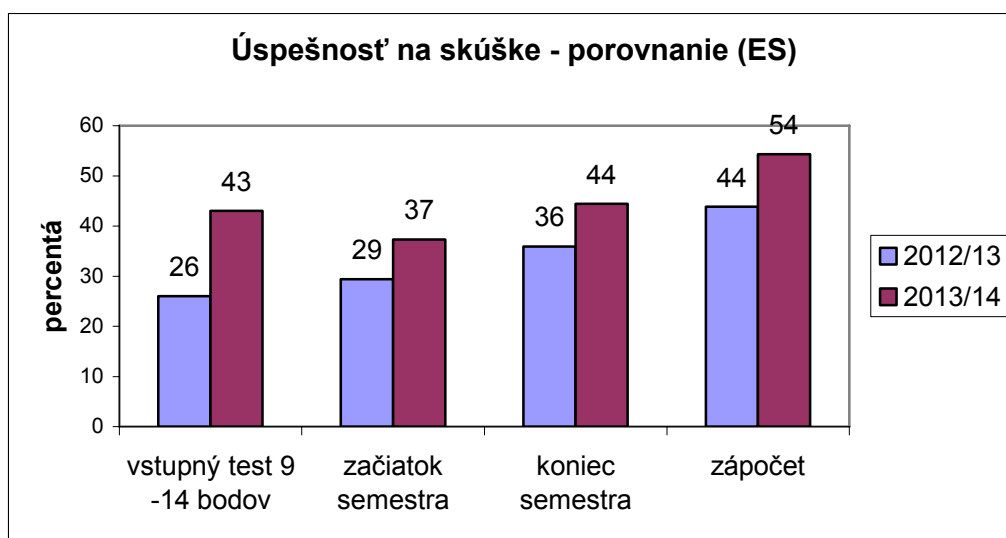


Vidíme, že počas semestra v experimentálnej skupine odišlo 16,1% študentov a v kontrolnej skupine 14,84%, čo nie je veľký rozdiel. Počet študentov, ktorí získali zápočet vzhľadom na začiatok semestra (Z) je v experimentálnej skupine vyšší o 2,23%. Nasledujúci graf porovnáva úspešnosť na skúške študentov experimentálnej a kontrolnej skupiny vzhľadom na počet študentov, ktorí boli na začiatku semestra, na konci semestra a vzhľadom na tých, ktorí získali zápočet z Matematiky I.



Z grafu je zrejmé, že úspešnosť na skúške je skoro rovnaká aj v experimentálnej aj kontrolnej skupine. Je však zarážajúce, že približne iba 37% zapísaných študentov do prvého ročníka ukončí predmet Matematika I. Úspešnosť tých študentov, ktorí získali zápočet je asi 55%. Tieto výsledky vonkoncom nie sú uspokojivé a určite budeme hľadať nové metódy, ako to vylepšiť.

Porovnali sme aj úspešnosť na skúške v experimentálnej skupine v akademickom roku 2012/13 a 2013/14.



Vzhľadom na to, že sa jednalo o skupinu, ktorá mala lešie výsledky vstupného testu (o 17% viac študentov napísalo vstupný test v akademickom roku 2013/14) aj úspešnosť

na skúške je lepšia, aj keď nie až o toľko percent ako pri vstupnom teste. Úspešnosť na skúške vzhľadom na počet študentov na začiatku aj konci semestra je v akademickom roku 2013/14 lepšia o 8%, vzhľadom na počet študentov, ktorí získali zápočet o 10%.

4. Záver

Experiment, ktorému sme sa venovali v dvoch akademických rokoch, nepriniesol očakávané výsledky. Ani v predchádzajúcom, ani v tomto akademickom roku priechodnosť v predmete Matematika I sa rapídne nezlepšila, dokonca v tomto roku je rovnaká aj v experimentálnej aj kontrolnej skupine. Minulý rok sme vyhodnotili aj dotazník, ktorý sme dali vyplniť študentom experimentálnej skupiny. Väčšina študentov vtedy kladne hodnotila systematickú prácu počas celého semestra, niektorí chceli ešte aj viac povinných domácich úloh a kontroliek. V tomto akademickom roku, sme dokonca na základe výsledkov daného dotazníka po skúške zverejnili aj príklady, ktoré boli na skúške spolu aj s výsledkami, resp. návodom na riešenie. Začíname sa zamýšľať už aj nad tým, či to, čo robíme pre študentov nie je kontraproduktívne. Naši študenti majú v AIS k dispozícii prednášky, príklady na cvičenia, každý učiteľ má v týždni dve hodiny konzultácií, na ktorých je zvyčajne sám, existujú skriptá so vzorovo riešenými príkladmi....Možno, že aj tu by mohlo platiť, že niekedy menej je viac. Tiež je otáznosť, či sa študenti vedú správne pripravovať na skúšku? Nespočíva ich príprava iba pri čítaní prednášok a cvičení? Nemá vplyv na úspešnosť aj fakt, že medzi koncom semestra a začiatkom skúškového obdobia sú vianočné prázdniny? Nie je demotivujúce aj to, že vyšší bodový základ z cvičení, ktorý mali študenti experimentálnej skupiny, pri príprave na skúšku študenta nenúti intenzívnejšie pracovať?

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AN EVALUATION OF POSSIBILITIES OF PUTTING A NEW PRODUCT ON THE MARKET BY A FOOD ENTERPRISE

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Abstract: The article presents the results of research conducted among consumers, the aim of which was to design a new product that would allow extending the offer of a big local bakery, operating in the western part of the Silesian Province. The research included the following aspects:

- identification of criteria influencing consumers' choice of bread,
- exploring consumers' preferences by means of a survey,
- designing a process of organic bread production using the QFD method.

An analysis of the obtained results allowed designing a new type of „fitness“ bread and developing a production process in accordance with the preferences of the enterprise customers.

Introduction

Looking at the current situation on the market, it is not necessary to possess extensive knowledge about the industry to conclude that it is increasingly hard for a company to keep their advantage. It comes as no surprise that the customers' needs undergo radical changes, as the trends and tendencies in the industry also change. Companies often wonder how they should rationally talk about “tomorrow”. It is made possible by a constant analysis of both the company's and industry's situation in order to compare trends and tendencies [26]. As stated in the source [23], food items in Poland have a “bright” future ahead of them; however, the crisis connected with a rise in prices of resources, as well as the fact that consumers focus on the healthiness of products, happens to strike the baking and confectionery industries. They modify their diets, choosing lighter snacks and baking their own bread. An effect of this decrease in demand for bread is bankruptcy of bakeries, mostly among the smaller ones. To avoid this, some of them try to increase their revenue by putting unique prices on their products, using the weak and strong sides of their competitors to strengthen the position of their own goods. They also decide to periodically introduce new products as a quick response to the changing needs of the consumers [15]. The aim of this paper is to conduct an analysis with regard to introducing a new organic, slim-line product in the bakery.

The subject of the study

A company which belongs to the small enterprise group has been chosen as the subject of this study. It is a bakery which specialises in the production of bread, pastry goods and confectionery. The products are distributed through 4 outlets located in the Silesian and Opole

Provinces. The company also cooperates with a few restaurants. Most of the work in the company is done manually and traditional technologies are used.

The methods chosen and, consequently, the tools, which will allow for the completion of the stages of this study are: community interview, observation, survey questionnaire, QFD analysis.

Choice of the research sample

We started by gathering the consumers' opinions on the products offered at the time. To do that, a survey on customers' preferences was conducted. The most important task was to determine the size of the research sample. With market studies, the size of the general population is quite big; the number of households reported by the Statistical Office in 2012 was 14,148,307. For the survey to be reliable, typical individuals were chosen. The research was limited to the Kedzierzyn-Koźle and Gliwice counties due to the fact that the company operates only within these two regions and the probability of individuals from these areas taking part in a survey is very high. Therefore, a non-random draft of the research sample was made, based on objective data about the structure of the studied population. The number of participants was chosen on the basis of the data from 2012 disclosed by the Statistical Offices pertaining to the number of households in the counties. They all amounted to 67987 households.

The calculations were performed using the formula for a research sample n in a finite population:

$$n = \frac{N^2 p(1-p)}{e^2 + N p(1-p)}$$

where:

p – proportion in the population (estimated),

e - permissible error,

N - size of the population,

$Z=1.96$ for $e = 0.05$.

The estimation of the value of p (proportion in the population) has been made using the weekly record of customers based on receipts.

The average number of customers equals approximately 865 per day. Having assumed a confidence level of 95%, for which $Z = 1.96$, and a permissible error e of 5%, it was possible to determine the size of the research sample. If we put the values into the equation, we get the value of p and the size of the research sample.

$$p = \frac{86500}{67987} \cong 1.27\%$$

$$n = \frac{0.0127(1-0.0127)}{\frac{0.05^2}{1.96^2} + \frac{0.0127(1-0.0127)}{67987}} \cong 141$$

The survey research has been conducted on 141 interviewees chosen from all outlets in the period of 21.01.-25.01.2014. The respondents were asked to give a rating for all the consumer attributes. The conducted survey was aimed at determining the factors which influence the customers' choice of bread as well as their overall preferences.

Research analysis

Based on the research conducted in the company, 15 customer attributes were distinguished and put as subject for evaluation in the survey.

The respondents were asked to rate every attribute on a 1-10 scale, on which 10 was given to an attribute regarded as very important by the customer and 1 to one which was completely unimportant. The results of the survey have been presented in Table 1:

Table 1

Weights of individual consumer attributes

No.	Consumer attributes	Rating	Weigh
1	Low amount of artificial additives	1091	8
2	Low caloric value	860	4
3	Long shelf life	859	4
4	Low glycemc index	658	1
5	Low fat content	722	3
6	Low salt content	740	3
7	Type of flour	980	6
8	Condition of the crust	985	5
9	Shape	699	2
10	Colour	834	4
11	Smell	982	5
12	Bread structure	978	5
13	Price	1013	7
14	Packaging	632	1
15	Slicing	802	4

Table 1 presents the clearly defined consumer demands. The ones rated the highest are those connected with price and healthiness of the offered product, i.e.: ‘low amount of artificial additives’, ‘type of flour’. A little below that are the attributes connected with nutritional value, such as ‘low calories’. Organoleptic features are also important to the customers, comprising: ‘condition of the crust’, ‘smell’ and ‘bread structure’. Long shelf life is a very important requirement as well.

In order to determine which technical attributes have the biggest influence on the designed product, it was necessary to calculate the product of the weight given to a consumer attribute and the value of relationship with the technical attribute, according to the assumed relationships (strong – 9, semi-strong – 3 and weak – 1). Table 2 shows the technical attributes arranged in order.

Table 2

Technical attributes

No.	Technical attribute	Relative significance	Absolute significance [%]
1	Pre-ferment	267	21
2	PN-A-74103:1993 standard requirements – mixed bread	227	18
3	Proper preparation and baking conditions	177	14
4	Cooling time and manner	118	9
5	Yeast dosing	117	9
6	Fermentation time	114	9
7	Furnace steaming	102	8
8	Expansion	81	6
9	Flour power (PN-A-74032:2002 standard - Cereals)	58	5

At the process design stage, target values are specified, by means of which we can monitor the individual processes and measure their efficiency. Reaching these values will allow us to satisfy customers’ needs and contribute to increased competitiveness of the product. After that, we define the value which the given attributes are approaching as well as the aims and objectives, which are supposed to help them reach the target value.

Thanks to the aforementioned actions and the defining of target values, it was possible to fill in all the spaces in the “quality house” (Fig 1).

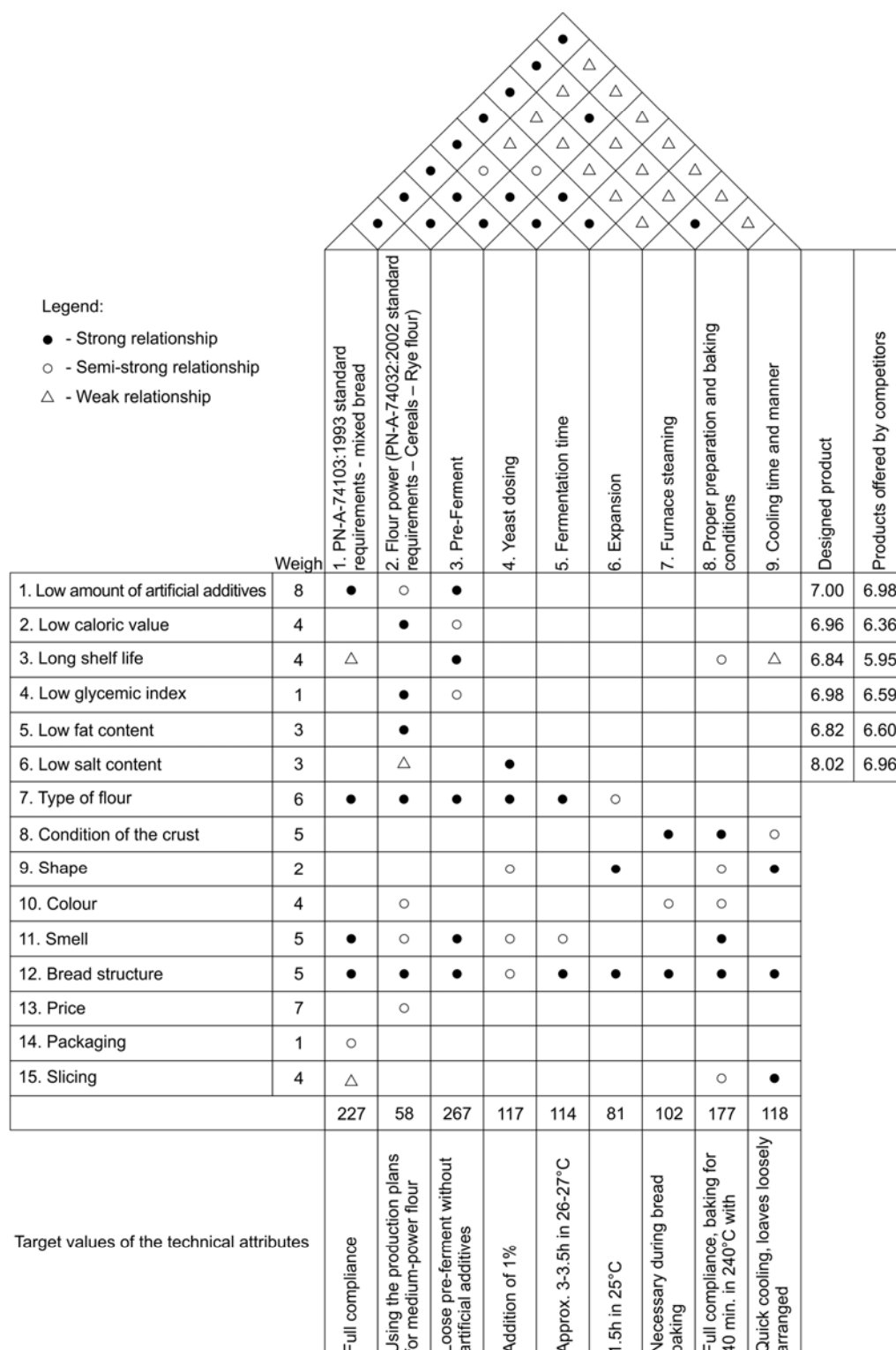


Fig. 1. Quality house

The analysis of the data concerning the customers' demands with regard to the offered bread as well as the impact of those needs on the technical parameters allowed us to obtain information thanks to which it was possible to design a production process for white, organic

bread – bread with bran, classified as a special high quality type of bread, enriched with special cereal products or other additives. The company has at their disposal the machines and devices used in the production of various types of bread, so no further costs involved in the purchase of new equipment need to be incurred.

Description of the production process

The production process of the proposed bread begins with preparing a pre-ferment in room temperature in the bakery.

Next, the pre-ferment is subjected to fermentation. The fermentation time can be regulated by dosing the yeast. In the designed process, due to time saving, as the production of the new product does not affect the currently realised production plan of the night shift, 3% dosing has been proposed, which should result in a fermentation time of 1.5h in 25°C.

After the fermentation is over, the pastry needs to be divided and formed into 600g pieces (of any shape). It should be placed in trays sprinkled with bran and leave for expansion for 2.5h in 25°C. Directly before putting it into the furnace, the top should be sprinkled with bran and 5 parallel cuts should be made on the surface. This bread should be very well baked. The baking temperature and time are 240°C for 40min. with steaming. Approx. 5 minutes before the end, the door of the furnace should be opened to allow the bread to dry up a bit. The ducts are closed in order for the bread to get its crispy, brown crust. Drying up the bread at the final stage of baking has an impact on the quality of its structure; thanks to that the bread is not too damp. After being cooled down, the loaves can be cut, packaged properly, marked and made ready for transport.

Conclusions

Conducting the QFD analysis allowed us, thanks to the information obtained from the customers, to translate the customers' needs into technical requirements and determine their significance for the production process. The tool has allowed us to translate the customers' demands regarding the bread into conditions the studied company has to meet at every stage of the production process. Parts which are the most important to the process include: preparing a good pre-ferment, actions compliant with the current requirements of standards pertaining to mixed bread as well as providing the right conditions in the facility (preparation and baking conditions). The least important part is the fulfilment of requirements of the standard pertaining to the power of rye flour. A comparison with the competitors has allowed us to conclude that both the company in which the study was conducted and the competing companies offer products of high quality. It is, therefore, important to design a new product very carefully so as to attract new customers and strengthen the company's position on the market.

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Využití GeoGebry ve výuce matematiky a geometrie

Workshop na konferenci 3 μ 2014

Horní Lomná, 2. – 4. června 2014

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Co je GeoGebra?

GeoGebra je volný a multiplatformní dynamický software pro všechny úrovně vzdělávání, neboť spojuje geometrii, algebru, tabulky, znázornění grafů, statistiku a infinitezimální počet, to vše v jednom balíčku. Tento program získal četná ocenění pro vzdělávací software v Evropě a USA.

- Grafika, algebra a tabulky jsou propojeny a plně dynamické
- Jednoduše použitelné uživatelské prostředí, mnohé výkonné funkce
- Autorizační nástroje k vytvoření výukového materiálu na webové stránce
- Přístupné milionům uživatelů na celém světě v mnoha jazycích
- Free a open source software

<http://www.geogebra.org>

Kolektiv autorů děkuje za podporu Katedře matematiky a deskriptivní geometrie, Vysoké školy báňské - Technické univerzity Ostrava.

Co se naučíte na našem workshopu?

Šifrování jako aplikace lineární algebry v GeoGebře

Jana Volná, Petr Volný (jana.volna@vsb.cz, petr.volny@vsb.cz)

V GeoGebře si vytvoříme jednoduchou aplikaci, která nám umožní zašifrovat text. Bude se jednat o aplikaci lineární algebry, konkrétně o operace s maticemi.

Skriptování v GeoGebře

Jana Bělohávková (jana.belohlavkova@vsb.cz)

Seznámíme se se základy skriptování a ukážeme si, jak se skriptování dá využít při tvorbě studijního materiálu nebo ke zpestření přednášek.

Klasifikace kuželoseček

Radomír Paláček (radomir.palacek@vsb.cz)

Cílem této lekce je ukázat, jak lze využít GeoGebru při klasifikaci kuželoseček pomocí invariantů.

Výuka náhodných veličin s využitím GeoGebry

Petra Schreiberová (petra.schreiberova@vsb.cz)

Cílem lekce je ukázat si způsob, jak lze ve cvičeních využít GeoGebru pro lepší pochopení pojmu rozdělení náhodné veličiny a významu hodnot distribuční funkce. Tuto lekci lze využít jako snadné cvičení pro studenty.



Využití GeoGebry ve výuce matematiky a geometrie

3 μ 2014

Šifrování jako aplikace lineární algebry v GeoGebře

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Abstrakt: V GeoGebře si vytvoříme jednoduchou aplikaci, která nám umožní zašifrovat text. Bude se jednat o aplikaci lineární algebry, konkrétně o operace s maticemi.

V okamžiku, kdy si lidé začali předávat zprávy písemnou formou, vynořila se potřeba utajení informací obsažených ve zprávě před nepovolanou osobou. Bylo nutné zvolit takovou formu utajení informace obsažené ve zprávě, aby se osoba, která nese zprávu či se k této zprávě ať už náhodným nebo cíleným způsobem dostane, nebyla schopna přečtením zprávy utajenou informaci získat. Ovšem adresát zprávy musel být se způsobem utajení informace seznámen, musel mít tzv. klíč. Jinak by samozřejmě nebyl schopen informaci obsaženou ve zprávě získat.

Mezi nejjednodušší možnosti utajení informací patřily „neviditelné inkousty“. Do zprávy, která obsahovala banální informace se vepsala zpráva nová inkoustem, který po uschnutí zmizel. Inkoust bylo možné zviditelnit nejčastěji vystavením listu zprávy tepelnému působení, nebo se používala odpovídající chemikálie, kterou adresát zprávy doručený list potřel.

Mezi sofistikovanější způsoby patřilo přeházení písmen textu podle dohodnutého klíče. Klíč mohl být číselný, bylo možné také použít dohodnutou sadu znaků či symbolů.

My si v lekci ukážeme jednoduchou aplikaci, pomocí které zašifrujeme zadaný text. Pro vlastní šifrování použijeme „vhodnou“ regulární matici, pro dešifrování zprávy matici inverzní modulo 26.

Skript byl vytvořen v GeoGebře verze 4.4.36.0.

Pro jednoduchost se budeme zabývat zašifrováním jednoho slova, které bude tvořeno malými písmeny anglické abecedy, tzn. nebudeme používat diakritická znaménka.

Princip kódování je následující:

zadanému slovu přiřadíme číselnou řadu $\rightarrow$ z číselné řady budeme vybírat n -tice čísel, pro náš případ to budou čtveřice $\rightarrow$ každou čtveřici vynásobíme zadanou regulární maticí čtvrtého řádu $\rightarrow$ novou číselnou řadu poskládanou ze získaných čtveřic převedeme zpátky na text.

Dekódování se realizuje zcela analogicky, pouze k násobení používáme maticí inverzní modulo 26 k zadané regulární maticí čtvrtého řádu.

Konstrukce

1.		V okně Algebra klikneme na pravé tlačítko myši, zvolíme Auxiliary Objects.
2.	Vstup:	Do vstupního pole zadáme prázdný textový řetězec, OriginalText = "", skryjeme.
3.	Vstup:	Do vstupního pole zadáme prázdný textový řetězec CodedText = "", skryjeme.
4.	Vstup:	Do vstupního pole zadáme prázdný textový řetězec DecodedText = "", skryjeme.
5.	a = 1	Vytvoříme textové pole, Caption=OriginalText, Linked Object=OriginalText = "".
6.	a = 1	Vytvoříme textové pole, Caption=CodedText, Linked Object=CodedText = "".
7.	a = 1	Vytvoříme textové pole, Caption=DecodedText, Linked Object=DecodedText = "".
8.	Vstup:	Vytvoříme matici čtvrtého řádu, $B = \{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$.
9.	OK	Vytvoříme tlačítko, Caption=Encrypt.
10.	OK	Vytvoříme tlačítko, Caption=Decrypt.

Tu jednodušší část máme za sebou. Nyní je třeba nastavit pro naše tlačítka Encrypt a Decrypt skript, který zajistí požadované kódování a dekodování. Účastníci lekce dostanou k dispozici textové soubory obsahující zmíněné skripty.

Na tlačítko Encrypt klikneme pravým tlačítkem myši a vybereme Object Properties, dále položku Scripting, OnClick a GeoGebraScript.

Skript - kódování

```
OriginalList={};
CodedList={};
```

Zavedeme pomocné objekty, které se nastaví na prázdné číselné seznamy.

```
SetValue[OriginalList,TextToUnicode[OriginalText]];
```

Převod zadaného textu na číselný seznam, písmenům a–z se přiřadí číselné hodnoty v Unicode, tj. 97–122.

```
lengthA=Length[OriginalText];
```

Změříme délku původního textového řetězce.

```
SetValue[OriginalList, Join[OriginalList,
Sequence[122, j, 1, Mod[4-Mod[lengthA, 4], 4]]]];
```

Nastavíme novou hodnotu číselného seznamu OriginalList, a to tak, že k původní sadě čísel přidáme takový počet čísel 122, aby délka nového seznamu byla dělitelná čtyřmi, tzn. buď se nepřidá nic, je-li délka původního seznamu dělitelná čtyřmi, a nebo se přidá posloupnost délky 1 nebo 2 nebo 3 tvořená čísly 122 (nezáleží na tom, které číslo zvolíme, 122 odpovídá hodnotě písmena z v Unicode) v závislosti na tom, jaký je zbytek po dělení délky seznamu modulo 4. Toto je třeba ošetřit, protože jsme se na začátku rozhodli, že kódování budeme provádět pomocí matice čtvrtého řádu.

```
lengthB=Length[OriginalList];
```

Znovu změříme délku číselného seznamu.

```
SetValue[CodedList, Join[Sequence[Element[
{Take[OriginalList, 4*j+1, 4*j+4]}*B, 1], j, 0, (lengthB-3)/4]]];
```

Vytváříme vektory tvořené postupně následnými čtveřicemi čísel z číselného seznamu. Každý vektor vynásobíme zadanou maticí a výsledky násobení spojujeme do kódovaného číselného seznamu.

```
SetValue[CodedList,
Sequence[97+Mod[Element[CodedList, j]+7, 26], j, 1, lengthB]]];
```

Realizujeme přeškálování kódovaného číselného seznamu. Procházíme jednotlivá čísla z kódovaného číselného seznamu a jejich hodnotu pomocí modulo 26 a kalibrace pomocí čísel 97 a 7 přesouváme do intervalu 97–122. Přičtením čísla 7 se vzhledem k jednotkové matici přiřadí písmenu a písmeno a, $(97 + 7) \bmod 26 = 0$.

```
SetValue[CodedText, UnicodeToText[CodedList]]];
```

Kódovaný číselný seznam se převede na kódovaný textový řetězec.

Na tlačítko Decrypt klikneme pravým tlačítkem myši a vybereme Object Properties, dále položku Scripting, OnClick a GeoGebraScript.

Skript - dekodování

Lze převzít celý skript-kódování s několika změnami. Především určíme inverzní matici $\text{inv}B$ k B modulo 26, ne každá matice je vhodná pro kódování. Princip je takový, že určíme determinant matice B , ten násobíme čísly od 0 do 25 modulo 26. Pokud mezi takto získanými čísly je alespoň jedno číslo 1, matice je pro kódování vhodná.

```

DetB=Determinant[B];
invDet=0;
ListModulo=Sequence[Mod[j*DetB,26],j,0,25];
If[IsDefined[IndexOf[1,ListModulo]],
SetValue[invDet,IndexOf[1,ListModulo]-1]];
Adjung=Invert[B]*DetB;
invB=Adjung*invDet;

CodedList={};
DecodedList={};

SetValue[CodedList,TextToUnicode[CodedText]];

lengthB=Length[CodedText];

SetValue[CodedList,Join[CodedList,
Sequence[122,j,1,Mod[4-Mod[lengthB,4],4]]]];
lengthC=Length[CodedList];

SetValue[DecodedList,Join[Sequence[Element[
{Take[CodedList,4*j+1,4*j+4]}*invB,1],j,0,(lengthC-3)/4]]];

SetValue[DecodedList,
Sequence[97+Mod[Element[DecodedList,j]-97,26],j,1,lengthC]];

SetValue[DecodedText,UnicodeToText[DecodedList]];

```

Poznámka: Je možné kopírovat jednotlivé řádky skriptu ze souboru pdf. Aby skript fungoval, je nutné zrušit zalomení přetékačích řádků skriptu. Řádek vždy končí až středníkem.

Úloha: Zkuste dekódovat následující kódovaný text

keqkogmwdvqpcsrilxqehndd vzhledem ke kódovací matici $B = \begin{pmatrix} 5 & 0 & 7 & 9 \\ 1 & 1 & 8 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 8 & 0 & 1 \end{pmatrix}$.

Zdroj

<http://practicalcryptography.com/ciphers/classical-era/hill/>



Využití GeoGebry ve výuce matematiky a geometrie

3μ 2014

Skriptování v GeoGebře

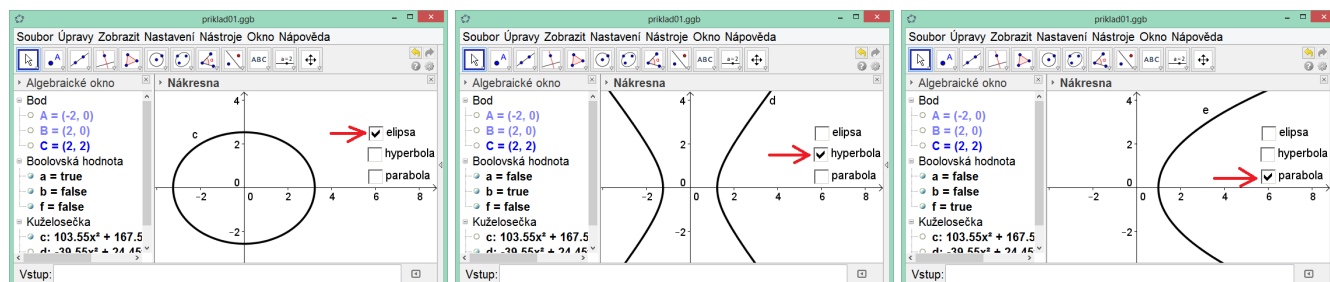
Jana Bělohávková

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Seznámíme se se základy skriptování a ukážeme si, jak se skriptování dá využít při tvorbě studijního materiálu nebo ke zpestření přednášek.

Příklad 1:

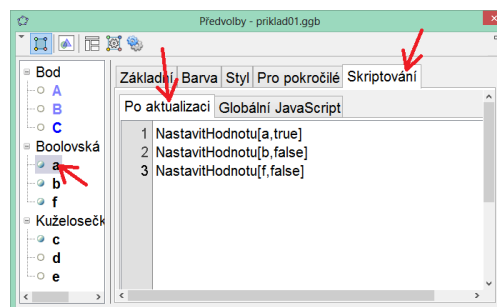
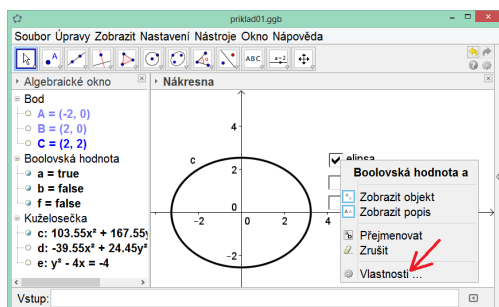
Zadání: Kliknutím postupně vybereme právě jednu ze tří možností. Viz obrázek 1.



Obrázek 1: Výběr jedné ze tří možností

Konstrukce

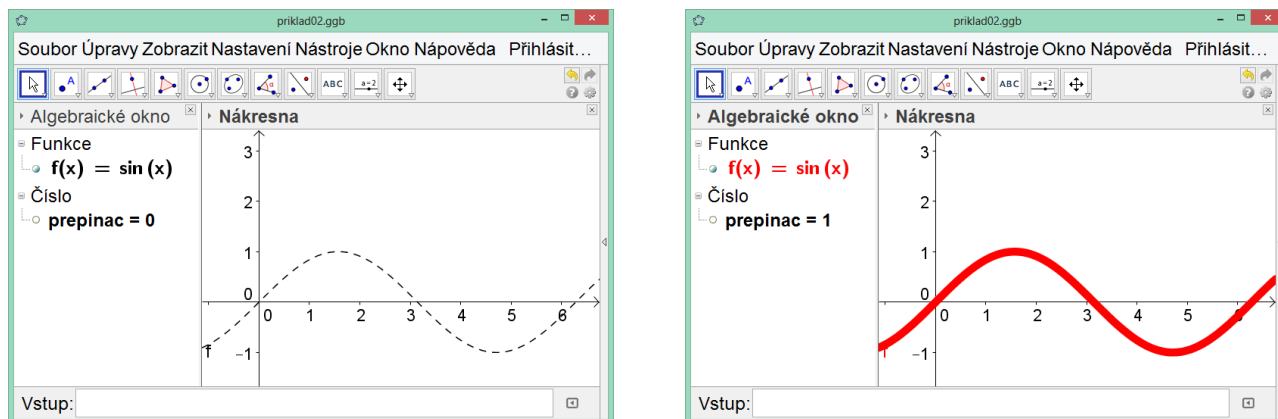
1.		Z panelu nástrojů vybereme nástroj <i>Elipsa</i> , třikrát klikneme do nákrešny. Sestrojíme tak tři body A , B , C , a tím i elipsu c .
2.		Podobně sestrojíme hyperbolu d : klikneme na již existující body A, B, C .
3.		Parabolu sestrojíme kliknutím na bod B a osu y .
4.		Z panelu vybereme nástroj <i>Zaškrťovací políčko</i> a klikneme do nákrešny. Otevře se dialogové okno, ve kterém doplníme <i>Popisek</i> : elipsa, z rozbalovacího menu vybereme <i>Elipsa c</i> a úpravy potvrdíme kliknutím na tlačítko <i>Použít</i> . Stejně vytvoříme zaškrťovací políčka b a f i pro hyperbolu d a parabolu e .
5.		Pravým tlačítkem klikneme na zaškrťovací políčko elipsy a ze zobrazeného menu vybereme <i>Vlastnosti</i> . Otevře se nové dialogové okno a v něm do záložky <i>Sriptování / Po aktualizaci</i> napíšeme postupně zvlášť na řádky <code>NastavitHodnotu[a,true]</code> , <code>NastavitHodnotu[b,false]</code> , <code>NastavitHodnotu[f,false]</code> podle obrázku 2.



Obrázek 2: Menu a dialogové okno

Příklad 2:

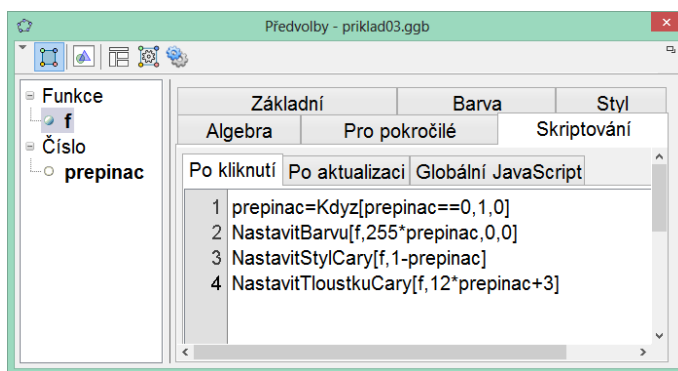
Zadání: Klikáním na zvolený objekt bude objekt měnit barvu, tloušťku a styl čáry. Viz obrázek 3.



Obrázek 3: Změna vlastností

Konstrukce

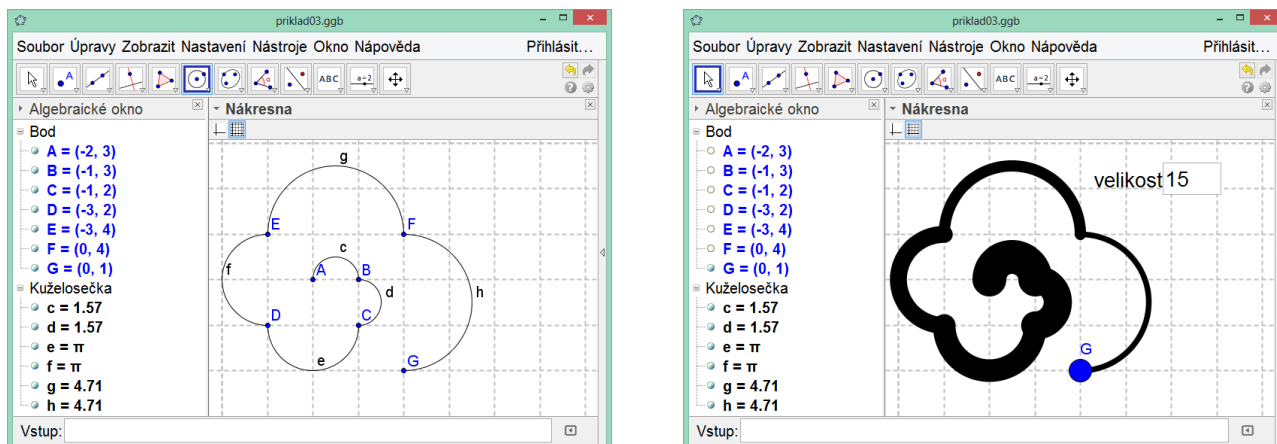
1.	Vstup:	Do vstupního pole zadáme $f(x) = \sin(x)$
2.	Vstup:	Do vstupního pole zadáme <code>prepinac=0</code>
3.		V dialogovém okně funkce f v záložce <i>Skriptování</i> / Po kliknutí napíšeme postupně zvlášť na řádky <code>prepinac=Kdyz[prepinac==0,1,0]</code> , <code>NastavitBarvu[f,255*prepinac]</code> , <code>NastavitStylCary[f,1-prepinac]</code> , <code>NastavitTloustkuCary[f,12*prepinac+3]</code> , podle obrázku 4.



Obrázek 4: Dialogové okno pro skriptování

Příklad 3:

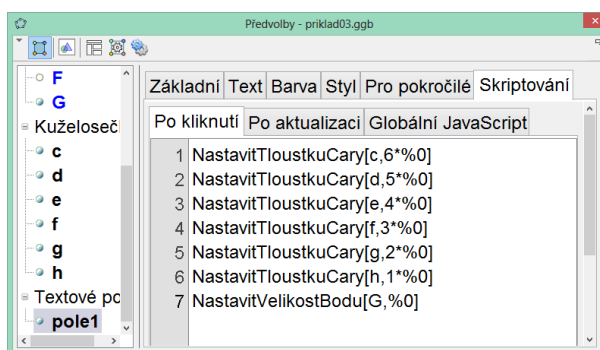
Zadání: Jedním vstupem můžeme změnit několik různých hodnot najednou. Například můžeme nastavit různé tloušťky čar u různých objektů. Viz obrázek 5.



Obrázek 5

Konstrukce

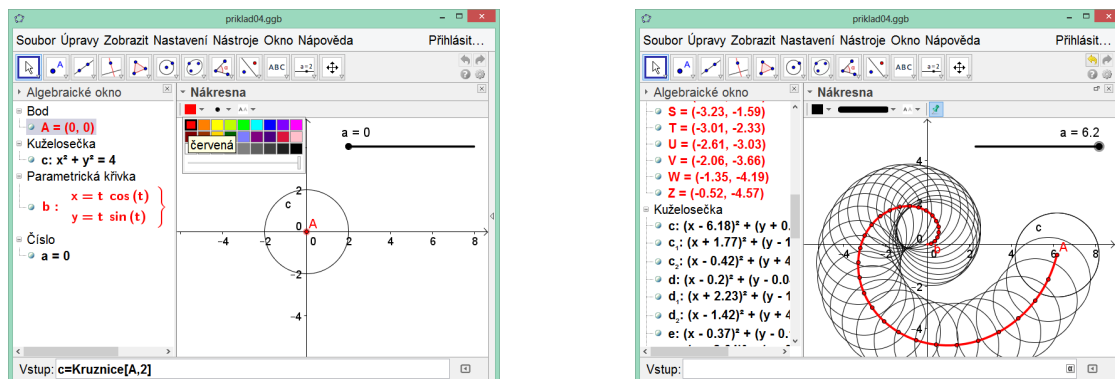
1.		Kliknutím na malou šipku v liště okna Nákrasna se otevře formátovací panel. Skryjeme osy a zobrazíme mřížku.
2.		Z hlavního menu vybereme nástroj <i>Polokružnice nad dvěma body</i> a podle obrázku 5 nebo podle své fantazie vytvoříme šest půlkružnic c až h .
3.		Vybereme nástroj <i>Textové pole</i> a klikneme do nákrasny. Otevře se okno <i>Textové pole</i> , do kterého vložíme <i>Popisek</i> : velikost a potvrdíme kliknutím na tlačítko <i>Použít</i> .
4.		Do záložky textového pole <i>Skriptování</i> / <i>Po kliknutí</i> napíšeme <code>NastavitTloušťkuCary[c, 6*%0], NastavitTloušťkuCary[d, 5*%0], ...NastavitTloušťkuCary[h, %0], NastavitVelikostBodu[G, %0]</code> .



Obrázek 6: Menu a dialogové okno

Příklad 4:

Zadání: Vytvoříme objekty, které budou kopírovat stopu objektu. Viz obrázek 7.



Obrázek 7

Konstrukce

1.		Do vstupního pole postupně zadáme $a = \text{Posuvnik}[0, 2\pi, 0.2]$, $A = (a \cdot \cos(a), a \cdot \sin(a))$, $b = \text{Krivka}[t \cos(t), t \sin(t), t, 0, a]$, $c = \text{Kruznice}[A, 2]$. Sestrojený bod A a křivku b můžeme obarvit na červeno.
2.		Do záložky bodu A <i>Skriptovani/Po aktualizaci</i> napíšeme <code>KopirovatVolnyObjekt[A]</code> , do záložky kružnice c <i>Skriptovani/Po aktualizaci</i> napíšeme <code>KopirovatVolnyObjekt[c]</code> .
3.		Pohneme posuvníkem a .

Příklad 5:

Zadání: Vytvoříme dva body, které budou volné a závislé zároveň.

Konstrukce

1.		Vytvoříme dva body A a B .
2.		Do záložky bodu A <i>Skriptovani/Po aktualizaci</i> napíšeme <code>NastavitHodnotu[B, (x(A)+1, y(A)+1)]</code> , do záložky bodu B <i>Skriptovani/Po aktualizaci</i> napíšeme <code>NastavitHodnotu[A, (x(B)-1, y(B)-1)]</code> .
3.		Pohneme bodem A , pohneme bodem B .

Zdroj

<http://www.geogebra.org/material/show/id/17721>



Využití GeoGebry ve výuce matematiky a geometrie

3μ 2014

Klasifikace kuželoseček

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Cílem této lekce je ukázat, jak lze využít GeoGebru při klasifikaci kuželoseček pomocí invariantů.

Klasifikace kuželoseček

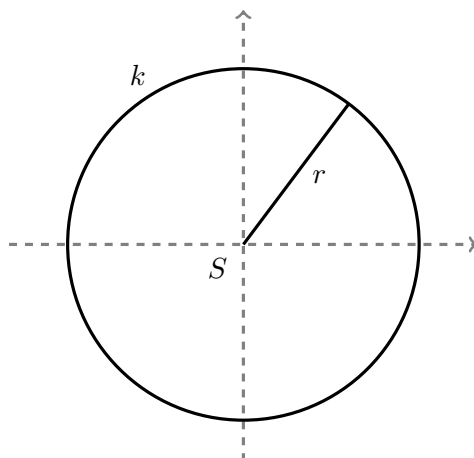
Vytvořte aplet na klasifikaci kuželoseček zadaných obecnou rovnicí.

Malé opakování na úvod

Typy kuželoseček

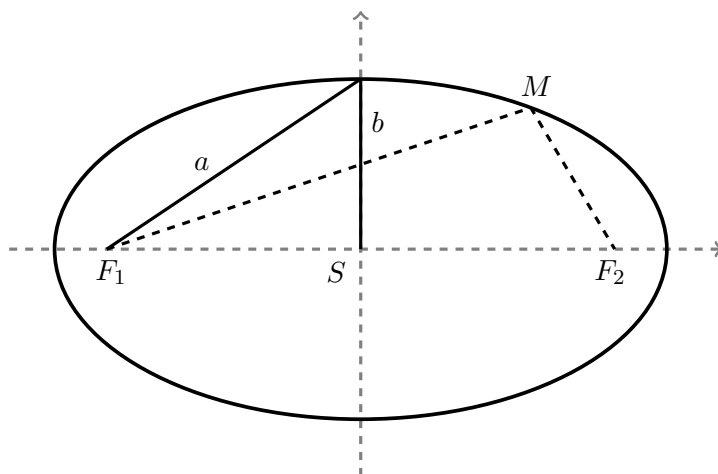
Kružnice

Kružnice je množina všech bodů roviny, které mají od daného bodu S (středu kružnice) stejnou vzdálenost r (poloměr kružnice). Kružnice je speciálním případem elipsy. V tomto případě ohniska splynou v jeden bod (střed S) a pro velikosti poloos platí, že $a = b$ a velikost excentricity $e = 0$.



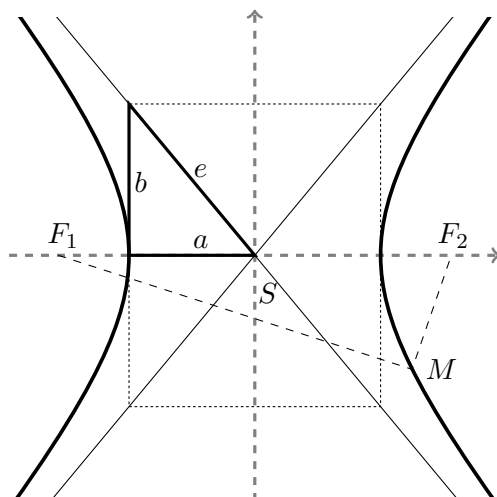
Elipsa

Elipsa je množina všech bodů roviny, které mají od dvou různých bodů F_1, F_2 konstantní součet vzdáleností rovný $2a$, který je větší než vzdálenost bodů F_1, F_2 . Číslo a je velikost hlavní poloosy, číslo b je velikost vedlejší poloosy a musí platit, že $a, b > 0$. Body F_1, F_2 se nazývají ohniska elipsy. Vzdálenost ohniska elipsy od jejího středu se nazývá excentricita elipsy. Vzdálenost ohnisek F_1, F_2 je rovna $2e$ a nazývá se ohnisková vzdálenost.



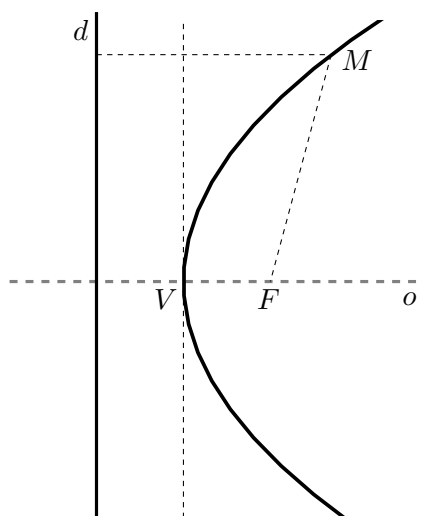
Hyperbola

Hyperbola je množina všech bodů roviny, které mají od dvou různých pevně daných bodů F_1, F_2 konstantní kladný rozdíl vzdáleností rovný $2a$, který je menší než vzdálenost bodů F_1, F_2 . Body F_1, F_2 se nazývají ohniska hyperboly. Bod S je střed hyperboly. Číslo b je délka hlavní poloosy hyperboly, číslo b je délka vedlejší poloosy hyperboly, e nazýváme excentricitou hyperboly.



Parabola

Parabola je množina všech bodů roviny, jejichž vzdálenost od bodu F je rovna vzdálenosti od přímky d . Danou přímku nazýváme řídící přímkou a značíme ji d . Bod F je ohnisko paraboly. Přímka, která je kolmá k řídící přímce d a prochází ohniskem F je osa o paraboly. Bod V , který je vrcholem paraboly leží na ose a pólí vzdálenost bodu F od řídící přímky d .



Příklad 6: Klasifikace kuželoseček

Zadání:

Vytvořte aplet na klasifikaci kuželoseček zadaných obecnou rovnicí.

Budeme uvažovat tzv. obecnou rovnici ve tvaru

$$k: a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2 + 2a_{01}x_1 + 2a_{02}x_2 + a_{00} = 0,$$

kde $a_{ij} \in \mathbb{R}$ a $(a_{11}, a_{12}, a_{22}) \neq (0, 0, 0)$.

Při transformaci souřadnic se nemění některé charakteristické veličiny této algebraické rovnice kuželosečky. Tyto veličiny se označují jako *invarianty*.

Uvedená rovnice má tři invarianty:

- determinant matice kuželosečky

$$\det(A) = \begin{vmatrix} a_{00} & a_{01} & a_{02} \\ a_{01} & a_{11} & a_{12} \\ a_{02} & a_{12} & a_{22} \end{vmatrix},$$

- determinant kvadratických členů

$$\det(B) = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix},$$

- třetím invariantem je

$$S = a_{11} + a_{22}.$$

Jestliže je $\det(A) \neq 0$, potom říkáme, že k je regulární kuželosečka, v opačném případě je singulární.

Klasifikaci kuželoseček provedeme podle invariantů rovnice kuželosečky. Podrobnosti jsou uvedeny v tabulce:

$\det(B) < 0$	hyperbola	
$\det(B) = 0$	parabola	
$\det(B) > 0$	$S \cdot \det(A) < 0, a_{11} = a_{22}$	kružnice
	$S \cdot \det(A) < 0, a_{11} \neq a_{22}$	elipsa

Tabulka 1: Klasifikace kuželoseček

5.		Označíme myší hodnoty $A1-C3$, klikneme pravým tlačítkem myši a vybereme Vytvořit – Matice. Přejmenujeme ji na matici A.
6.		Označíme myší hodnoty $B2-C3$, klikneme pravým tlačítkem myši a vybereme Vytvořit – Matice. Přejmenujeme ji na matici B.
7.	Vstup:	Do vstupu zapíšeme $S = a_{11} + a_{22}$.
8.	Vstup:	Vypočítáme determinanty • $\det A = \text{Determinant}[A]$ • $\det B = \text{Determinant}[B]$ a přetáhneme je do nákresny.
9.		Do nákresny vložíme text $S \cdot \det A =$, který propojíme s libovolným objektem a jeho obsah přepíšeme na $S \det A$.
10.		Do nákresny umístíme přes sebe následující texty a nastavíme u každého z nich podmínky pro zobrazení objektu: • Kružnice podmínky: $(\det B > 0) \wedge (S \det A < 0) \wedge (a_{11} = a_{22})$ • Elipsa podmínky: $(\det B > 0) \wedge (S \det A < 0) \wedge (a_{11} \neq a_{22})$ • Parabola podmínky: $\det B = 0$ • Hyperbola podmínky: $\det B < 0$

Klasifikaci pomocí invariantů jsme tímto úspěšně udělali, nicméně konstrukci kuželosečky můžeme ještě obohatit o vizuální znázornění některých jejích základních vlastností. Následující popis konstrukce ukazuje jen některé z nich a je na čtenáři, aby si další doplnil podle vlastního uvážení.

11.		Do vstupu postupně zapíšeme následující příkazy: • <code>r=Polomer[k]</code> - vypočítá velikost poloměru (u kružnice), • <code>S=Stred[k]</code> - zobrazí střed kuželosečky, • <code>Ohnisko[k]</code> - zobrazí ohniska kuželosečky, • <code>Vrchol[k]</code> - zobrazí vrchol kuželosečky, • <code>Osy[k]</code> - zobrazí obě osy kuželosečky, • <code>HlavniOsa[k]</code> - zobrazí pouze hlavní osu kuželosečky, • <code>VedlejsiOsa[k]</code> - zobrazí pouze vedlejší osu kuželosečky, • <code>Asymptota[k]</code> - zobrazí asymptoty kuželosečky, • <code>RidiciPrimka[k]</code> - zobrazí řídící přímku kuželosečky.
12.		Pro každý prvek z bodu 11., pro který existuje grafická reprezentace v nákresně, vytvoříme zaškrťovací políčko. Popisek volíme podle povahy prvku. (poloměr <code>r</code> je pouze číslo a samotnou konstrukci budeme muset udělat ručně)

Nyní provedeme konstrukci poloměru na kružnici. Nejprve nastavíme koeficienty a_{11} , a_{12} , a_{21} , a_{01} , a_{02} , a_{00} tak, abychom v nákresně dostali kružnici.

13.		Sestrojíme úsečku, klikneme na bod <code>s</code> a poté na kružnici. Dáme ji popisek poloměr.
14.		Vytvoříme zaškrťovací políčko pro poloměr.

Obdobným způsobem můžeme vytvořit například velkou a malou poloosu u elipsy.

Pozn.: V případě, že nejprve máme zaškrťovací políčko a teprve poté děláme konstrukci, nebo chceme něco k již existujícímu políčku přidat, potom u každého objektu tvořící konstrukci v záložce *Pro pokročilé* zapíšeme do *Podmínky zobrazení objektu* název daného zaškrťovacího políčka.

Je zřejmé, že ne všechny prvky z bodu 11. mají smysl pro každý typ kuželosečky, např. není důvod počítat poloměr pro elipsu, hyperbolu nebo parabolu. Proto bychom chtěli, aby se po provedení klasifikace a zobrazení textu určujícího typ kuželosečky zobrazovaly jen některé prvky.

Toho docílíme tak, že v záložce *Pro pokročilé* u každého zaškrťovacího políčka nastavíme *Podmínky zobrazení objektu*. Ty budou stejné jako podmínky pro zobrazení typu kuželosečky. Pokud by se měly zobrazovat u více typů kuželoseček, provedeme spojení podmínek pomocí logické spojky $\vee$ (nebo).

Posledním krokem bude vytvoření nápovědy ve formě textu, který bude ve stručnosti vypovídat o tom, jak aplet funguje a podle jakých kritérií se klasifikace provádí.

15.	ABC	Do nákresny vložíme Text. Zaškrtneme LaTeX vzorec a vepíšeme následující obsah: Vysvětlivky : $\det(B) < 0$ hyperbola $\det(B) = 0$ parabola $\det(B) > 0$ $(S \cdot \det(A) < 0), a_{11} = a_{22}$ kružnice $(S \cdot \det(A) < 0), a_{11} \neq a_{22}$ elipsa

Zdroj

1. JANYŠKA, Josef a Anna SEKANINOVÁ. Analytická teorie kuželoseček a kvadrik. 2. vyd. Brno: Masarykova univerzita, 2001, 178 s., ISBN 80-210-2604-9.
2. <http://www.matematika.cz/kuzelosecky>



Využití GeoGebry ve výuce matematiky a geometrie **3 μ 2014**

Výuka náhodných veličin s využitím GeoGebry

Petra Schreiberová

Katedra matematiky a deskriptivní geometrie, VŠB-TU Ostrava

Cílem lekce je ukázat si způsob, jak lze ve cvičeních využít GeoGebru pro lepší pochopení pojmu rozdělení náhodné veličiny a významu hodnot distribuční funkce. Tuto lekci lze využít jako snadné cvičení pro studenty.

První úloha

Ukážeme si způsob, jak lze využít GeoGebru k vizualizaci a určení hodnot v tabulce normovaného normálního rozdělení.

Druhá úloha

Využijeme GeoGebru k pochopení významu parametrů u normálního a binomického rozdělení.

Příklad 7: Normované normální rozdělení

Zadání: Zakreslete graf funkce hustoty a určete hodnoty distribuční funkce normovaného normálního rozdělení, které jsou dány v tabulce daného rozdělení.

Řešení:

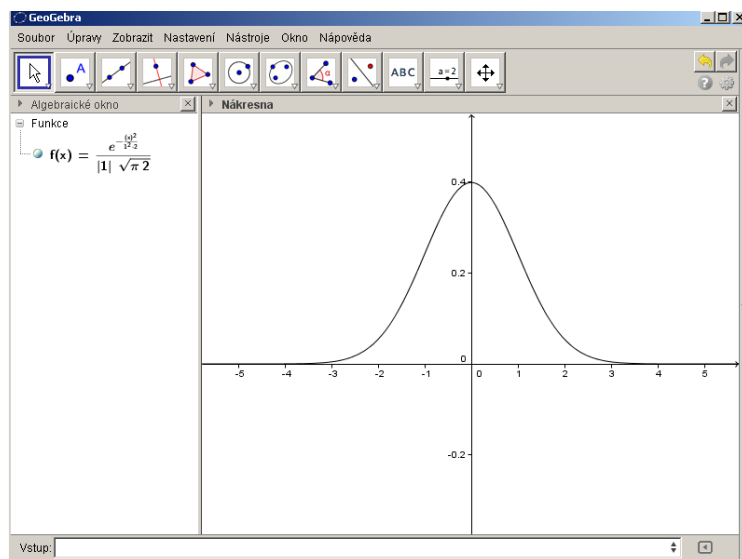
1.	Otevřeme GeoGebru
2.	Vykreslíme graf funkce hustoty normovaného normálního rozdělení. Klikneme do vstupu a zadáme příkaz.

Vstup: **NormalniRozdeleni** [<Střední Hodnota> <Směrodatná odchylka>, x]

3.	Kde střední hodnota je rovna 0, tabulátorem se posuneme na možnost směrodatné odchylky, kde zvolíme 1. Dáme Enter. Dostali jsme matematickou formuli pro výpočet funkce hustoty a graf.

Pro lepší přehlednost si upravíme měřítko os a vycentrujeme graf.

4.	Myší najedeme na osu y , stiskneme CTRL a pohybem myši upravíme měřítko.
5.	Držíme CTRL, klikneme kdekoliv do Nákresny a pohybem myši posuneme graf na střed.



Obsah plochy pod křivkou funkce hustoty je roven 1. Studenti si toto mohou snadno ověřit.

6.	Vstup: <code>Integral[f, -∞, ∞]</code>	Do vstupu zadáme příkaz Integrál.

Tabulka normovaného normálního rozdělení používá 3 desetinná místa u hodnot distribuční funkce a 2 desetinná místa u hodnot z (v tabulce x).

Tabulka 1: Distribuční funkce normovaného normálního rozdělení

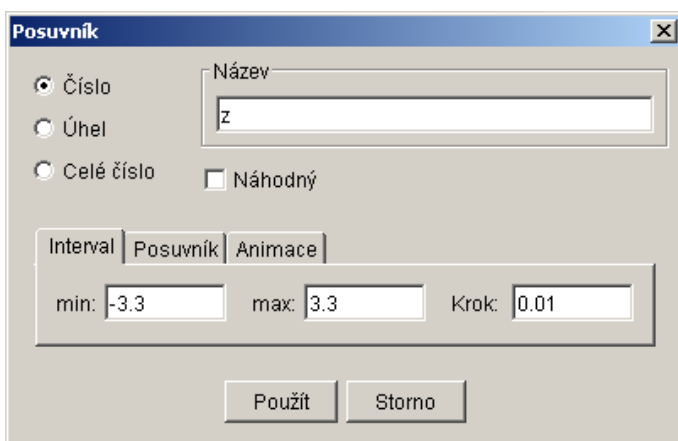
Pro $x < 0$ užijte vztahu: $\Phi(-x) = 1 - \Phi(x)$

Pro kvantily norm. normálního rozdělení platí: $x_p = -x_{1-p}$

x	0	1	2	3	4	5	6	7	8	9
0	0,5	0,504	0,508	0,512	0,516	0,520	0,524	0,528	0,532	0,536
0,1	0,540	0,544	0,548	0,552	0,556	0,560	0,564	0,567	0,571	0,575
0,2	0,579	0,583	0,587	0,591	0,595	0,599	0,603	0,606	0,610	0,614
0,3	0,618	0,622	0,626	0,629	0,633	0,637	0,641	0,644	0,648	0,652
0,4	0,655	0,659	0,663	0,666	0,670	0,674	0,677	0,681	0,684	0,688
0,5	0,691	0,695	0,698	0,702	0,705	0,709	0,712	0,716	0,719	0,722
0,6	0,726	0,729	0,732	0,736	0,739	0,742	0,745	0,749	0,752	0,755
0,7	0,758	0,761	0,764	0,767	0,770	0,773	0,776	0,779	0,782	0,785
0,8	0,788	0,791	0,794	0,797	0,800	0,802	0,805	0,808	0,811	0,813
0,9	0,816	0,819	0,821	0,824	0,826	0,829	0,831	0,834	0,836	0,839
1	0,841	0,844	0,846	0,848	0,851	0,853	0,855	0,858	0,860	0,862
1,1	0,864	0,867	0,869	0,871	0,873	0,875	0,877	0,879	0,881	0,883
1,2	0,885	0,887	0,889	0,891	0,893	0,894	0,896	0,898	0,900	0,901
1,3	0,903	0,905	0,907	0,908	0,910	0,911	0,913	0,915	0,916	0,918
1,4	0,919	0,921	0,922	0,924	0,925	0,926	0,928	0,929	0,931	0,932
1,5	0,933	0,934	0,936	0,937	0,938	0,939	0,941	0,942	0,943	0,944
1,6	0,945	0,946	0,947	0,948	0,949	0,951	0,952	0,953	0,954	0,954
1,7	0,955	0,956	0,957	0,958	0,959	0,960	0,961	0,962	0,962	0,963
1,8	0,964	0,965	0,966	0,966	0,967	0,968	0,969	0,969	0,970	0,971
1,9	0,971	0,972	0,973	0,973	0,974	0,974	0,975	0,976	0,976	0,977
2	0,977	0,978	0,978	0,979	0,979	0,980	0,980	0,981	0,981	0,982
2,1	0,982	0,983	0,983	0,983	0,984	0,984	0,985	0,985	0,985	0,986
2,2	0,986	0,986	0,987	0,987	0,987	0,988	0,988	0,988	0,988	0,989
2,3	0,989	0,990	0,990	0,990	0,990	0,991	0,991	0,991	0,991	0,992
2,4	0,992	0,992	0,992	0,992	0,993	0,993	0,993	0,993	0,993	0,994
2,5	0,994	0,994	0,994	0,994	0,994	0,995	0,995	0,995	0,995	0,995
2,6	0,995	0,995	0,996	0,996	0,996	0,996	0,996	0,996	0,996	0,996
2,7	0,997	0,997	0,997	0,997	0,997	0,997	0,997	0,997	0,997	0,997
2,8	0,997	0,998	0,998	0,998	0,998	0,998	0,998	0,998	0,998	0,998
2,9	0,998	0,998	0,998	0,998	0,998	0,998	0,998	0,999	0,999	0,999
3	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999
3,1	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999
3,2	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999
3,3	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

Proto si v GeoGebře toto upravíme.

7.		V Menu v nabídce Nastavení zvolíme Zaokrouhlování a klikneme na 3 desetinných míst pro hodnoty $\phi(z)$.
8.		Klik na ikonu posuvníku, následně na nákresnu a vytvoříme si posuvník pro hodnoty z - název z a interval zvolíme od -3.3 do 3.3 s krokem 0.01.

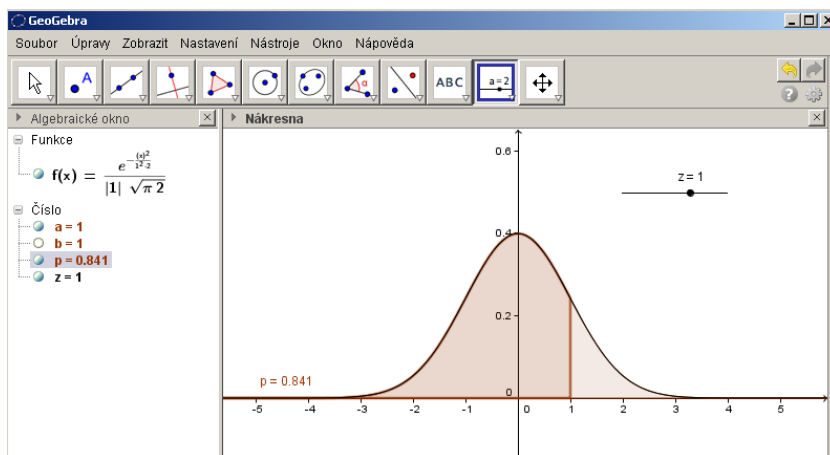


9.	Dáme použít.

Nyní můžeme začít počítat hodnoty distribuční funkce, což není nic jiného než obsah plochy do zvolené hodnoty.

10.		Do vstupu zadáme výpočet integrálu do naší zvolené hodnoty.

Vidíme, že hodnota p pro $z = 1$ vyšla 0.841. Porovnáme s tabulkou. S využitím posuvníku můžeme dynamicky měnit hodnotu na ose x a tudíž i plochu pod grafem funkce hustoty.



Pomocí posuvníku můžeme určit další hodnoty z tabulky, např. pro $z = 1.65$ nebo i přímo hodnoty distribuční funkce pro záporné hodnoty, které v tabulce nejsou, ale lze je spočítat pomocí vztahu uvedeného nahoře tabulky.

Postřehy a poznámky

Hodnoty distribuční funkce lze v GeoGebře vypočítat přímo pomocí pravděpodobnostní kalkulačky.

1.	V Menu zvolíme možnost Zobrazit Tabulka.
2.	Klikem na šipku u ikony  zvolíme poslední možnost - pravděpodobnostní kalkulačka.

Příklad 8: Normální a binomické rozdělení

Zadání: Zakreslete graf funkce hustoty normálního rozdělení a pravděpodobnostní funkce pro binomické rozdělení a posuďte vliv jednotlivých parametrů na graf.

Řešení:

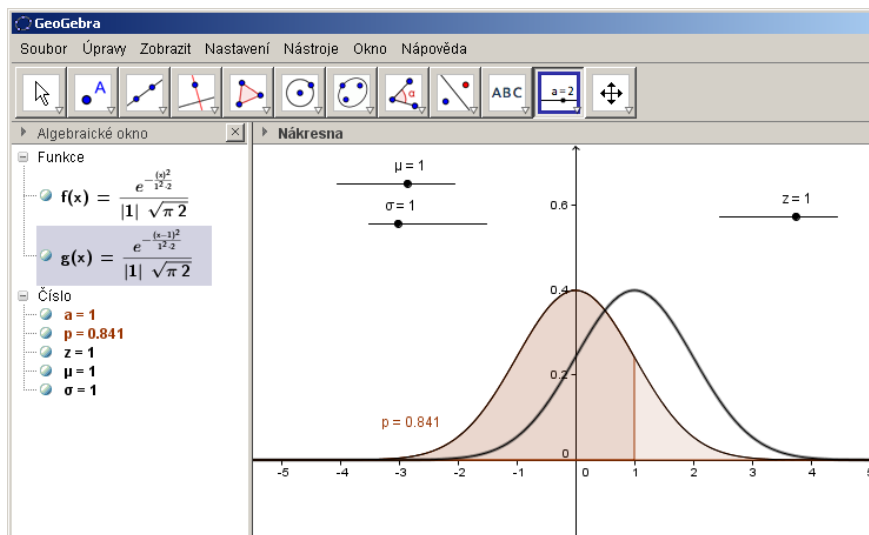
Ke znázornění grafu funkce hustoty potřebujeme vytvořit 2 posuvníky pro parametry normálního rozdělení μ a σ .

1.		Klik na ikonu posuvníku, následně na nákresnu a vytvoříme si posuvník pro hodnoty μ - název μ a interval zvolíme od -5 do 5 s krokem 0.5.
2.		Klik na nákresnu a vytvoříme si posuvník pro hodnoty σ - název σ a interval zvolíme od 0 do 4 s krokem 0.01.

Znázorníme graf funkce hustoty.

3.	Vstup: <code>NormalniRozdeleni[μ,σ, x]</code>	Do vstupu zadáme příkaz pro Normální rozdělení s danými parametry.

Vykreslil se graf a zobrazil se předpis pro funkci hustoty normálního rozdělení s parametry $\mu = 1$ a $\sigma = 1$.



Změnou parametrů pomocí posuvníku lze hned vidět, jak se mění graf funkce.

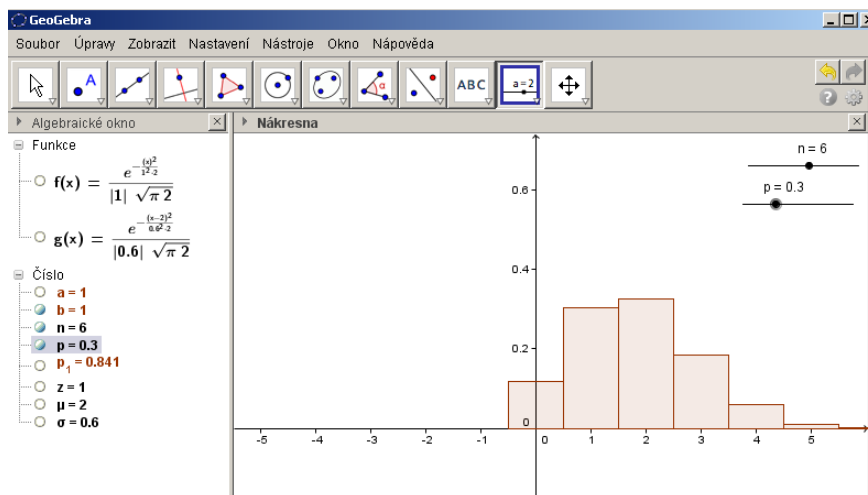
Totéž si ukážeme i pro případ diskrétního rozdělení, konkrétně binomického, které má také 2 parametry, a to n a p . Jako první si opět vytvoříme posuvníky.

4.		Klik na ikonu posuvníku, následně na nákresnu a vytvoříme si posuvník pro hodnoty n - název n a interval zvolíme od 1 do 10 s krokem 1.
5.		Klik na nákresnu a vytvoříme si posuvník pro hodnoty p - název p a interval zvolíme od 0.00001 do 1 s krokem 0.01.

Znázorníme graf pravděpodobnostní funkce.

6.	Vstup: <code>Binomické[n,p]</code>	Do vstupu zadáme příkaz pro Binomické rozdělení s danými parametry.

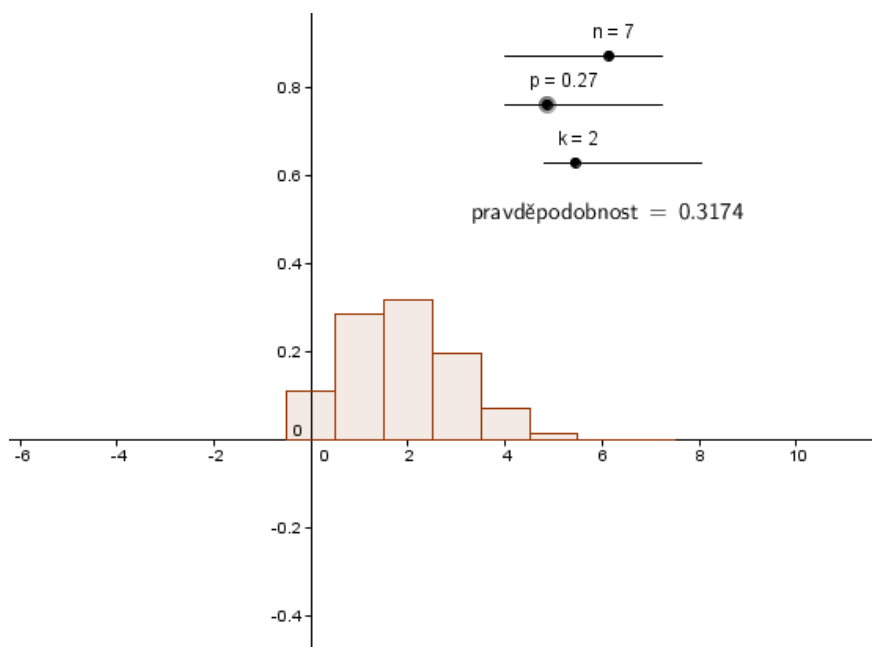
Vykreslil se graf pravděpodobnostní funkce s parametry $n = 1$ a $p = 1$. Pomocí posuvníku upravujeme parametry a opět lze hned vidět, jak se mění pravděpodobnost.



Konkrétní hodnoty pravděpodobnostní funkce si můžeme také lehce znázornit. Vytvoříme si další posuvník pro počet úspěchů.

7.		Klik na ikonu posuvníku, následně na nákresnu a vytvoříme si posuvník pro hodnoty k - název k a interval zvolíme od 0 do 10 s krokem 1.
8.	Vstup: <code>pravděpodobnost=Binomické[n,p,k,false]</code>	Do vstupu zadáme příkaz pro Binomické rozdělení s danými parametry a volbou false.

Pomocí myši přetáhneme hodnotu pravděpodobnosti na nákresnu. Opět pomocí posuvníku a změnou parametrů či počtu úspěchů lze dynamicky demonstrovat vliv na hodnotu pravděpodobnosti.



Postřehy a poznámky

Stejným způsobem lze vytvořit např. i Poissonovo rozdělení a porovnáním s Binomickým rozdělením lze znázornit, při jakých hodnotách parametrů lze Binomické rozdělení aproximovat Poissonovým.

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